Harold’s Series Convergence Tests
Cheat Sheet

22 September 2025

1
Divergence or n*th* Term Test

Series: $\sum_{n=1}^{\infty} a_n$

<u>Condition(s) of Convergence:</u>
None. This test cannot be used to show convergence.

<u>Condition(s) of Divergence:</u>
$$\lim_{n\to\infty} a_n \neq 0$$

2
Geometric Series Test

Series: $\sum_{n=0}^{\infty} ar^n$

<u>Condition of Convergence:</u>
$$|r| < 1$$

Sum: $\mathbf{S} = \lim_{n\to\infty} \frac{a(1-r^n)}{1-r} = \frac{a}{1-r}$

<u>Condition of Divergence:</u>
$$|r| \geq 1$$

3
p - Series Test

Series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$

<u>Condition of Convergence:</u>
$$p > 1$$

<u>Condition of Divergence:</u>
$$p \leq 1$$

4
Alternating Series Test

Series: $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$

<u>Condition of Convergence:</u>
$$0 < a_{n+1} \leq a_n$$
$$\lim_{n\to\infty} a_n = 0$$
or if $\sum_{n=0}^{\infty} |a_n|$ is convergent

<u>Condition of Divergence:</u>
None. This test cannot be used to show divergence.

* Remainder: $|R_n| \leq a_{n+1}$

5
Integral Test

Series: $\sum_{n=1}^{\infty} a_n$
when $a_n = f(n) \geq 0$
and $f(n)$ is continuous, positive and decreasing

<u>Condition of Convergence:</u>
$\int_1^{\infty} f(x)dx$ converges

<u>Condition of Divergence:</u>
$\int_1^{\infty} f(x)dx$ diverges

* Remainder: $0 < R_N \leq \int_N^{\infty} f(x)dx$

6
Ratio Test

Series: $\sum_{n=1}^{\infty} a_n$

<u>Condition of Convergence:</u>
$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$$

<u>Condition of Divergence:</u>
$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$$

* Test *inconclusive* if
$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$$

7
Root Test

Series: $\sum_{n=1}^{\infty} a_n$

<u>Condition of Convergence:</u>
$$\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$$

<u>Condition of Divergence:</u>
$$\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$$

* Test *inconclusive* if
$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$$

8
Direct Comparison Test
$(a_n, b_n > 0)$

Series: $\sum_{n=1}^{\infty} a_n$

<u>Condition of Convergence:</u>
$$0 < a_n \leq b_n$$
and $\sum_{n=0}^{\infty} b_n$ is absolutely convergent

<u>Condition of Divergence:</u>
$$0 < b_n \leq a_n$$
and $\sum_{n=0}^{\infty} b_n$ diverges

9
Limit Comparison Test
$(\{a_n\}, \{b_n\} > 0)$

Series: $\sum_{n=1}^{\infty} a_n$

<u>Condition of Convergence:</u>
$$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$$
and $\sum_{n=0}^{\infty} b_n$ converges

<u>Condition of Divergence:</u>
$$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$$
and $\sum_{n=0}^{\infty} b_n$ diverges

10
Telescoping Series Test

Series: $\sum_{n=1}^{\infty} (a_{n+1} - a_n)$

<u>Condition of Convergence:</u>
$$\lim_{n\to\infty} a_n = L$$

<u>Condition of Divergence:</u> None

NOTE:
1) May need to reformat with partial fraction expansion or log rules.
2) Expand first 5 terms. n=1,2,3,4,5.
3) Cancel duplicates.
4) Determine limit L by taking the limit as $n \to \infty$.
5) Sum: $S = a_1 - L$

NOTE: These tests prove convergence and divergence, not the actual limit L or sum **S**.

Sequence: $\lim_{n\to\infty} a_n = L$
$(a_n, a_{n+1}, a_{n+2}, \ldots)$

Series: $\sum_{n=1}^{\infty} a_n = \mathbf{S}$
$(a_n + a_{n+1} + a_{n+2} + \cdots)$

Choosing a Convergence Test for Infinite Series

Courtesy David J. Manuel