**Harold’s Series Convergence Tests**

**Cheat Sheet**

22 September 2025

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| **1**  **Divergence or n*th* Term Test**  Series:  Condition(s) of Convergence:  None. This test cannot be used to show convergence.  Condition(s) of Divergence: | **2**  **Geometric Series Test**  Series:  Condition of Convergence:  Sum:  Condition of Divergence: | **3**  ***p* - Series Test**  Series:  Condition of Convergence:  Condition of Divergence: |
| **4**  **Alternating Series Test**  Series:  Condition of Convergence:  or if is convergent  Condition of Divergence:  None. This test cannot be used to show divergence.  \* Remainder: | **5**  **Integral Test**  Series:  when  and is continuous, positive and decreasing  Condition of Convergence:  converges  Condition of Divergence:  diverges  \* Remainder: | **6**  **Ratio Test**  Series:  Condition of Convergence:  Condition of Divergence:  \* Test *inconclusive* if |
| **7**  **Root Test**  Series:  Condition of Convergence:  Condition of Divergence:  \* Test *inconclusive* if | **8**  **Direct Comparison Test**  Series:  Condition of Convergence:  and is absolutely convergent  Condition of Divergence:  and diverges | **9**  **Limit Comparison Test**  Series:  Condition of Convergence:  and converges  Condition of Divergence:  and diverges |
| **10**  **Telescoping Series Test**  Series:  Condition of Convergence:  Condition of Divergence: None | **NOTE:**  1) May need to reformat with partial fraction expansion or log rules.  2) Expand first 5 terms. n=1,2,3,4,5.  3) Cancel duplicates.  4) Determine limit L by taking the limit as.  5) Sum: | **NOTE:** These tests prove convergence and divergence, not the actual limit or sum **S**.  Sequence:  ()  Series: |

**Choosing a Convergence Test for Infinite Series**

Courtesy David J. Manuel

**Do**

**the individual No terms approach 0?**

**Series Diverges by**

**the Divergence Test**

**Yes**

**Does the series alternate**

**signs?**

**No**

**Yes**

**Do individual terms have factorials or exponentials?**

**No**

**Yes**

**Use Ratio Test**

(Ratio of consecutive terms)

**Is the individual term easy to integrate?**

**No**

**Yes**

**Use Integral Test**

(May be challenging)

**Use Alternating**

**Series Test**

(Does absolute value of terms go to 0?)

**Do individual terms involve fractions with powers of n?**

**No**

**Yes**

**Use Comparison Test or Limit Comp. Test**

(Look at dominating terms)