**Harold’s Infinite Products**

**Cheat Sheet**

22 September 2025

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| **Notation** | **Expanded Form** |
| **Product** |  |
| **Sum** |  |
| **Binary AND Operation** |  |
| **Binary OR Operation** |  |

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| **Property** | **Expanded Form** |
| **Sum of Infinite Products** |  |
| **Scalar Product of Infinite Products** |  |
| **Product of Infinite Products** |  |
| **Infinite Series → Product Equivalence** | where and is a root of . |
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| **Convergence Criteria** | The product of positive real numbers ∏n=1∞anconverges to a nonzero real number if and only if the sum∑n=1∞log⁡(an) of the logarithm of each term converges. |
| **Canonical Product Representation** | This can be regarded as a generalization of the [fundamental theorem of algebra](https://en.wikipedia.org/wiki/Fundamental_theorem_of_algebra), since for polynomials, the product becomes finite and ϕ(z) is constant. |

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| **Infinite Products** | | |
| **Integers** | | |
|  | **Harold’s Trivial Product** |  |
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| **Odd Primes** | The numerator and denominator differ by 1, sum to the odd primes, and the numerator is even. |
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| **Rationals** | | |
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| **Irrationals** | | |
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|  | **François Viète's Formula** |  |
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|  | **John Wallis’ Product** |  |
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| **François Viète's Formula** |  |
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|  | **Leonhard Euler Series** | A summation, but the reciprocal of the value below. |
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|  |  | Product of and above. |
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| **Eugène Charles Catalan** |  |
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|  | **Carl Friedrich Gauss** | |
|  | **Nicholas Pippenger** |  |
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|  | **Golden Ratio** |  |
| **Infinitely Nested Radicals** |  |
|  | **Philipp Ludwig von Seidel** |  |
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| **Algebraic** | | |
|  | **Polynomial** | where are roots of . |
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|  | **Simple Pole** | For |

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| **Trigonometric** | | |
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| **Canonical Product** | |
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|  | **Sinc Function** |  |
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| **Hyperbolic** | | |
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| **Miscellaneous** | | |
|  | **Binomial Coefficient – Non-Infinite** |  |
|  | **Binomial Coefficient – Infinite** |  |
|  | **Reciprocal Combination – Special Case** |  |
|  | **Gamma Function** | |
|  | **Reciprocal Gamma Function** | |

**Sources**

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**See Also**

* [Harold’s Infinite Series Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Infinite_Series_Cheat_Sheet.pdf)
* [Harold’s Taylor Series Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Taylor_Series_Cheat_Sheet.pdf)