# Harold’s
# Simplex Tableau (Linear Optimization)
# Cheat Sheet

18 August 2025

## How to Optimize using the Simplex Method

**Steps**

1. Read the word problem at least 4 times
2. Assign non-basic variables ($x_1$, $x_2$, …)
3. List optimization function, z = ________, that will be maximized
4. List inequalities (constraints)
5. Add basic variables, also called slack variables, ($s_1$, $s_2$, …), to turn inequalities into equations
    a. ≤ means $s_n$ is positive (default)
    b. ≥ means $s_n$ is negative (seldom used)
    c. Column has all zeros (0) except for one (1) for the slack variable
6. Organize the equation and inequalities into a matrix, with variables for the columns
7. Construct a simplex tableau corresponding to the system
    a. Rows 1-n are the inequalities
    b. Last row (**indicator row**) is the z equation solved to equal zero (0)
        i. Example: if $z = 5x_1 + 7x_2$, then $-5x_1 - 7x_2 + z = 0$, or -5 -7 1 | 0
8. If the indicator row coefficients are all positive, then the problem is solved, otherwise …
9. Find pivot
    a. Pivot Column is the most negative value in indicator row on bottom
    b. Pivot Row is the smallest positive ratio of pivot column coefficient to b value on far right
10. Pivot (perform matrix row operations) to create a new simplex tableau
    a. Example: $R_1 = R_1 - 2R_2$
    b. All values in column should be turned into zeros (0) except the pivot element (like the Identity matrix)
    c. Pivot element should be turned into one (1) using division afterwards to avoid working with fractions
    d. Column b should always be positive when maximizing
11. Repeat steps 8 - 10 until no more negatives in the indicator row on bottom
12. Maximum objective function value is in the simplex tableau’s bottom right corner

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| **Example** | Objective Function:<br>$z = x_1 + 2x_2 - x_3$<br>Subject To:<br>$2x_1 + x_2 + x_3 \le 14$<br>$4x_1 + 2x_2 + 3x_3 \le 28$<br>$2x_1 + 5x_2 + 5x_3 \le 30$<br>$x_1 \ge 0; x_2 \ge 0; x_3 \ge 0$ |
| **Simplex Tableau** | Adding slack variables gives:<br>$2x_1 + x_2 + x_3 + s_1 = 14$<br>$4x_1 + 2x_2 + 3x_3 + s_2 = 28$<br>$2x_1 + 5x_2 + 5x_3 + s_3 = 30$<br>where all variables $x_n \ge 0$ (e.g., not negative)<br><br>Simplex Tableau before Pivoting:<br>$\begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \\ - \\ R_4 \end{array} \begin{bmatrix} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \mid & b \\ 2 & 1 & 1 & 1 & 0 & 0 & 0 & \mid & 14 \\ 4 & 2 & 3 & 0 & 1 & 0 & 0 & \mid & 28 \\ 2 & 5 & 5 & 0 & 0 & 1 & 0 & \mid & 30 \\ - & - & - & - & - & - & - & - & - \\ -1 & -2 & 1 & 0 & 0 & 0 & 1 & \mid & 0 \end{bmatrix}$<br><br>Pivot Determination:<br>The -2 is the most negative on the bottom row, so pivot column is 2.<br>Ratios are row 1: 14/1 = 14, row 2: 28/2 = 14, row 3: 30/5 = 6.<br>The smallest positive ratio is 6.<br>So, the pivot is at column 2, row 3 = 5. |

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<table>
<tr>
<td>After Pivot #1</td>
<td>
Row Operations:<br>
Pivot element is Col 2, Row 3.<br>
$R_1 = 5\ R_1 - R_3$<br>
$R_2 = 5\ R_2 - 2\ R_3$<br>
$R_4 = 5\ R_4 + 2\ R_3$<br>
$R_3 = (1/5)\ R_3$<br>
<br>
Simplex Tableau after Pivot #1:<br>
$$\begin{array}{cc} & \begin{array}{cccccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & | & b \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} & \left[\begin{array}{ccccccc|c} 8 & 0 & 0 & 5 & 0 & -1 & 0 & 40 \\ 16 & 0 & 5 & 0 & 5 & -2 & 0 & 80 \\ \frac{2}{5} & 1 & 1 & 0 & 0 & \frac{1}{5} & 0 & 6 \\ \hline -1 & 0 & 15 & 0 & 0 & 2 & 5 & 60 \end{array}\right] \end{array}$$
<br>
Pivot Determination:<br>
The -1 is the most negative on the bottom row, so pivot column is 1.<br>
Ratios are row 1: 40/8 = 5, row 2: 80/16 = 5, row 3: 6/(2/5) = 15.<br>
The smallest positive ratio is 5.<br>
So, the pivot is at column 1, row 1 = 8. Row 2 also works.
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<tr>
<td>After Pivot #2</td>
<td>
Next Pivot element is Col 1, Row 2.<br>
$R_1 = 2\ R_1 - R_2$<br>
$R_3 = 16\ R_3 - (2/5)\ R_2$<br>
$R_4 = 16\ R_4 + R_2$<br>
$R_2 = (1/16)\ R_2$<br>
<br>
Final Tableau after Pivot #2:<br>
$$\begin{array}{cc} & \begin{array}{cccccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & | & b \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} & \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & \frac{5}{8} & -5 & 0 & -\frac{1}{8} & 5 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -\frac{1}{4} & 0 & \frac{1}{4} & 0 & 4 \\ \hline 0 & 0 & 3 & \frac{1}{8} & 0 & \frac{3}{8} & 0 & 13 \end{array}\right] \end{array}$$
<br>
Note: All indicators in bottom row are now zero or larger.<br>
13 is not an indicator. It is the maximum solution.
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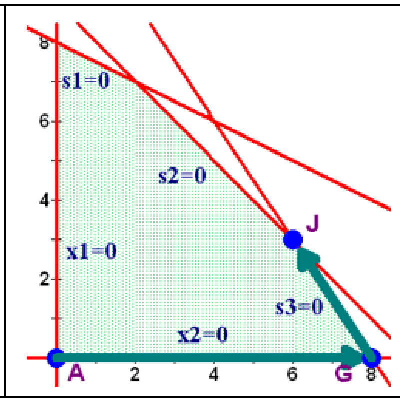
| **Basic Feasible Solution** | $x_1 = 5$ Choose 5 $x_1$s<br>$x_2 = 4$ Choose 4 $x_2$s<br>$x_3 = 0$ Choose no $x_3$s<br>$s_1 = 0$<br>$s_2 = 0$<br>$s_3 = 0$<br>$z = 13$ Objective function value of 13.<br><br>Since all slack variables $s_n \geq 0$, this solution is optimal. |  |

**Sources:**

- https://math.uww.edu/~mcfarlat/s-prob.htm
- http://simplex.tode.cz/en/#steps

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