Harold's Infinite Products
Cheat Sheet

22 September 2025

Notation	Expanded Form
Product $\prod_{k=1}^{n} a_k$	$a_1 \times a_2 \times a_3 \times a_4 \times a_5 \times \ldots \times a_n$
Sum $\sum_{k=1}^{n} a_k$	$a_1 + a_2 + a_3 + a_4 + a_5 + \cdots + a_n$
Binary AND Operation $\bigwedge_{p \in S} p$	$p_1 \wedge p_2 \wedge \ldots \wedge p_N$
Binary OR Operation $\bigvee_{p \in S} p$	$p_1 \vee p_2 \vee \ldots \vee p_N$

Property	Expanded Form
Sum of Infinite Products	$\prod_{n=1}^{\infty} a_n + \prod_{n=1}^{\infty} b_n \neq \prod_{n=1}^{\infty} a_n + b_n$
Scalar Product of Infinite Products	$c \prod_{n=1}^{\infty} a_n \neq \prod_{n=1}^{\infty} c\, a_n$
Product of Infinite Products	$\prod_{n=1}^{\infty} a_n \times \prod_{n=1}^{\infty} b_n = \prod_{n=1}^{\infty} a_n\, b_n$
Infinite Series → Product Equivalence	$f(x) = \sum_{n=0}^{\infty} a_n x^n = \prod_{n=1}^{\infty} \left[1 - \left(\frac{x}{x_n}\right)\right]$ where $f(0) = 1$ and x_n is a root of $f(x)$.
	$ln \prod_{n=1}^{\infty} a_n = \sum_{n=0}^{\infty} ln(a_n)$
Convergence Criteria	$\prod_{n=1}^{\infty} a_n \rightarrow \sum_{n=1}^{\infty} log(\mathrm{a_n})$ The product of positive real numbers converges to a nonzero real number if and only if the sum of the logarithm of each term converges.

Canonical Product Representation	$$f(z) = z^m e^{\varphi(z)} \prod_{n=1}^{\infty} \left(1 - \frac{z}{u_n}\right)$$ This can be regarded as a generalization of the fundamental theorem of algebra, since for polynomials, the product becomes finite and $\varphi(z)$ is constant.

Infinite Products

Integers

1	**Harold's Trivial Product** $\prod_{n=1}^{\infty} 1$	$1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times \dots$
2	$\prod_{n=1}^{\infty} \frac{(1+n^{-1})^2}{1+2n^{-1}}$	$\left(\frac{4}{3}\right) \times \left(\frac{9}{8}\right) \times \left(\frac{16}{15}\right) \times \left(\frac{25}{24}\right) \times \left(\frac{36}{35}\right) \times \left(\frac{49}{48}\right) \times \left(\frac{64}{63}\right) \times \left(\frac{81}{80}\right) \times \dots$
	$\prod_{n=2}^{\infty} \left(\frac{n^2}{n^2-1}\right)$	$\left(\frac{4}{3}\right) \times \left(\frac{9}{8}\right) \times \left(\frac{16}{15}\right) \times \left(\frac{25}{24}\right) \times \left(\frac{36}{35}\right) \times \left(\frac{49}{48}\right) \times \left(\frac{64}{63}\right) \times \dots$
	Odd Primes $\prod_{n=1}^{\infty} \left(\frac{Odd\ prime \mp 1}{Odd\ prime}\right)$	$\left(\frac{2}{1}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{4}{3}\right) \times \left(\frac{6}{5}\right) \times \left(\frac{6}{7}\right) \times \left(\frac{8}{9}\right) \times \left(\frac{10}{9}\right) \times \left(\frac{12}{11}\right) \times \dots$ The numerator and denominator differ by 1, sum to the odd primes, and the numerator is even.
6	$\prod_{n=3}^{\infty} \left(\frac{n^2}{n^2-4}\right)$	$\left(\frac{9}{5}\right) \times \left(\frac{16}{12}\right) \times \left(\frac{25}{21}\right) \times \left(\frac{36}{32}\right) \times \left(\frac{49}{45}\right) \times \left(\frac{64}{60}\right) \times \left(\frac{81}{77}\right) \times \dots$

Rationals

$\frac{1}{6}$	$\prod_{n=3}^{\infty} \left(1 - \frac{4}{n^2}\right)$	$\left(1 - \frac{4}{9}\right) \times \left(1 - \frac{4}{16}\right) \times \left(1 - \frac{4}{25}\right) \times \left(1 - \frac{4}{36}\right) \times \dots$
$\frac{1}{2}$	$\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right)$	$\left(1 - \frac{1}{4}\right) \times \left(1 - \frac{1}{9}\right) \times \left(1 - \frac{1}{16}\right) \times \left(1 - \frac{1}{25}\right) \times \dots$
	$\prod_{n=1}^{\infty} \left(\frac{n+1+(-1)^n}{n+1}\right)$	$\frac{1}{2} \times \left[\left(\frac{4}{3}\right) \times \left(\frac{3}{4}\right)\right] \times \left[\left(\frac{6}{5}\right) \times \left(\frac{5}{6}\right)\right] \times \left[\left(\frac{8}{7}\right) \times \left(\frac{7}{8}\right)\right] \times \dots$ $= \frac{1}{2} \times \left[\frac{1}{1}\right] \times \left[\frac{1}{1}\right] \times \left[\frac{1}{1}\right] \times \dots$
$\frac{2}{3}$	$\prod_{n=2}^{\infty} \left(\frac{n^3-1}{n^3+1}\right) = \prod_{n=2}^{\infty} \left(\frac{(n-1)(n^2+n+1)}{(n+1)(n^2-n+1)}\right)$	$\left(\frac{1}{3} \cdot \frac{7}{3}\right) \times \left(\frac{2}{4} \cdot \frac{13}{7}\right) \times \left(\frac{3}{5} \cdot \frac{21}{13}\right) \times \left(\frac{4}{6} \cdot \frac{31}{21}\right) \times \left(\frac{5}{7} \cdot \frac{43}{31}\right) \times \dots$ $= \frac{2}{3} \times \left(\frac{1}{1}\right) \times \left(\frac{1}{1}\right) \times \left(\frac{1}{1}\right) \times \left(\frac{1}{1}\right) \times \dots$

$\frac{3}{2}$	$\prod_{n=2}^{\infty}\left(\frac{n^3+1}{n^3-1}\right)=$ $\prod_{n=2}^{\infty}\left(\frac{(n+1)(n^2-n+1)}{(n-1)(n^2+n+1)}\right)$	$\left(\frac{3}{1}\cdot\frac{3}{7}\right)\times\left(\frac{4}{2}\cdot\frac{7}{13}\right)\times\left(\frac{5}{3}\cdot\frac{13}{21}\right)\times\left(\frac{6}{4}\cdot\frac{21}{31}\right)\times\left(\frac{7}{5}\cdot\frac{31}{43}\right)\times\ldots$ $=\frac{3}{2}\times\left(\frac{1}{1}\right)\times\left(\frac{1}{1}\right)\times\left(\frac{1}{1}\right)\times\left(\frac{1}{1}\right)\times\ldots$
$\frac{2}{5}$	$\prod_{p\in prime}^{\infty}\left(\frac{p^2-1}{p^2+1}\right)$	$\left(\frac{2^2-1}{2^2+1}\right)\times\left(\frac{3^2-1}{3^2+1}\right)\times\left(\frac{5^2-1}{5^2+1}\right)\times\left(\frac{7^2-1}{7^2+1}\right)\times\ldots$
$\frac{5}{2}$	$\prod_{p\in prime}^{\infty}\left(\frac{p^2+1}{p^2-1}\right)$	$\left(\frac{2^2+1}{2^2-1}\right)\times\left(\frac{3^2+1}{3^2-1}\right)\times\left(\frac{5^2+1}{5^2-1}\right)\times\left(\frac{7^2+1}{7^2-1}\right)\times\ldots$
$\frac{6}{7}$	$\prod_{p\in prime}^{\infty}\left(\frac{p^4-1}{p^4+1}\right)$	$\left(\frac{2^4-1}{2^4+1}\right)\times\left(\frac{3^4-1}{3^4+1}\right)\times\left(\frac{5^4-1}{5^4+1}\right)\times\left(\frac{7^4-1}{7^4+1}\right)\times\ldots$
$\frac{7}{6}$	$\prod_{p\in prime}^{\infty}\left(\frac{p^4+1}{p^4-1}\right)$	$\left(\frac{2^4+1}{2^4-1}\right)\times\left(\frac{3^4+1}{3^4-1}\right)\times\left(\frac{5^4+1}{5^4-1}\right)\times\left(\frac{7^4+1}{7^4-1}\right)\times\ldots$
$\frac{715}{691}$	$\prod_{p\in prime}^{\infty}\left(\frac{p^6+1}{p^6-1}\right)$	$\left(\frac{2^6+1}{2^6-1}\right)\times\left(\frac{3^6+1}{3^6-1}\right)\times\left(\frac{5^6+1}{5^6-1}\right)\times\left(\frac{7^6+1}{7^6-1}\right)\times\ldots$
$\frac{7{,}293}{7{,}234}$	$\prod_{p\in prime}^{\infty}\left(\frac{p^8+1}{p^8-1}\right)$	$\left(\frac{2^8+1}{2^8-1}\right)\times\left(\frac{3^8+1}{3^8-1}\right)\times\left(\frac{5^8+1}{5^8-1}\right)\times\left(\frac{7^8+1}{7^8-1}\right)\times\ldots$
$\frac{524{,}875}{523{,}833}$	$\prod_{p\in prime}^{\infty}\left(\frac{p^{10}+1}{p^{10}-1}\right)$	$\left(\frac{2^{10}+1}{2^{10}-1}\right)\times\left(\frac{3^{10}+1}{3^{10}-1}\right)\times\left(\frac{5^{10}+1}{5^{10}-1}\right)\times\left(\frac{7^{10}+1}{7^{10}-1}\right)\times\ldots$

Irrationals		
$\sqrt{2}$	$\prod_{n=1}^{\infty}\frac{(4n-2)^2}{(4n-2)^2-1}$	$\left(\frac{2^2}{2^2-1}\right)\times\left(\frac{6^2}{6^2-1}\right)\times\left(\frac{10^2}{10^2-1}\right)\times\left(\frac{14^2}{14^2-1}\right)\times\ldots$
	$\prod_{n=1}^{\infty}\left(\frac{Paired\ Even\ Seq.}{Even\ Numerator\mp 1}\right)$	$\left(\frac{2}{1}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{6}{5}\right)\times\left(\frac{6}{7}\right)\times\left(\frac{10}{9}\right)\times\left(\frac{10}{11}\right)\times\left(\frac{14}{13}\right)\times\left(\frac{14}{15}\right)\times\ldots$
$\sqrt{3}$	$\prod_{n=1}^{\infty}\frac{(6n-3)^2-1}{(6n-3)^2-4}$	$\left(\frac{3^2-1}{3^2-4}\right)\times\left(\frac{9^2-1}{9^2-4}\right)\times\left(\frac{15^2-1}{15^2-4}\right)\times\left(\frac{21^2-1}{21^2-4}\right)\times\ldots$
π	**François Viète's Formula** $\prod_{n=1}^{\infty}\frac{2}{\sqrt{2+\cdots}}$	$2\times\frac{2}{\sqrt{2}}\times\frac{2}{\sqrt{2+\sqrt{2}}}\times\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}}\times\ldots$

πz	$\prod_{n=1}^{\infty}\left(\frac{n}{n-\frac{1}{2}}\right)^2\cdot\left(\frac{n-1+z}{n+z}\right)$	$\left(\frac{1}{1-\frac{1}{2}}\right)^2\cdot\left(\frac{1-1+z}{1+z}\right)\times\left(\frac{2}{2-\frac{1}{2}}\right)^2\cdot\left(\frac{2-1+z}{2+z}\right)\times\left(\frac{3}{3-\frac{1}{2}}\right)^2\cdot\left(\frac{3-1+z}{3+z}\right)\times\ldots$ $=\left(\frac{2}{1}\right)^2\cdot\left(\frac{z}{1+z}\right)\times\left(\frac{4}{3}\right)^2\cdot\left(\frac{1+z}{2+z}\right)\times\left(\frac{6}{5}\right)^2\cdot\left(\frac{2+z}{3+z}\right)\times\ldots$
$\frac{\pi}{2}$	**John Wallis' Product** $\prod_{n=1}^{\infty}\left(\frac{2n}{2n-1}\cdot\frac{2n}{2n+1}\right)$	$\left(\frac{2}{1}\cdot\frac{2}{3}\right)\times\left(\frac{4}{3}\cdot\frac{4}{5}\right)\times\left(\frac{6}{5}\cdot\frac{6}{7}\right)\times\left(\frac{8}{7}\cdot\frac{8}{9}\right)\times\left(\frac{10}{9}\cdot\frac{10}{11}\right)\times\ldots$
	$\prod_{p\,\in\,odd\ prime}^{\infty}\left(\frac{p}{p+(-1)^{\frac{(p-1)}{2}}}\right)$	$\left(\frac{3}{2}\right)\times\left(\frac{5}{6}\right)\times\left(\frac{7}{6}\right)\times\left(\frac{11}{10}\right)\times\left(\frac{13}{14}\right)\times\left(\frac{17}{18}\right)\times\left(\frac{19}{18}\right)\times\ldots$
	$\prod_{n=1}^{\infty}\left(\frac{evens}{primes}\right)^{\frac{1}{2^n}}$	$\left(\frac{2}{1}\right)^{\frac{1}{2}}\left(\frac{2^2}{1\cdot 3}\right)^{\frac{1}{4}}\left(\frac{2^3\cdot 4}{1\cdot 3^3}\right)^{\frac{1}{8}}\left(\frac{2^4\cdot 4^4}{1\cdot 3^6\cdot 5}\right)^{\frac{1}{16}}\ldots$
$\frac{2}{\pi}$	$\prod_{n=1}^{\infty}\left(1-\frac{1}{(2n)^2}\right)$	$\left(1-\frac{1}{4}\right)\times\left(1-\frac{1}{16}\right)\times\left(1-\frac{1}{36}\right)\times\left(1-\frac{1}{64}\right)\times\ldots$
	$\prod_{n=1}^{\infty}cos\left(\frac{\pi}{2^{n+1}}\right)$	$cos\left(\frac{\pi}{4}\right)\times cos\left(\frac{\pi}{8}\right)\times cos\left(\frac{\pi}{16}\right)\times cos\left(\frac{\pi}{32}\right)\times\ldots$
	François Viète's Formula $\prod_{n=1}^{\infty}\frac{\sqrt{2+\cdots}}{2}$	$\frac{\sqrt{2}}{2}\times\frac{\sqrt{2+\sqrt{2}}}{2}\times\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}\times\ldots$
$\frac{\pi}{3}$	$\prod_{n=1}^{\infty}\left(\frac{(6n)^2}{(6n)^2-1}\right)$	$\left(\frac{36}{35}\right)\times\left(\frac{144}{143}\right)\times\left(\frac{324}{323}\right)\times\left(\frac{576}{575}\right)\times\left(\frac{900}{899}\right)\times\ldots$
$\frac{3}{\pi}$	$\prod_{n=1}^{\infty}\left(1-\frac{1}{(6n)^2}\right)$	$\left(1-\frac{1}{36}\right)\times\left(1-\frac{1}{144}\right)\times\left(1-\frac{1}{324}\right)\times\left(1-\frac{1}{576}\right)\times\ldots$
$\frac{\pi}{4}$	$\prod_{n=1}^{\infty}\left(1-\frac{1}{(2n+1)^2}\right)$	$\left(1-\frac{1}{9}\right)\times\left(1-\frac{1}{25}\right)\times\left(1-\frac{1}{49}\right)\times\left(1-\frac{1}{81}\right)\times\ldots$
	$\prod_{p\,\in\,odd\ prime}^{\infty}\left(\frac{p}{p-(-1)^{\frac{(p-1)}{2}}}\right)$	$\left(\frac{3}{4}\right)\times\left(\frac{5}{4}\right)\times\left(\frac{7}{8}\right)\times\left(\frac{11}{12}\right)\times\left(\frac{13}{12}\right)\times\left(\frac{17}{16}\right)\times\left(\frac{19}{20}\right)\times\ldots$
$\frac{\pi^2}{6}$	**Leonhard Euler Series** $\sum_{n=1}^{\infty}\frac{1}{n^2}$	A summation, but the reciprocal of the value below.
	$\prod_{p\,\in\,prime}^{\infty}\left(\frac{p^2}{p^2-1}\right)$	$\left(\frac{2^2}{2^2-1}\right)\times\left(\frac{3^2}{3^2-1}\right)\times\left(\frac{5^2}{5^2-1}\right)\times\left(\frac{7^2}{7^2-1}\right)\times\ldots$
$\frac{6}{\pi^2}$	$\prod_{n=1}^{\infty}\left(1-\frac{5}{18n^2}+\frac{1}{144n^4}\right)$	Product of $\frac{2}{\pi}$ and $\frac{3}{\pi}$ above.

$\frac{\pi^2}{15}$	$\prod_{p \in prime}^{\infty}\left(\frac{p^2}{p^2+1}\right)$	$\left(\frac{2^2}{2^2+1}\right)\times\left(\frac{3^2}{3^2+1}\right)\times\left(\frac{5^2}{5^2+1}\right)\times\left(\frac{7^2}{7^2+1}\right)\times \dots$
$\frac{\pi^4}{90}$	$\prod_{p \in prime}^{\infty}\left(\frac{p^4}{p^4-1}\right)$	$\left(\frac{2^4}{2^4-1}\right)\times\left(\frac{3^4}{3^4-1}\right)\times\left(\frac{5^4}{5^4-1}\right)\times\left(\frac{7^4}{7^4-1}\right)\times \dots$
$\frac{\pi^4}{105}$	$\prod_{p \in prime}^{\infty}\left(\frac{p^4}{p^4+1}\right)$	$\left(\frac{2^4}{2^4+1}\right)\times\left(\frac{3^4}{3^4+1}\right)\times\left(\frac{5^4}{5^4+1}\right)\times\left(\frac{7^4}{7^4+1}\right)\times \dots$
$\frac{\pi}{2e}$	$\prod_{n=1}^{\infty}\left(1+\frac{2}{n}\right)^{(-1)^{n+1}n}$	$\left(1+\frac{2}{1}\right)^{1}\times\left(1+\frac{2}{2}\right)^{-2}\times\left(1+\frac{2}{3}\right)^{3}\times\left(1+\frac{2}{4}\right)^{-4}\times \dots$
	$\prod_{n=1}^{\infty}\left(\frac{e_n+1}{e_n}\right)$ $where\ e_1=1,$ $e_{n+1}=(n+1)(e_n+1)$	$\left(\frac{2}{1}\right)\times\left(\frac{4}{3}\right)\times\left(\frac{15}{14}\right)\times\left(\frac{64}{63}\right)\times\left(\frac{321}{320}\right)\times\left(\frac{1{,}921}{1{,}920}\right)\times\dots$
e	**Eugène Charles Catalan** $\prod_{n=1}^{\infty}\left(\frac{Even\ Seq.}{Odd\ Seq.}\right)^{\frac{1}{2^{n-1}}}$	$\left(\frac{2}{1}\right)^{\frac{1}{1}}\left(\frac{4}{3}\right)^{\frac{1}{2}}\left(\frac{6}{5}\cdot\frac{8}{7}\right)^{\frac{1}{4}}\left(\frac{10}{9}\cdot\frac{12}{11}\cdot\frac{14}{13}\cdot\frac{16}{15}\right)^{\frac{1}{8}}\dots$
	$\prod_{n=1}^{\infty}\left(\frac{2^n\cdot 4^{(n-2)^2}\cdot\dots}{1\cdot 3^{\frac{(n-1)n}{2}}\cdot 5^{n-3}\cdot\dots}\right)^{\frac{1}{n}}$	$\left(\frac{2}{1}\right)^{\frac{1}{1}}\left(\frac{2^2}{1\cdot 3}\right)^{\frac{1}{2}}\left(\frac{2^3\cdot 4}{1\cdot 3^3}\right)^{\frac{1}{3}}\left(\frac{2^4\cdot 4^4}{1\cdot 3^6\cdot 5}\right)^{\frac{1}{4}}\dots$
e^{γ}	$\prod_{n=1}^{\infty}\left(\frac{2^n\cdot 4^{(n-2)^2}\cdot\dots}{1\cdot 3^{\frac{(n-1)n}{2}}\cdot 5^{n-3}\cdot\dots}\right)^{\frac{1}{n+1}}$	$\left(\frac{2}{1}\right)^{\frac{1}{2}}\left(\frac{2^2}{1\cdot 3}\right)^{\frac{1}{3}}\left(\frac{2^3\cdot 4}{1\cdot 3^3}\right)^{\frac{1}{4}}\left(\frac{2^4\cdot 4^4}{1\cdot 3^6\cdot 5}\right)^{\frac{1}{5}}\dots$ $where\ \gamma\ is\ Euler's\ constant$ $\gamma \cong$ 0.57721 56649 01532 86060 65120 90082 40243 …
$e^{\frac{\pi}{2}}$	**Carl Friedrich Gauss** $\prod_{k,n=0}^{\infty}\left(1-\frac{1}{\left(k+\frac{1}{2}+i\left(n+\frac{1}{2}\right)\right)^4}\right)$	
$\frac{e}{2}$	**Nicholas Pippenger** $\prod_{n=1}^{\infty}\left(\frac{even}{odd}\right)^{\frac{1}{2^n}}$	$\left(\frac{2}{1}\right)^{\frac{1}{2}}\left(\frac{2}{3}\cdot\frac{4}{3}\right)^{\frac{1}{4}}\left(\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{6}{7}\cdot\frac{8}{7}\right)^{\frac{1}{8}}\left(\frac{8}{9}\cdot\frac{10}{9}\cdot\frac{10}{11}\cdot\frac{12}{11}\cdot\frac{12}{13}\cdot\frac{14}{13}\cdot\frac{14}{15}\cdot\frac{16}{15}\right)^{\frac{1}{16}}\dots$
$\frac{e}{4}$	$\prod_{n=1}^{\infty}\left(\frac{even}{odd}\right)^{\frac{1}{2^n}}$	$\left(\frac{2}{3}\right)^{\frac{1}{2}}\left(\frac{4}{5}\cdot\frac{6}{7}\right)^{\frac{1}{4}}\left(\frac{8}{9}\cdot\frac{10}{11}\cdot\frac{12}{13}\cdot\frac{14}{15}\right)^{\frac{1}{8}}\dots$
$\sqrt{e}$	$\prod_{n=1}^{\infty}\left(\frac{even}{odd}\right)^{\frac{1}{2^n}}$	$\frac{2}{1}\left(\frac{2}{3}\right)^{\frac{1}{2}}\left(\frac{6}{5}\cdot\frac{6}{7}\right)^{\frac{1}{4}}\left(\frac{10}{9}\cdot\frac{10}{11}\cdot\frac{14}{13}\cdot\frac{14}{15}\right)^{\frac{1}{8}}\dots$
φ	**Golden Ratio** $\prod_{n=1}^{\infty}\frac{(5n-3)(5n-2)}{(5n-4)(5n-1)}$	$\frac{1+\sqrt{5}}{2}=\left(\frac{2}{1}\cdot\frac{3}{4}\right)\times\left(\frac{7}{6}\cdot\frac{8}{9}\right)\times\left(\frac{12}{11}\cdot\frac{13}{14}\right)\times\dots$

	Infinitely Nested Radicals $\frac{1+\sqrt{5}}{2} \cong 1.61803398874 \ldots$	$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}$
$\frac{log(x)}{x-1}$	**Philipp Ludwig von Seidel** $\prod_{n=1}^{\infty}\left(\frac{2}{1+x^{\frac{1}{2^n}}}\right)$	$\frac{2}{1+\sqrt{x}} \times \frac{2}{1+\sqrt{\sqrt{x}}} \times \frac{2}{1+\sqrt{\sqrt{\sqrt{x}}}} \times \frac{2}{1+\sqrt{\sqrt{\sqrt{\sqrt{x}}}}} \times \ldots$
$log(2)$	$\prod_{n=1}^{\infty}\left(\frac{2}{1+2^{\frac{1}{2^n}}}\right)$	$\frac{2}{1+\sqrt{2}} \times \frac{2}{1+\sqrt{\sqrt{2}}} \times \frac{2}{1+\sqrt{\sqrt{\sqrt{2}}}} \times \frac{2}{1+\sqrt{\sqrt{\sqrt{\sqrt{2}}}}} \times \ldots$
$log(\sqrt{3})$	$\prod_{n=1}^{\infty}\left(\frac{2}{1+3^{\frac{1}{2^n}}}\right)$	$\frac{2}{1+\sqrt{3}} \times \frac{2}{1+\sqrt{\sqrt{3}}} \times \frac{2}{1+\sqrt{\sqrt{\sqrt{3}}}} \times \frac{2}{1+\sqrt{\sqrt{\sqrt{\sqrt{3}}}}} \times \ldots$

Algebraic		
$P(x)$	**Polynomial** $c\prod_{n=1}^{N}\left(1-\frac{x}{a_n}\right)$	$b_N x^N + \cdots + b_0 = c\left(1-\frac{x}{a_1}\right)\left(1-\frac{x}{a_2}\right)\ldots\left(1-\frac{x}{a_N}\right)$ where a_n are roots of $P(x)$.
x	$\prod_{n=1}^{\infty}\left(x^{\frac{1}{2^n}}\right)$	$\left(x^{\frac{1}{2}}\right)\times\left(x^{\frac{1}{4}}\right)\times\left(x^{\frac{1}{8}}\right)\times\left(x^{\frac{1}{16}}\right)\times\left(x^{\frac{1}{32}}\right)\times \ldots$
$1-x$	$\prod_{n=1}^{\infty}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2^n}}$	$\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}\times\left(\frac{1-x}{1+x}\right)^{\frac{1}{4}}\times\left(\frac{1-x}{1+x}\right)^{\frac{1}{8}}\times\left(\frac{1-x}{1+x}\right)^{\frac{1}{16}}\times \ldots$
$\frac{1}{1-x}$	**Simple Pole** $\prod_{n=0}^{\infty}\left(1+x^{2^n}\right)$	$(1+x)\times(1+x^2)\times(1+x^4)\times(1+x^8)\times(1+x^{16})\times \ldots$ For $\|x\| < 1$

Trigonometric		
$sin(x)$	$x\prod_{n=1}^{\infty}\left(1-\frac{x^2}{n^2\pi^2}\right)$	$x\left(1-\frac{x^2}{\pi^2}\right)\times\left(1-\frac{x^2}{4\pi^2}\right)\times\left(1-\frac{x^2}{9\pi^2}\right)\times\left(1-\frac{x^2}{16\pi^2}\right)\times \ldots$
	Canonical Product $x\prod_{n=1}^{\infty}\left(1-\frac{x}{\pi n}\right)e^{\frac{x}{\pi n}}$	
	$x\prod_{n=1}^{\infty}cos\left(\frac{\pi}{2^n}\right)$	$x \times cos\left(\frac{\pi}{2}\right)\times cos\left(\frac{\pi}{4}\right)\times cos\left(\frac{\pi}{8}\right)\times cos\left(\frac{\pi}{16}\right)\times \ldots$
$sin(\pi x)$	$\prod_{n=1}^{\infty}\left(1-\left(\frac{2x-1}{2n-1}\right)^2\right)$	$\left(1-\frac{(2x-1)^2}{1}\right)\times\left(1-\frac{(2x-1)^2}{9}\right)\times\left(1-\frac{(2x-1)^2}{25}\right)\times \ldots$

$\frac{sin(\pi x)}{\pi x}$	**Sinc Function** $\prod_{n=1}^{\infty}\left(1-\frac{x^2}{n^2}\right)$	$(1-x^2)\times\left(1-\frac{x^2}{4}\right)\times\left(1-\frac{x^2}{9}\right)\times\left(1-\frac{x^2}{16}\right)\times \ldots$
$cos(x)$	$\prod_{n=1}^{\infty}\left(1-\frac{4x^2}{\pi^2(2n-1)^2}\right)$	$\left(1-\frac{4x^2}{\pi^2}\right)\times\left(1-\frac{4x^2}{9\pi^2}\right)\times\left(1-\frac{4x^2}{25\pi^2}\right)\times\left(1-\frac{4x^2}{49\pi^2}\right)\times \ldots$
$cos(\pi x)$	$\prod_{n=1}^{\infty}\left(1-\left(\frac{2x}{2n-1}\right)^2\right)$	$\left(1-\frac{4x^2}{1}\right)\times\left(1-\frac{4x^2}{9}\right)\times\left(1-\frac{4x^2}{25}\right)\times\left(1-\frac{4x^2}{49}\right)\times \ldots$
$tan(x)$	$x\prod_{n=1}^{\infty}\left(\frac{(2n-1)^2(\pi^2n^2-x^2)}{n^2(\pi^2(2n-1)^2-4x^2)}\right)$	
$tan(\pi x)$	$\prod_{n=1}^{\infty}\left(1+\frac{4x-1}{(2n-1)^2-4x^2}\right)$	
$csc(x)$	$\frac{1}{x}\prod_{n=1}^{\infty}\left(\frac{n^2\pi^2}{n^2\pi^2-x^2}\right)$	
$sec(x)$	$\prod_{n=1}^{\infty}\left(\frac{\pi^2(2n-1)^2}{\pi^2(2n-1)^2-4x^2}\right)$	
$cot(x)$	$x\prod_{n=1}^{\infty}\left(\frac{n^2(\pi^2(2n-1)^2-4x^2)}{(2n-1)^2(\pi^2n^2-x^2)}\right)$	
$cot(\pi x)$	$\prod_{n=1}^{\infty}\left(1-\frac{4x-1}{(2n-1)^2-(2x-1)^2}\right)$	

Hyperbolic		
$sinh(x)$	$x\prod_{n=1}^{\infty}\left(1+\frac{x^2}{\pi^2n^2}\right)$	$x\left(1+\frac{x^2}{\pi^2}\right)\times\left(1+\frac{x^2}{4\pi^2}\right)\times\left(1+\frac{x^2}{9\pi^2}\right)\times\left(1+\frac{x^2}{16\pi^2}\right)\times \ldots$
$sinh(\pi)$	$\pi\prod_{n=1}^{\infty}\left(1+\frac{1}{n^2}\right)$	$\pi\left(1+\frac{1}{1}\right)\times\left(1+\frac{1}{4}\right)\times\left(1+\frac{1}{9}\right)\times\left(1+\frac{1}{16}\right)\times \ldots$
	$4\pi\prod_{n=2}^{\infty}\left(1-\frac{1}{n^4}\right)$	$4\pi\left(1-\frac{1}{16}\right)\times\left(1-\frac{1}{81}\right)\times\left(1-\frac{1}{256}\right)\times\left(1-\frac{1}{625}\right)\times \ldots$
$cosh(x)$	$\prod_{n=1}^{\infty}\left(1+\frac{4x^2}{\pi^2(2n-1)^2}\right)$	$\left(1+\frac{4x^2}{\pi^2}\right)\times\left(1+\frac{4x^2}{9\pi^2}\right)\times\left(1+\frac{4x^2}{25\pi^2}\right)\times\left(1+\frac{4x^2}{49\pi^2}\right)\times \ldots$
$cosh\left(\frac{\sqrt{3}\,\pi}{2}\right)$	$\pi\prod_{n=1}^{\infty}\left(1+\frac{1}{n^3}\right)$	$\pi\left(1+\frac{1}{1}\right)\times\left(1+\frac{1}{8}\right)\times\left(1+\frac{1}{27}\right)\times\left(1+\frac{1}{64}\right)\times \ldots$
	$3\pi\prod_{n=2}^{\infty}\left(1-\frac{1}{n^3}\right)$	$3\pi\left(1-\frac{1}{8}\right)\times\left(1-\frac{1}{27}\right)\times\left(1-\frac{1}{64}\right)\times\left(1-\frac{1}{125}\right)\times \ldots$
$\frac{1+cosh(\sqrt{3}\pi}{12\pi^2}$	$\prod_{n=2}^{\infty}\left(1-\frac{1}{n^6}\right)$	$\left(1-\frac{1}{64}\right)\times\left(1-\frac{1}{729}\right)\times\left(1-\frac{1}{4{,}096}\right)\times\left(1-\frac{1}{15{,}625}\right)\times \ldots$

$csch(\pi)$	$\frac{1}{\pi}\prod_{n=2}^{\infty}\left(\frac{n^2-1}{n^2+1}\right)$	$\frac{1}{\pi}\left(\frac{3}{5}\right)\times\left(\frac{8}{10}\right)\times\left(\frac{15}{17}\right)\times\left(\frac{24}{26}\right)\times\left(\frac{35}{37}\right)\times\left(\frac{48}{50}\right)\times\ldots$
$\frac{1+coth(x)}{2}$	$\prod_{n=1}^{\infty}\left(1+\frac{1}{e^{2^n x}}\right)$	$\left(1+\frac{1}{e^{2x}}\right)\times\left(1+\frac{1}{e^{4x}}\right)\times\left(1+\frac{1}{e^{8x}}\right)\times\left(1+\frac{1}{e^{16x}}\right)\times\ldots$

Miscellaneous		
$\binom{r}{n}$	**Binomial Coefficient – Non-Infinite** $\prod_{k=1}^{n}\frac{r-k+1}{k}$	$\left(\frac{r-1+1}{1}\right)\times\left(\frac{r-2+1}{2}\right)\times\left(\frac{r-3+1}{3}\right)\times\ldots\times\left(\frac{r-n+1}{3}\right)$ $=\frac{r(r-1)(r-2)\ldots(r-n+1)}{n!}$
$\binom{a}{b}$	**Binomial Coefficient – Infinite** $\prod_{n=1}^{\infty}\frac{(n+b)}{n}\cdot\frac{(n+a-b)}{(n+a)}$	$\frac{(1+b)}{1}\cdot\frac{(1+a-b)}{(1+a)}\times\frac{(2+b)}{2}\cdot\frac{(2+a-b)}{(2+a)}\times\ldots$
$\frac{1}{\binom{2k}{k}}$	**Reciprocal Combination – Special Case** $\prod_{n=k+1}^{\infty}\left(\frac{n^2-k^2}{n^2}\right)$	$\left(\frac{2k+1}{(k+1)^2}\right)\times\left(\frac{4k+4}{(k+2)^2}\right)\times\left(\frac{6k+9}{(k+3)^2}\right)\times\left(\frac{8k+16}{(k+4)^2}\right)\times\ldots$
$\Gamma(z)$	**Gamma Function** $\left[z\,e^{\gamma z}\prod_{n=1}^{\infty}\left(1+\frac{z}{n}\right)e^{-\frac{z}{n}}\right]^{-1} = \frac{1}{z}\prod_{n=1}^{\infty}\left(\frac{\left(1+\frac{1}{n}\right)^z}{1+\frac{z}{n}}\right)$	
$\frac{1}{\Gamma(z)}$	**Reciprocal Gamma Function** $z\,e^{\gamma z}\prod_{n=1}^{\infty}\left(1+\frac{z}{n}\right)e^{-\frac{z}{n}} = z\prod_{n=1}^{\infty}\left(\frac{1+\frac{z}{n}}{\left(1+\frac{1}{n}\right)^z}\right)$	

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See Also

- Harold's Infinite Series Cheat Sheet
- Harold's Taylor Series Cheat Sheet