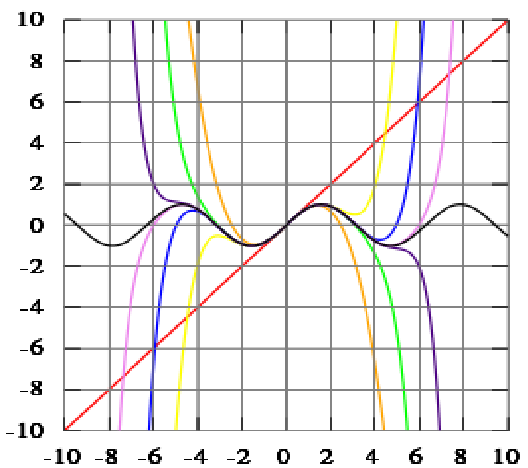
Harold's Taylor Series
Cheat Sheet
12 November 2024

Power Series	
Power Series About Zero Geometric Series if $a_n = a$	$\sum_{n=0}^{+\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$
Power Series	$\sum_{n=0}^{+\infty} a_n (x-c)^n = a_0 + a_1 (x-c) + a_2 (x-c)^2 + \cdots$

Approximation Polynomial	
	$\boldsymbol{f(x) = P_n(x) + R_n(x)}$ $P_n(x) = n^{th}$ degree polynomial approximation $R_n(x) = \pm\, Error$ NOTE: $P_n(x)$ is easy to integrate and differentiate

Maclaurin Series	
Maclaurin Series Taylor Series centered about $x = 0$	$\boldsymbol{f(x) \approx P_n(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(0)}{n!} x^n}$
Maclaurin Series Remainder	$R_n(x) = \frac{f^{(n+1)}(x^*)}{(n+1)!} x^{n+1}$ where $x \leq x^* \leq max$ and $\lim_{x \to +\infty} R_n(x) = 0$

Taylor Series	
Taylor Series Maclaurin Series if $c = 0$	$f(x) \approx P_n(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(c)}{n!} (x-c)^n$
Taylor Series Remainder	$R_n(x) = \frac{f^{(n+1)}(x^*)}{(n+1)!} (x-c)^{n+1}$ where $x \leq x^* \leq c$ and $\lim_{x \to +\infty} R_n(x) = 0$

Key Maclaurin Series	Expanded Form
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all x	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!} + \cdots$
$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$ for $\lvert x\rvert < 1$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} - \frac{x^8}{8} + \cdots$
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $\lvert x\rvert < 1$	$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + \cdots$
$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ for all x	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!} - \frac{x^{15}}{15!} + \cdots$
$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$ for all x	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!} - \frac{x^{14}}{14!} + \cdots$
$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} x^{2n+1}$ for $\lvert x\rvert < 1$	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots$ *for* $-1 < x < 1$ $\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \frac{1}{7x^7} - \frac{1}{9x^9} + \cdots$ *for* $x \geq 1$ $-\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \frac{1}{7x^7} - \frac{1}{9x^9} + \cdots$ *for* $x < 1$
$\sinh(x) = \frac{e^x - e^{-x}}{2} = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$ for all x	$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \frac{x^{11}}{11!} + \frac{x^{13}}{13!} + \frac{x^{15}}{15!} + \cdots$
$\cosh(x) = \frac{e^x + e^{-x}}{2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$ for all x	$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \frac{x^{10}}{10!} + \frac{x^{12}}{12!} + \frac{x^{14}}{14!} + \cdots$

See Harold's Infinite Series Cheat Sheet.

