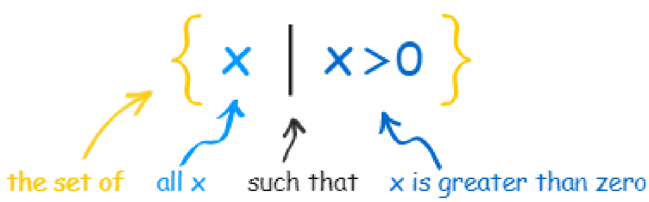
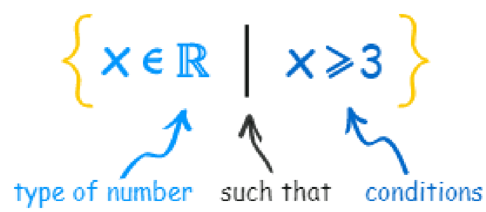
Harold's Sets
Cheat Sheet
18 August 2025

Set Definitions

Term	Definition	Examples
Set	A well-defined collection of distinct mathematical objects	C = {2, 4, 5} denotes a set of three numbers: 2, 4, and 5 D = {(2, 4), (−1, 5)} denotes a set of two ordered pairs of numbers
Element	Objects, members	a, 3, (x, y)
Pair	Ordered pair. An element with two members. Order matters.	(x, y)
Tuple	Ordered tuple. A column of three mathematical objects. Order matters.	$(a, b, c)\ or\ \begin{bmatrix} a \\ b \\ c \end{bmatrix}$
n-Tuple	Ordered n-tuple. $\mathbb{Z}^3$ is the set of all **3-tuples** whose entries are integers. Order matters.	$\mathbb{Z}^3 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a, b, c \in \mathbb{Z} \right\}$
Set-Builder Notation	Set $Uppercase\_letter = \{number\_type\ [: or\ \|] formula \wedge restrictions\ or\ conditions\}$	$F = \{n \in \mathbb{Z} : n^3 \wedge 1 \leq n \leq 100\}$ The set of cubes of the first 100 positive integers.
Roster Notation	A list of the elements enclosed in curly braces with the individual elements separated by commas	A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

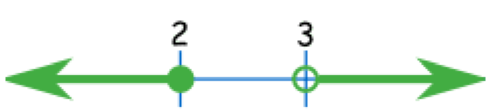
Set-Builder Notation





Set-Builder Notation: $\{ x \in \mathbb{R} \mid x \leq 2 \text{ or } x > 3 \}$

Number Line:

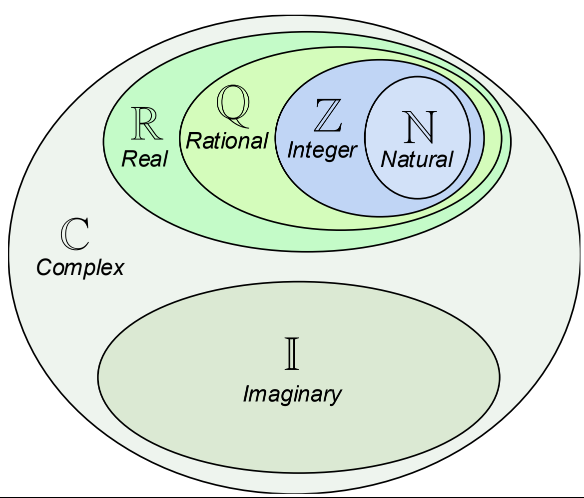


Interval Notation: $(-\infty, 2] \cup (3, +\infty)$

Number Sets

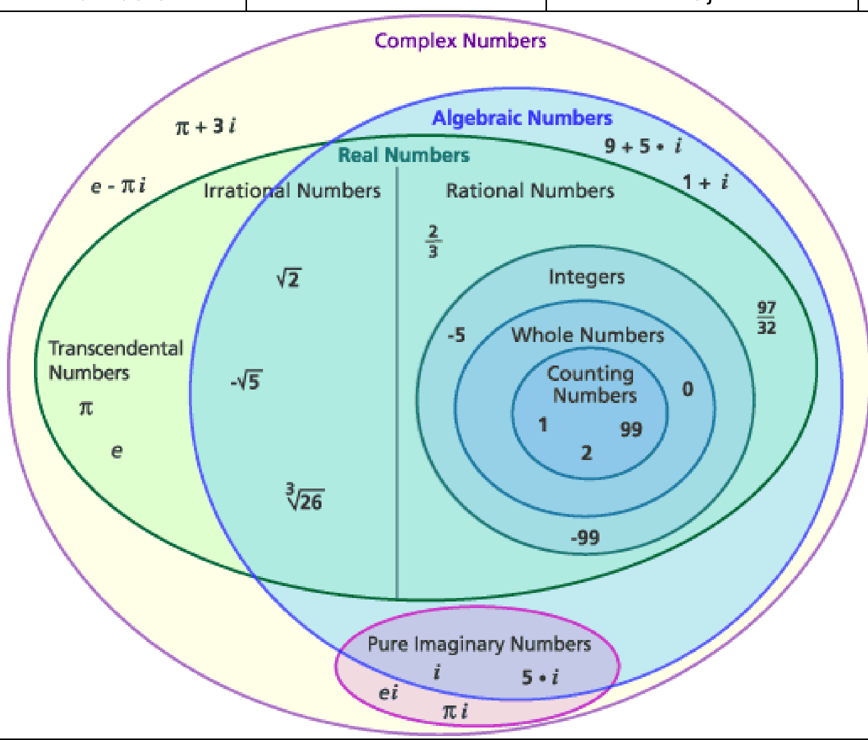
Symbol	Definition	Set Notation	Examples	Equations
∅	**Empty** or null set	{ }	∅ ∈ {∅}	$1 = 2$
ℙ	**Prime** numbers	{a, b ∈ $\mathbb{Z}^+$: (p\ab ⟶ p\a ∨ p\b)}	{2, 3, 5, 7, 11, 13, …}	$\gcd(n, m) = 1$
ℕ $\mathbb{N}_0$	**Natural** numbers	{x ∈ ℤ : x ≥ 0}	{0, 1, 2, 3, 4, …} (per ISO 80000-2 2-7.1)	$x - 3 = 0$
𝕎	**Whole numbers**	{x ∈ ℤ : x ≥ 0}	{0, 1, 2, 3, …}	n ≥ 0
ℤ	**Integers**	{x : x = ±ℕ ∨ x = 0}	{…, −3, −2, −1, 0, 1, 2, 3, …}	$x + 7 = 0$
ℚ	**Rational** numbers	{p/q : p, q ∈ ℤ ∧ q ≠ 0}	{0, ¼, ½, ¾, 1}	$4x - 1 = 0$
𝕀	**Irrational** numbers	{x ∈ ℝ : x ∉ ℚ}	{0, ¼, ½, ¾, 1}	$4x - 1 = 0$
𝔸	**Algebraic** numbers	{ x ∈ ℝ : x = root of a one-variable polynomial ∧ coefficients ∈ ℚ}	{5, -7, ½, $\sqrt{2}$}	$2x^2 + 4x - 7 = 0$
𝕋	**Transcendental** numbers	{x ∈ ℝ : x ∉ 𝔸, x ∉ ℚ}	{π, e, e^π, sin(x), $\log_b$ a}	𝕋 = 𝕌 – 𝔸
ℝ	**Real** numbers	{x : x corresponds to a number on the number line}	{ π , 3.1415, -1, ⅞, $\sqrt{2}$}	$x^2 - 2 = 0$
𝕀	**Imaginary** numbers	{b : bi where $i = \sqrt{-1}$}	{2i, $\sqrt{-1}$}	$x^2 + 1 = 0$
ℂ	**Complex** numbers	{a, b ∈ ℝ : a + bi}	{1 + 2i, -3.4i, ⅝}	$x^2 - 4x + 5 = 0$
𝕄	**Matrix**	{A ∈ $\mathbb{M}_n$ ∧ n ∈ ℕ : $n \times n$ matrix}	$\lvert \mathbb{M}_n \rvert = \lvert A \rvert \neq 0$	$\lvert A \rvert = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} = 2$
𝕌	**Universal** set	all possible values in a particular context		

∅ ⊂ ℙ ⊂ ℕ ⊂ ℤ ⊂ ℚ ⊂ 𝔸 ⊂ ℝ ⊂ ℂ ⊂ 𝕌



Special Number Sets

Symbol	Definition	Set Notation	Examples	Equations
$\mathbb{N}^*$ $\mathbb{N}_1$	**Non-zero naturals**	$\{x \in \mathbb{N} : x > 0\}$	{1, 2, 3, ...}	n = 1
$\mathbb{Z}^*$ $\mathbb{Z} - \{0\}$	**Non-zero integers**	$\{x \in \mathbb{Z} : x \neq 0\}$	{..., −3, −2, −1, 1, 2, 3, ...}	n ≠ 0
$\mathbb{Z}^+$	**Positive integers**	$\{x \in \mathbb{Z} : x > 0\}$	{1, 2, 3, ...}	n > 0
$\{0\}$	**Zero integer**	$\{x \in \mathbb{Z} : x = 0\}$	{0}	n = 0
$\mathbb{Z}^-$	**Negative integers**	$\{x \in \mathbb{Z} : x < 0\}$	{..., −3, −2, −1}	n < 0
$\mathbb{N}$	**Non-negative integers**	$\{x \in \mathbb{Z} : x \geq 0\}$	{0, 1, 2, 3, ...}	n ≥ 0
$\mathbb{Z}^- \cup \{0\}$	**Non-positive integers**	$\{x \in \mathbb{Z} : x \leq 0\}$	{..., −3, −2, −1, 0}	n ≥ 0
$\{0\}, \mathbb{R}^\times$	**Zero real**	$\{x \in \mathbb{R} : x = 0\}$	{0.0}	x = 0
$\mathbb{R}^*$ $\mathbb{R} - \{0\}$ $\mathbb{R} \setminus \{0\}$	**Non-zero real** numbers	$\{x \in \mathbb{R} : x \neq 0\}$	{-0.001, 0.002}	x ≠ 0
$\mathbb{R}^+$ $(0, \infty)$	**Positive real** numbers	$\{x \in \mathbb{R} : x > 0\}$	{0.0001, 0.0002, ...}	x > 0
$\mathbb{R}^-$ $(-\infty, 0)$	**Negative real** numbers	$\{x \in \mathbb{R} : x < 0\}$	{..., −0.0002, −0.0001}	x < 0
$[0, \infty)$	**Non-negative real** numbers	$\{x \in \mathbb{R} : x \geq 0\}$	{0, 0.0001, 0.0002, ...}	x ≥ 0
$(-\infty, 0]$	**Non-positive real** numbers	$\{x \in \mathbb{R} : x \leq 0\}$	{..., −0.0002, −0.0001, 0}	x ≤ 0



Set Laws

Law	Union Example	Intersection Example
Idempotent Laws	$A \cup A = A$	$A \cap A = A$
Associative Laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity Laws	$A \cup \emptyset = A$	$A \cap \mathbb{U} = A$
Domination Laws	$A \cup \mathbb{U} = \mathbb{U}$	$A \cap \emptyset = \emptyset$
Double Complement Law	$(A^c)^c = A$	
Complement Laws	$A \cup A^c = \mathbb{U}$	$A \cap A^c = \emptyset$
Complements of 𝕌 and ∅	$\mathbb{U}^c = \emptyset$	$\emptyset^c = \mathbb{U}$
De Morgan's Laws	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$
Absorption Laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Set Difference Law		$A \setminus B = A \cap B^c$ $A - B = A \cap B^c$

Set Properties

Property	Description	Examples
Composition	Objects may be of various types. A set may contain elements of different varieties.	A = {2, strawberry, monkey}
Order	The order in which the elements are listed is unimportant	A = { 10, 6, 4, 2 }
Duplicates	Repeating an element does not change the set	A = { 2, 2, 4, 6, 10 }
Notation	Typically, capital letters will be used as variables denoting sets, and lowercase letters will be used for elements in the set	A = {a, b}
Range	Every set A	$\emptyset \subseteq A \subseteq \mathbb{U}$
Empty Set	Set with no members.	∅ is a subset of every set.

Set Notation

Term	Definition	Examples
{ } { }	Denotes a **set**	A = {a, e, i, o, u}
| :	'**Such tha**t' or 'for which'	$B = \{x \mid x \in \mathbb{N} \text{ and } x \le 5\}$ $B = \{x : x \in \mathbb{N} \text{ and } x \le 5\}$
$\Rightarrow$ $\equiv$	Is **equivalent** or identical to	$(C \cap E) \Rightarrow (x \in C \wedge x \in E)$
|A| $n(A)$	The **cardinality** of A, the number of elements in set A	if A = {(1,2), (3,4), (5,6)}, then |A| = 3
A = B	If and only if they have precisely the **same** elements. A is **equal** to b.	if A = {4, 9} and B = $\{n^2 : n=2 \text{ or } n=3\}$, then A = B
$A \subseteq B$	If and only if every element of A is also an element of B. A is a **subset** of B.	$\{1, 8, 1107\} \subseteq \mathbb{N}$
$A \nsubseteq B$	A is not a **subset** of B. A is **not contained** in B.	$\{-1, -8, -1107\} \nsubseteq \mathbb{N}$
$A \subset B$	A is a **proper subset** of B. A is a subset of B that is not equal to B.	$\{1, 8, 1107\} \subset \mathbb{N}$
$A \not\subset B$	A is not a **proper subset** of B. A is **not contained** in B.	$\{-1, -8, -1107\} \not\subset \mathbb{N}$
$B \supseteq A$	If and only if every element of A is in B. B is a **superset** of A.	$\{1, 8, 1107\} \subseteq \mathbb{N}$
$a \in A$ $A \in B$ $a \in A$	A is a member of, an **element** of, or in A	$¾ \in \mathbb{Q}$
$a \notin A$	A is not a member of A, is **not** an **element** of A	$3.14 \notin \mathbb{Z}$
$A \cap B$ $A \cap B$ $A \cap B$	The set contains elements that are in both A and B. A∩B is the **intersection** of A and B.	if A = {1, 2} and B = {2 ,3}, then $A \cap B = \{2\}$
$A \cup B$ $A \cup B$ $A \cup B$	The set contains elements that are in either A or B or both. A∪B is the **union** of A and B.	if A = {1, 2} and B = {2, 3}, then $A \cup B = \{1, 2, 3\}$
$A \setminus B$ $A - B$	**Set difference**. The set contains elements that are in A **but not** in B. A\B is "A drop B". A–B is "A difference B".	if A = {1, 2} and B = {2, 3}, then $A \setminus B = \{1\}$
$A \oplus B$	The **symmetric difference** is the set of elements that are a member of exactly one of A and B, but not both	$A \oplus B = (A - B) \cup (B - A)$
$A \cap B = \emptyset$	A and B are **disjoint** sets. No elements in common.	$A \cap B = \emptyset$
A^k	Cartesian product of a set A with itself	$A^k = A \times A \times \ldots \times A$ k times

Logical Form of Set Notation

Set Notation	Logical Statement	Description
A	$x \in A$ $\forall x \{x \in A\}$	• Is an element of
$\neg A$	$x \notin A$ $\forall x \{x \notin A\}$	• Is not an element of
$A = B$ $A = B$	$A \leftrightarrow B$ $\forall x\, [(x \in A \longrightarrow x \in B) \wedge (x \in B \longrightarrow x \in A)]$ $A \subseteq B \wedge B \subseteq A$	• Equal • Equivalence • Iff • $\stackrel{def}{=}$
$A \neq B$ $A \neq B$	$\forall x\, (x \in A \wedge x \notin B)$	• Not equal
$A \subseteq B$	$\forall x\, (x \in A \longrightarrow x \in B)$ $\forall x \in A\, (x \in B)$ $x \notin A \setminus B$	• Subset of • $A \cap B = A \longrightarrow A \subseteq B$
$A \nsubseteq B$	$\exists x\, (x \in A \wedge x \notin B)$	• Not a subset of
$A \cap B$	$\forall x\, (x \in A \wedge x \in B)$	• Intersection
$A \cup B$	$\forall x\, (x \in A \vee x \in B)$	• Union
$A \setminus B$	$\forall x\, (x \in A \wedge x \notin B)$	• Difference • But Not
$A \oplus B$	$\forall x \{ x \in A - B \vee x \in B - A \}$	• Exactly one
$A \longrightarrow B$	$\forall x\, (x \notin A \vee x \in B)$	• If – Then
$A \cap B = \emptyset$	$\neg \exists x\, (x \in A \wedge x \in B)$ $\forall x\, \neg (x \in A \wedge x \in B)$ $\forall x\, (x \notin A \vee x \notin B)$ $\forall x\, (x \in A \longrightarrow x \notin B)$	• A and B are disjoint, having no elements in common
$\mathcal{F}$	$\{A_i \mid i \in I\}$	• Family of sets
$x \in \cap\mathcal{F}$	$\{x \mid \forall A \in \mathcal{F}\, (x \in A)\}$ $\{x \mid \forall A\, (A \in \mathcal{F} \longrightarrow x \in A)\}$	• Intersection of family of sets
$x \in \cup\mathcal{F}$	$\{x \mid \exists A \in \mathcal{F}\, (x \in A)\}$ $\{x \mid \exists A\, (A \in \mathcal{F} \wedge x \in A)\}$	• Union of a family of sets
$\cap\mathcal{F}$	$\bigcap_{i \in I} A_i = \{x \mid \forall i \in I\, (x \in A_i)\}$ $\bigcap_{i \in I} A_i = A_1 \cap A_2 \cap A_3 \cap A_4 \cap \ldots$	• Intersection of an indexed family of sets
$\cup\mathcal{F}$	$\bigcup_{i \in I} A_i = \{x \mid \exists i \in I\, (x \in A_i)\}$ $\bigcup_{i \in I} A_i = \{x \in I \mid \exists i \in I\, A\, (i, x)\}$ $\bigcap_{i \in I} A_i = A_1 \cup A_2 \cup A_3 \cup A_4 \cup \ldots$	• Union of an indexed family of sets
$x \in \wp(A)$	$x \subseteq A$ $\forall y\, (y \in x \longrightarrow y \in A)$	• Power Set • All subsets of set A, including $\emptyset$ • $\lvert P(A) \rvert = 2^{\lvert A \rvert}$

Logical Form of Numbers

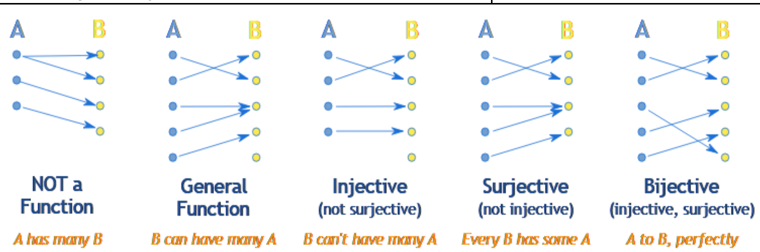
Definition	Logical Statement	Description
Even	$\exists k \in \mathbb{Z}\ (x = 2k)$ Set $E = \{2k : k \in \mathbb{Z}\}$ $2\mathbb{Z}$	• Definition of Even
Odd	$\exists k \in \mathbb{Z}\ (x = 2k + 1)$ Set $O = \{2k + 1 : k \in \mathbb{Z}\}$	• Definition of Odd
Prime	$\forall a, b \in \mathbb{Z}^{+} \mid (p \backslash ab \longrightarrow p \backslash a \lor p \backslash b)$	• A positive integer $p > 1$ that has no positive integer divisors other than 1 and p itself is prime. • Here \ means "is a divisor of"
Not Prime	$\exists a, b \in \mathbb{Z}^{+}\ (ab = n \land a < n \land b < n)$	• a and b are factors of n, so not prime
Divides	$x \mid y \longleftrightarrow \exists k \in \mathbb{Z}\ (y = kx)$	• Divisibility • Divides • Divides into • x divides y evenly • x | y to mean "x divides y," • $x \nmid y$ means "x does not divide y"
Rational	$r \in \mathbb{R}\ \exists x, y \in \mathbb{Z}\ ((y \neq 0) \land (r = x/y)) \longrightarrow r \in \mathbb{Q}$	• Definition of a Rational number • A fraction composed of two integers, but no division by 0
Irrational	$\forall x \mid x \in \mathbb{R} \land x \notin \mathbb{Q}$	• Definition of Irrational number

Logical Form of Geometry

Definition	Logical Statement	Description
Line	$\{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = mx + b\}$ $= \{(0, b), (1, m + b), (2, 2m + b), \ldots\}$	• You can think of the graph of the equation as a picture of its truth set! • $\mathbb{R}^1 = \mathbb{R}$
Plane	$\mathbb{R} \times \mathbb{R} = \{(x, y) \mid$ x and y are real numbers$\}$	• These are the coordinates of all the points in the plane • $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$
3D Space	$\mathbb{R}^3 = \{(x, y, z) \mid$ x, y and z are real numbers$\}$	• These are the coordinates of all the points in 3D space • $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$
Spacetime	$\mathbb{R}^4 = \{(x, y, z, t) \mid$ x, y, z and t are real numbers$\}$	• These are the coordinates of all the points in 3D space and 1D time • $\mathbb{R}^4 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$

Logical Form of Functions

Definition	Logical Statement	Description
Function	$f: x \longrightarrow y$ $\forall x \in X\ \exists! y \in Y\ ((x, y) \in f)$ $f = \{(a, b) \in A \times B \mid b = f(a)\}$	• General f • f is a relation frrom A to B • Example : $f = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = x^2\}$
Domain	$\mathrm{Dom}(f)$ X	• Domain of f
Range	$\mathrm{Ran}(f)$ $\{f(a) \mid a \in A\}$ Y	• Range ⊆ co-domain • Co-domain • Image of f (linear algebra term)
Injection	$f = \forall y \in Y\ \{\exists$ at **most** one $x \in X$ such that $f(x) = y\}$ $\neg \exists a_1 \in A\ \exists a_2 \in A\ (f(a_1) = f(a_2) \wedge a_1 \neq a_2)$ $\boldsymbol{\forall a_1, a_2 \in A \mid (f(a_1) = f(a_2) \longrightarrow a_1 = a_2)}$ $f(x) = f(y) \leftrightarrow x = y$ $f(x) \neq f(y) \leftrightarrow x \neq y$	• **One-to-one** • Injective f • For any y there is at most one x • Passes the horizontal line test • Can have orphan y's • *e.g., y is either married or single*
Surjection	$f = \forall y \in Y\ \{\exists$ at **least** one $x \in X$ such that $f(x) = y\}$ $\forall y \in Y, \exists x \in X \mid (f(x) = y)$ $\mathrm{Ran}(f) = Y$	• **Onto** • Surjective f • Every y is mapped to by at least one x • No orphan y's • Entire range of y is covered • *e.g., y is dating at least one x*
Bijection	$f =$ iff $\forall y \in Y\ \{\exists$ a **unique** $x \in X$ such that $f(x) = y\}$ $f(Y) = y \leftrightarrow f^{-1}(y) = Y$	• Bijective = surjective and injective • One-to-one correspondence • Bijective f • Invertible f • Iff has a well-defined inverse (f^{-1}) • Iff both surjective and injective • One-to-one and onto • *e.g., Everyone is married to a spouse*
Inverse	$f^{-1}: B \longrightarrow A$ $\forall b \in B\ \exists! a \in A\ ((b, a) \in f^{-1})$ $f(g(x)) = x$ $f^{-1} \circ f = i_A$ and $f \circ f^{-1} = i_B$	• Inverse f
k-to-1 Correspondence	*Let X and Y be finite sets. The function f:X⟶Y is a k-to-1 correspondence if, for every y ∈ Y, there are exactly k different x ∈ X such that f(x) = y.*	• Bijection is k = 1



Cartesian Product

Set Notation	Logical Statement	Description
A × B A × B A × B A ⨯ B	{(a, b) | a ∈ A ∧ b ∈ B} {(a, b) | a ∈ A, b ∈ B}	• Cartesian product • Cross product • Set of all ordered pairs in which the first entry is in A and the second entry is in B

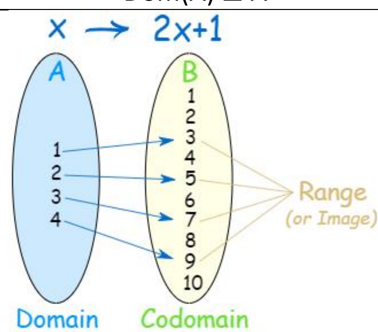
Properties of Cartesian Products

Law	Logical Statement	Operator Mnemonic
Distributive	A × (B ⋃ C) = (A × B) ⋂ (A × C)	• ×⋂
	A × (B ⋃ C) = (A × B) ⋃ (A × C)	• ×⋃
Commutative	(A × B) ⋂ (C × D) = (A ⋂ C) × (B ⋂ D)	• ×⋂×
	(A × B) ⋃ (C × D) ⊆ (A ⋃ C) × (B ⋃ D)	• ×⋃×
Domination	A × ∅ = ∅ ∅ × A = ∅	• ×∅

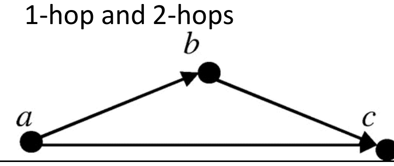
Relations

Property	i_A	Equivalence (=)	Partial Order (Poset)	Total Order (Linear)
Reflexive	✓	✓	✓	
Symmetric	✓	✓		
Anti-Symmetric			✓	✓
Asymmetric				
Transitive	✓	✓	✓	✓
Total				✓
Density				
Binary Relation		✓		

Set Relations (xRy)

Set Notation	Logical Statement	Description
Relation	R ⊆ A × B ∀x (x ∈ R ⟶ x ∈ A × B) R = {(a, b) ∈ A × B | conditions} xRy = (x, y) ∈ R Example: D_r = {(x, y) ∈ ℝ × ℝ | x and y differ by less than r} ⇒ |x − y| < r}	• Relation from A to B • R is a subset of the cross product
Domain	Dom(R) {a ∈ A | ∃b ∈ B ((a, b) ∈ R)} Dom(A) ⊆ A	• The domain of R is the set containing all the first coordinates of its ordered pairs
Codomain (Target)		• All possible values in the range set • Ran(R) is a subset of the Target • The set of <u>possible</u> output values of a function • The definition of a function
Range (Image)	Ran(R) {b ∈ B | ∃a ∈ A ((a, b) ∈ R)} Ran(B) ⊆ B	• The range of R is the set containing all the second coordinates of its ordered pairs • The <u>actual</u> or most accurate output values of a function • The image of a function
Inverse (R^{-1})	{(y, x) ∈ Y × X | (x, y) ∈ R} (y, x) ∈ R^{-1} ⟷ (x, y) ∈ R (x, y) ∈ R^{-1} ⟶ (x, y) ∈ R	• The inverse of R is the relation R^{-1} from B to A with the order of the coordinates of each pair reversed
Composition (S ∘ R)	S ∘ R = (a, c) ∈ S ∘ R ⟷ ∃b | (a, b) ∈ R and (b, c) ∈ S {(a, c) ∈ A × C | ∃b ∈ B ((a, b) ∈ R and (b, c) ∈ S)} aRb and bSc {(a, c) ∈ A × C | ∃b ∈ B (aRb ∧ bSc)}	• The composition of S and R is the relation S ∘ R from A to C • aRb and bSc, meaning R:a ⟶ R:b ⟶ S:b ⟶ S:c, so (R:a, S:c) • Ring operator
Identity (i_A)	{(x, y) ∈ A × A | x = y} {(x, x) | x ∈ A}	• Identity relation

Order Properties of Binary Relations with Two Sets

Property	Logical Statement	Description
Reflexive	xRx $(x, x) \in R$ $\forall x \in A\ (xRx)$ $\forall x \in A\ ((x, x) \in R)$	• $i_A \subseteq R$ where i_A is the identity relation of set A or $i_A = \{(x, x) \mid x \in A\}$ • **Directed graph**: Loop 
Anti-Reflexive	$\neg\ (xRx)$ $\forall x \in A\ \neg\ (xRx)$	• **Directed graph**: No loops
Symmetric	$xRy \longrightarrow yRx$ $\forall x \in A\ \forall y \in A\ (xRy \longrightarrow yRx)$	• $R = R^{-1}$ • **Directed graph**: 2-way arrow (edges come in pairs) or no arrows
Anti-Symmetric	$(xRy \land yRx) \longrightarrow (x = y)$ $(x \neq y) \longrightarrow \neg\ (xRy) \lor \neg\ (yRx)$ $\forall x \in A\ \forall y \in A\ ((xRy \land yRx) \longrightarrow (x = y))$	• Equivalence • **Directed graph**: An arrow from x to y implies that there is no arrow from y to x No: 
Asymmetric	$xRy \longrightarrow \neg\ (yRx)$ $\forall x \in A\ \forall y \in A\ \forall z \in A\ (xRy \longrightarrow \neg\ (yRx))$	• Fails the vertical line test, so not a proper function, f(x) • **Directed graph**: 1-way arrow
Transitive	$(xRy \land yRz) \longrightarrow xRz$ $\forall x\ \forall y\ \forall z\ ((xRy \land yRz) \longrightarrow xRz)$ $\forall x \in A\ \forall y \in A\ \forall z \in A\ ((xRy \land yRz) \longrightarrow xRz)$	• $R \circ R \subseteq R$ • Similar to $S \circ R$ • **Directed graph**: Two routes from every vertex A to every vertex B, 1-hop and 2-hops 
Total	$xRy \lor yRx$ $\forall x \in A\ \forall y \in A\ (xRy \lor yRx)$	• Either-or
Density	$xRy \longrightarrow \exists z \mid xRz \land zRy$ $\forall x \in A\ \forall y\ (xRy) \longrightarrow \exists z \mid xRz \land zRy$	• A middle-man exists
Binary	$R^{-1} \circ R$ = Relation on set A $R \circ R^{-1}$ = Relation on set C	• Relation on set <set> • Binary relation on set <set>
Identity	$i_A = \{(x, y) \in A \times A \mid x = y\}$ $i_A = \{(x, x) \mid x \in A\}$	• Similar to a diagonal matrix

Mathematical Number Sets → Computer Science Data Types

<table>
<tr><th>Symbol</th><th>Definition</th><th>C Data Type</th><th>C++ Data Type</th></tr>
<tr><td>∅</td><td>empty set,
set with no members</td><td colspan="2">void</td></tr>
<tr><td>ℕ</td><td>natural numbers</td><td colspan="2">enum
unsigned
unsigned char
unsigned short
unsigned int
unsigned long
unsigned long long</td></tr>
<tr><td>ℤ</td><td>integers</td><td colspan="2">char
short
int
long
long long</td></tr>
<tr><td>ℚ</td><td>rational numbers</td><td>NA</td><td>std::ratio<1, 10></td></tr>
<tr><td>ℝ</td><td>real numbers</td><td colspan="2">float
double
long double</td></tr>
<tr><td>𝕀</td><td>imaginary numbers</td><td>(see complex below)

double complex z1; im = cimag(z1);</td><td>(see complex below)

std::complex <double> z1;
im = std::imag(z1);</td></tr>
<tr><td>ℂ</td><td>complex numbers</td><td>#include <complex.h>
float complex
double complex
long double complex</td><td>#include <complex>
std::complex<float>
std::complex<double>
std::complex<long double></td></tr>
</table>

Sources:

- SNHU MAT 470 - Real Analysis, The Real Numbers and Real Analysis, Ethan D. Bloch, Springer New York, 2011.
- See also “Harold’s Logic Cheat Sheet”.
- https://www.storyofmathematics.com/set-notation
- https://math24.net/set-identities.html

