**Harold’s Rules of Circle Geometry**

**Cheat Sheet**

21 May 2023

**Terminology**

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| **Category** | **Examples** |
| **Arcs** | circular sector - Wiktionary  |
| **Angles & Sectors** | geometry - How to find x and y coordinates on a flipped circle using  javascript methods - Stack Overflow Circles - CBSE Tuts |
| **Lines & Chords** |  |
| **Tangents** | 9-5 Tangents - Geometry Mr Lee |

**Arcs and Angles in a Circle**

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| **Configuration** | **Rule / Formula** | **Diagram** |
| **Central Angle**(Angle at Center) | Equal to arc$$θ=x^{°}$$$$m∠ABC=m\hat{AC}$$ |  |
| **Inscribed Angle**(Angle in Same Segment) | Half the arc$$θ=\frac{1}{2}x^{°}$$ |  |
| **Inscribed Quadrilateral**(Opposite Angles of Cyclic Quadrilateral) | $$m∠A+m∠C=180^{°}$$$$m∠B+m∠D=180^{°}$$The opposite angles of cyclic quadrilaterals are supplementary (180°). |  |
| **Radius Ʇ Tangent** | The angle between the radius and a tangent is 90°. |  |
| **Two Chords****(Internal Angle)** | Half the sum$$θ=\frac{1}{2}\left(x^{°}+y^{°}\right)$$ |  |
| **Two Secants****(External Angle)** | Half the difference$$θ=\frac{1}{2}\left(x^{°}-y^{°}\right)$$$$m∠D=$$$$\frac{1}{2}\left(m\hat{EF}-m\hat{GH}\right)$$ |   |
| **Secant & Tangent****(External Angle)** | $$m∠Q=$$$$\frac{1}{2}\left(m\hat{RS}-m\hat{RT}\right)$$ |  |
| **Two Tangents****(External Angle)** | $$m∠L=$$$$\frac{1}{2}\left(m\hat{MPN}-m\hat{MN}\right)$$ |  |
| **Angle at Center** | $$2x^{°}vs. x^{°}$$The angle at the center is twice the angle standing on the same chord/arc. |  |
| **Angles Inscribed in a Semi-Circle** | Right Angles(90°)Angles on a semi-circle are 90°. |  |
| **Angles Inscribed in a Circle** | Angles from two points on a circle are equal. | Proving theorems about angles without angles |
| **Same Segment Theorem**(Two Inscribed Angles) | $$x^{°}=x^{°}$$$$y^{°}=y^{°}$$Angles on the same arc are equal. |  |
| **Alternate Segment Theorem** | $$x^{°}=x^{°}$$$$y^{°}=y^{°}$$The angle between a chord and a tangent is equal to the angle in the alternate segment. |  |
| **Tangent and Intersected Chord Theorem** | $$m∠1=\frac{1}{2}\left(m\hat{AC}\right) $$$$m∠2=\frac{1}{2}\left(m\hat{ADC}\right)$$If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc. |  |
| **Supplimentary Angles** | $$m∠1+m∠2=180°$$ |  |
| **Interior Angles** | $$θ=\frac{360°}{n}$$Sum of interior angles of a circle is always 360°. |  |

**Chords and Secants in a Circle**

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| **Configuration** | **Rule / Formula** | **Diagram** |
| **Perpendicular Bisector of Chord Passes Through Center** | The line from the center of a circle to the center of a chord is perpendicular to the chord. A perpendicular line from the chord to the center bisects the chord. |  |
| **Equal Chords Equidistant from Center** | Equal chords are equal distance from the center. Chords that are equal distance from the center are equal. |  |
| **Equal Arcs, Equal Chords** | Equal arc/chord subtend equal angles at the center. Equal angles stand on an equal arc/chord. |  |
| **Tangents from External Point** | Tangent segments drawn from an external point are equal. | Circle Theorem: Tangents from the Same Point Are the Same Length (Key Stage  3) |
| **Intersecting Chords Theorem** | $$a∙b=c∙d$$ |  |
| **Intersecting Secants Theorem** | $$a∙\left(a+b\right)=$$$$c∙(c+d)$$ |  |
| **Intersecting Secant-Tangent Theorem** | $$a(a+b)=c^{2}$$ |  |

**Area and Perimeter**

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| **Configuration** | **Rule / Formula** | **Diagram** |
| **Radius** | $$r$$The distance from the center or origin to a point on the circle. |  |
| **Diameter** | $$d=2r$$ |
| **Circumference** | $$C=2πr$$$$C=πd$$ |
| **Area of Circle** | $$A=πr^{2}$$ |  |
| **Area of a Sector** | $$A=\left(\frac{θ^{°}}{360^{°}}\right)∙πr^{2}$$$$where \left(\frac{θ^{°}}{360^{°}}\right)=\frac{area of sector}{area of circle}$$ |  |
| **Surface Area of Sphere** | $$SA=4πr^{2}$$ | Solid Geometry on SAT Math: The Complete Guide |

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| **Volume of Sphere** | $$V=\frac{4}{3}πr^{3}$$ |  |

**Sources**:

Kevin’s Online Maths, Rules of Circle Geometry

<http://kelvinsonlinemaths.blogspot.com/2011/03/rules-of-circle-geometry.html>

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<https://www.pinterest.com.mx/pin/817403401103649163/>

Online Math Learning.com, Angles and Intercepted Arcs

<https://www.onlinemathlearning.com/arc-angles.html>

ck-12, 9.7 Segments of Secants and Tangents

[https://www.ck12.org/book/ck-12-foundation-and-leadership-public-schools-college-access-reader%3a-geometry/section/9.7/](https://www.ck12.org/book/ck-12-foundation-and-leadership-public-schools-college-access-reader%3A-geometry/section/9.7/)

ck-12, Angles Outside a Circle

<https://www.ck12.org/c/geometry/angles-outside-a-circle/lesson/Angles-Outside-a-Circle-BSC-GEOM/>