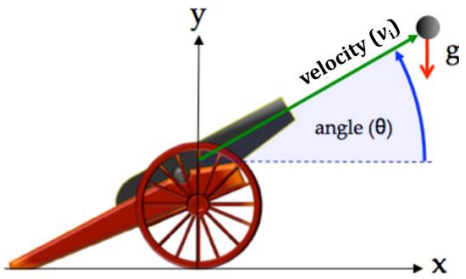
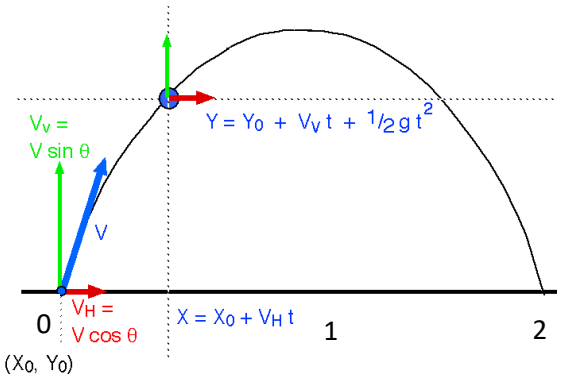
Harold's Physics of Projectiles
Cheat Sheet
26 September 2024

The Classic Cannonball Problem

Diagram





Givens

$\boldsymbol{v} = 40\ {}^{m}/_{s}$
$\theta = 30^{\circ}$ Degrees inclined from the horizontal

Unknowns

Horizontal (x-axis)	Vertical (y-axis)
❶ How far is it at time t? $(x(t))$	❷ How high is it at time t? $(y(t))$
❹ How far will it land? (x_{max})	❺ How high will it go? (y_{max})
❸ When will it land? (t_{max})	

Observations

Notes:
- Subscripts are dimensions, time, or both. Examples:
  - v_x is the velocity in the x direction.
  - x_0 is the initial horizontal position, or horizontal position at time = 0 s.
  - v_{y0} is the initial velocity in the y direction (vertical)
- Horizontal and vertical dimensions are orthogonal (independent from one another).
- Assume no wind resistance (drag). If we factor in wind resistance, then differential calculus is needed.
- The cannonball will reach its highest point exactly halfway through its journey. [t_1 and x_1]

$x_0 = 0, \quad x_1 = \frac{1}{2}x_2, \quad x_2 = x_{max}$ $v_x = v_{x0} = v_{x1} = v_{x2} = constant$ $a_x = 0$	$y_0 = 0, \quad y_1 = y_{max}, \quad y_2 = 0$ $v_{y0} = ?, \quad v_{y1} = 0, \quad v_{y2} = -v_{y0}$ $a_y = \boldsymbol{g} = -9.8\ {}^{m}/_{s^2}$
$t_0 = 0, \quad t_1 = \frac{1}{2}t_2, \quad t_2 = t_{max}$	

Equations

$\boldsymbol{x} = x_0 + v_{x0}t + \frac{1}{2}\boldsymbol{a_x}t^2$	$\boldsymbol{y} = y_0 + v_{y0}t + \frac{1}{2}\boldsymbol{a_y}t^2$
$v_x = \boldsymbol{v}\cos(\theta)$	$v_y = \boldsymbol{v}\,sin(\theta)$

We are now ready to solve for all 5 unknowns in the order 1, 2, 3, 4, 5.

First, determine the distance formulas.

	Horizontal (x-axis)	Vertical (y-axis)
Solve	$\boldsymbol{x} = \cancel{x_0 +} v_{x0}t \cancel{+ \frac{1}{2}\boldsymbol{a_x}t^2}$ $\boldsymbol{x} = v_{x0}t$	$\boldsymbol{y} = \cancel{y_0 +} v_{y0}t - \frac{1}{2}gt^2$ $\boldsymbol{y} = v_{y0}t - \frac{1}{2}gt^2$
	$x(t) = v_{x0}t = \boldsymbol{v}\cos(\theta)\, t$	$y(t) = \boldsymbol{v}\sin(\theta)\, t - \frac{1}{2}gt^2$
Substitute	$x(t) = 40\cos(30^\circ)\, t\ m$	$y(t) = 40\sin(30^\circ)\, t - 4.9\, t^2\ m$
Box Answer	❶ $\boxed{x(t) = 40\cos(30^\circ)\, t\ m}$ Distance travelled	❷ $\boxed{y(t) = 40\sin(30^\circ)\, t - 4.9\, t^2\ m}$ Height travelled

Next, determine time.

	Horizontal (x-axis)	Vertical (y-axis)
Solve		$\boldsymbol{y}(t_0) = y_0 = 0 = v_{y0}t - \frac{1}{2}gt^2$ $(t)\left(v_{y0} - \frac{1}{2}gt\right) = 0$ $t = t_0 = 0, \quad t = t_2 = \frac{2v_{y0}}{g}$ $t_{max} = t_2 = \frac{2v_{y0}}{g} = \frac{2(\boldsymbol{v}\sin(\theta))}{g}$
Substitute	$t_{max} = \frac{2(40\sin(30^\circ))}{9.8} = 4.08\ s$	
Box Answer	❸ $\boxed{t_{max} = 4.08\ s}$ Time the cannonball was in the air	

Finally, determine the farthest distances.

	Horizontal (x-axis)	Vertical (y-axis)
Solve	$x_{max} = v_{x0}\, t_{max} = \boldsymbol{v}\cos(\theta)\, t_{max}$	$y_{max} = y(t_1) = y\left(\frac{1}{2}t_{max}\right)$ $y_{max} = 40\sin(30^\circ)\left(\frac{1}{2}t_{max}\right) - 4.9\left(\frac{1}{2}t_{max}\right)^2$
Substitute	$x_{max} = 40\cos(30^\circ)\, 4.08 = 141.3\ m$	$y_{max} = 40\sin(30^\circ)\left(\frac{4.08}{2}\right) - 4.9\left(\frac{4.08}{2}\right)^2$ $= 20.41\ m$
Box Answer	❹ $\boxed{x_{max} = 141.3\ m}$ Farthest distance the cannonball travelled	❺ $\boxed{y_{max} = 20.41\ m}$ Highest distance the cannonball travelled