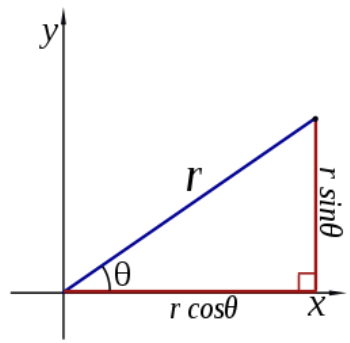
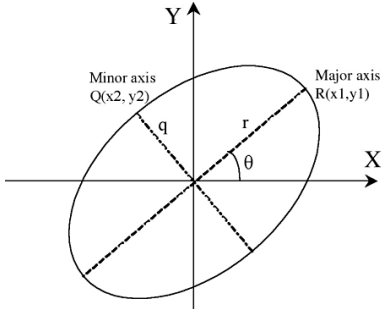
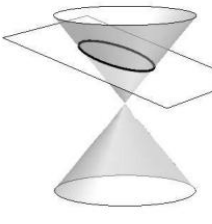
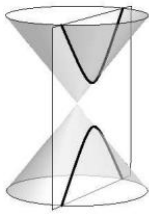
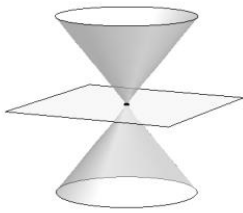
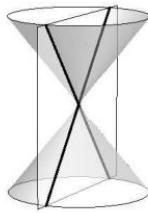


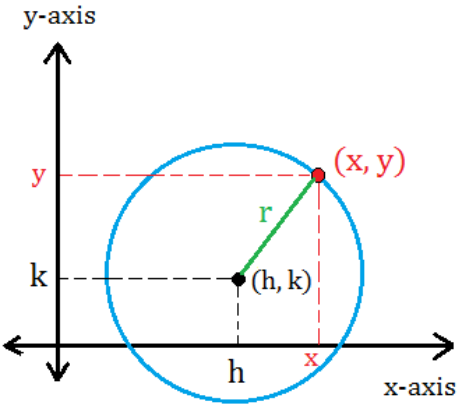
Harold's Precalculus Formulas (Rectangular – Polar – Parametric)

Cheat Sheet

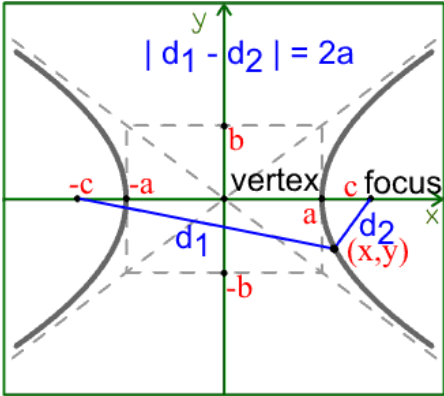
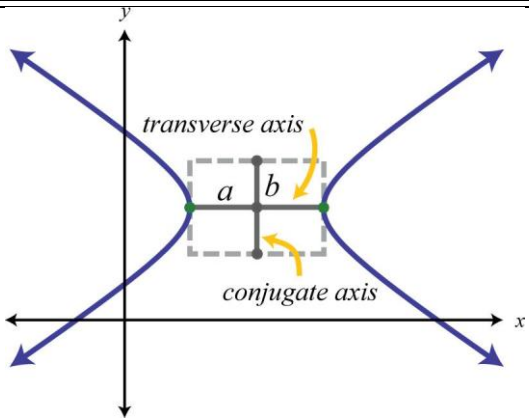
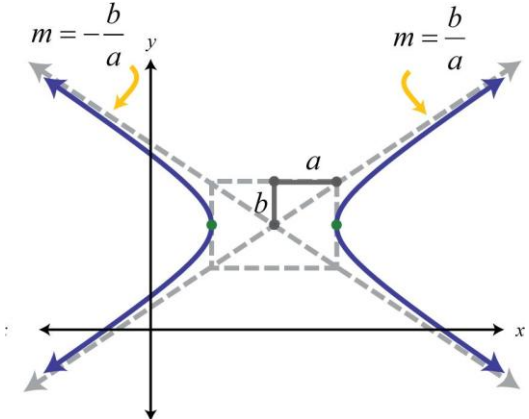
6 November 2025

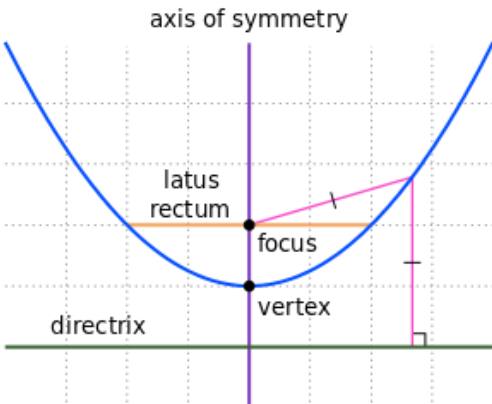
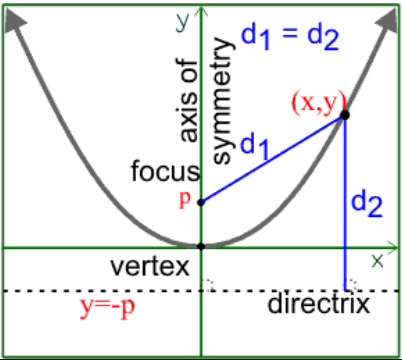
	Rectangular	Polar	Parametric
Point	$f(x) = y$ (x, y) (a, b) 	(r, θ) or $r \angle \theta$	<i>Point (a,b) in Rectangular:</i> $x(t) = a$ $y(t) = b$ $\langle a, b \rangle$ <i>t = 3rd variable, usually time,</i> <i>with 1 degree of freedom (df)</i>
		Polar $\rightarrow$ Rect. $x = r \cos \theta$ $y = r \sin \theta$ $\tan \theta = \left(\frac{y}{x}\right)$ Rect. $\rightarrow$ Polar $r^2 = x^2 + y^2$ $r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1} \left(\frac{y}{x}\right)$	
Line	<i>Slope-Intercept Form:</i> $y = mx + b$ <i>Point-Slope Form:</i> $y - y_0 = m (x - x_0)$ <i>Intercept Form:</i> $\frac{x}{a} + \frac{y}{b} = 1$ <i>Normal Form:</i> $Ax + By + C = 0$ 		$\langle x, y \rangle = \langle x_0, y_0 \rangle + t \langle a, b \rangle$ $\langle x, y \rangle = \langle x_0 + at, y_0 + bt \rangle$ <i>where</i> $\langle a, b \rangle = \langle x_2 - x_1, y_2 - y_1 \rangle$ $x(t) = x_0 + ta$ $y(t) = y_0 + tb$ $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{a}$
Plane	$n_x(x - x_0)$ $+ n_y(y - y_0)$ $+ n_z(z - z_0) = 0$	<i>Vector Form:</i> $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$	$\mathbf{r} = \mathbf{r}_0 + s\mathbf{v} + t\mathbf{w}$ $x = x_0 + su_1 + tv_1$ $y = y_0 + su_2 + tv_2$ $z = z_0 + su_3 + tv_3$

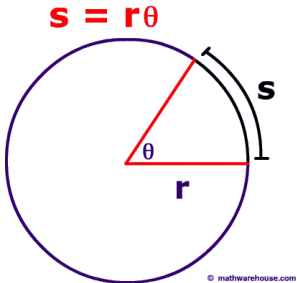
	Rectangular	Polar	Parametric
Conics	<i>General Equation for All Conics:</i> $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where Line: $A = B = C = 0$ Circle: $A = C$ and $B = 0$ Ellipse: $AC > 0$ or $B^2 - 4AC < 0$ Parabola: $AC = 0$ or $B^2 - 4AC = 0$ Hyperbola: $AC < 0$ or $B^2 - 4AC > 0$ Note: If $A + C = 0$, square hyperbola <i>Rotation:</i> If $B \neq 0$, then <u>rotate</u> the coordinate system: $\cot 2\theta = \frac{A - C}{B}$ $x = x' \cos \theta - y' \sin \theta$ $y = y' \cos \theta + x' \sin \theta$ New = (x', y'), Old = (x, y) rotates through angle θ from x-axis 	 Parabola  Ellipse  Circle  Hyperbola  Point  Line  Crossed Lines  Circle  Ellipse  Parabola  Hyperbola 	

	Rectangular	Polar	Parametric
Circle	$x^2 + y^2 = r^2$ $(x - h)^2 + (y - k)^2 = r^2$ Center: (h, k) Vertices: NA Focus: (h, k) 	Centered at Origin: $r = a$ (constant) $\theta = \theta [0, 2\pi]$ or $[0, 360^\circ]$	$x(t) = r \cos(t) + h$ $y(t) = r \sin(t) + k$ $[t_{min}, t_{max}] = [0, 2\pi)$ Center: (h, k) Focus: (h, k)

	Rectangular	Polar	Parametric
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ Center: (h, k) Vertices: $(h \pm a, k)$ Co-Vertices: $(h, k \pm b)$ Foci: $(h \pm c, k)$ Focus length, c, from center: $c^2 = a^2 - b^2$	Interesting Note: The <u>sum</u> of the distances from each focus to a point on the curve is constant. $d_1 + d_2 = k$	$x(t) = a \cos(t) + h$ $y(t) = b \sin(t) + k$ $[t_{min}, t_{max}] = [0, 2\pi]$ Center: (h, k) Rotated Ellipse: $x(t) = a \cos t \cos \theta - b \sin t \sin \theta + h$ $y(t) = a \cos t \sin \theta + b \sin t \cos \theta + k$ θ = the angle between the x-axis and the major axis of the ellipse

	Rectangular	Polar	Parametric
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ Center: (h, k) Vertices: $(h \pm a, k)$ Foci: $(h \pm c, k)$ Focus length, c, from center: $c^2 = a^2 + b^2$ 	  Interesting Note: The <u>difference</u> between the distances from each focus to a point on the curve is constant. $d_1 - d_2 = k$	<i>Left-Right Opening Hyperbola:</i> $x(t) = a \sec(t) + h$ $y(t) = b \tan(t) + k$ $[t_{\min}, t_{\max}] = [-c, c]$ (h, k) = vertex of hyperbola <i>Up-Down Opening Hyperbola:</i> $x(t) = a \tan(t) + h$ $y(t) = b \sec(t) + k$ $[t_{\min}, t_{\max}] = [-c, c]$ (h, k) = vertex of hyperbola <i>General Form:</i> $x(t) = At^2 + Bt + C$ $y(t) = Dt^2 + Et + F$ where A and D have different signs

	Rectangular	Polar	Parametric
Parabola	$y = ax^2 + bx + c$ $y = (x - h)^2 + k$ Vertical Axis of Symmetry: $x^2 = 4py$ $(x - h)^2 = 4p(y - k)$ Vertex: (h, k) Focus: $(h, k + p)$ Directrix: $y = k - p$ Horizontal Axis of Symmetry: $y^2 = 4px$ $(y - k)^2 = 4p(x - h)$ Vertex: (h, k) Focus: $(h + p, k)$ Directrix: $x = h - p$	 axis of symmetry latus rectum focus vertex directrix Vertical Axis of Symmetry: $x(t) = 2pt^2 + Et + F$ where A and D have the same sign Interesting Note: The distances from a point on the curve to the focus is the <u>same</u> as to the directrix.	Vertical Axis of Symmetry: $x(t) = 2pt + h$ $y(t) = pt^2 + k$ (opens upwards) or $y(t) = -pt^2 - k$ (opens downwards) $[t_{min}, t_{max}] = [-c, c]$ Vertex: (h, k) Horizontal Axis of Symmetry: $y(t) = 2pt + k$ $x(t) = pt^2 + h$ (opens to the right) or $x(t) = -pt^2 - h$ (opens to the left) $[t_{min}, t_{max}] = [-c, c]$ Vertex: (h, k) Projectile Motion: $x(t) = x_0 + v_x t$ $y(t) = y_0 + v_y t - 16t^2$ feet $y(t) = y_0 + v_y t - 4.9t^2$ meters $v_x = v \cos \theta$ $v_y = v \sin \theta$ General Form: $x = At^2 + Bt + C$ $y = Dt^2 + Et + F$ where A and D have the same sign
	 axis of symmetry focus vertex directrix $y = -p$ $d_1 = d_2$ (x, y) d_1 d_2		

	Rectangular	Polar	Parametric
Inverse Functions	$f(f^{-1}(x)) = f^{-1}(f(x)) = x$	if $y = \sin \theta$ then $\theta = \sin^{-1} y$ if $y = \cos \theta$ then $\theta = \cos^{-1} y$ if $y = \tan \theta$ then $\theta = \tan^{-1} y$ if $y = \csc \theta$ then $\theta = \csc^{-1} y$ if $y = \sec \theta$ then $\theta = \sec^{-1} y$ if $y = \cot \theta$ then $\theta = \cot^{-1} y$	or $\theta = \arcsin y$ or $\theta = \arccos y$ or $\theta = \arctan y$ or $\theta = \operatorname{arccsc} y$ or $\theta = \operatorname{arcsec} y$ or $\theta = \operatorname{arccot} y$
Arc Length		Circle: $L = s = r\theta$ Proof: $L = (\text{fraction of circumference}) \cdot \pi \cdot (\text{diameter})$ $L = \left(\frac{\theta}{2\pi}\right) \pi (2r) = r\theta$	
Perimeter	Square: $P = 4s$ Rectangle: $P = 2l + 2w$ Triangle: $P = a + b + c$	Circle: $C = \pi d = 2\pi r$ Ellipse: $C \approx \pi(a + b)$	
Area	Square: $A = s^2$ Rectangle: $A = lw$ Rhombus: $A = \frac{1}{2} ab$ Parallelogram: $A = Bh$ Trapezoid: $A = \frac{(B_1 + B_2)}{2} h$ Kite: $A = \frac{d_1 d_2}{2}$	Triangle: $A = \frac{1}{2} Bh$ Triangle: $A = \frac{1}{2} ab \sin(C)$ Triangle using Heron's Formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a + b + c}{2}$ Equilateral Triangle: $A = \frac{\sqrt{3}}{4} s^2$	Frustum: $A = \frac{1}{3} \left(\frac{B_1 + B_2}{2} \right) h$ Circle: $A = \pi r^2$ Circular Sector: $A = \frac{1}{2} r^2 \theta$ Ellipse: $A = \pi ab$
Lateral Surface Area	Cylinder: $SA = 2\pi rh$ Cone: $SA = \pi rl$		
Total Surface Area	Cube: $SA = 6s^2$ Rectangular Box: $SA = 2lw + 2wh + 2hl$ Regular Tetrahedron: $SA = 2bh$ Cylinder: $SA = 2\pi r(r + h)$	Cone: $SA = \pi r^2 + \pi rl = \pi r(r + l)$ Sphere: $SA = 4\pi r^2$ Ellipsoid: $SA = (\text{too complex})$	

	Rectangular	Polar	Parametric
Volume	<i>Cube:</i> $V = s^3$ <i>Rectangular Prism:</i> $V = lwh$ <i>Cylinder:</i> $V = \pi r^2 h$ <i>Triangular Prism:</i> $V = Bh$ <i>Tetrahedron:</i> $V = \frac{1}{3} Bh$	<i>Pyramid:</i> $V = \frac{1}{3} Bh$ <i>Cone:</i> $V = \frac{1}{3} bh = \frac{1}{3} \pi r^2 h$ <i>Sphere:</i> $V = \frac{4}{3} \pi r^3$ <i>Ellipsoid:</i> $V = \frac{4}{3} \pi abc$	