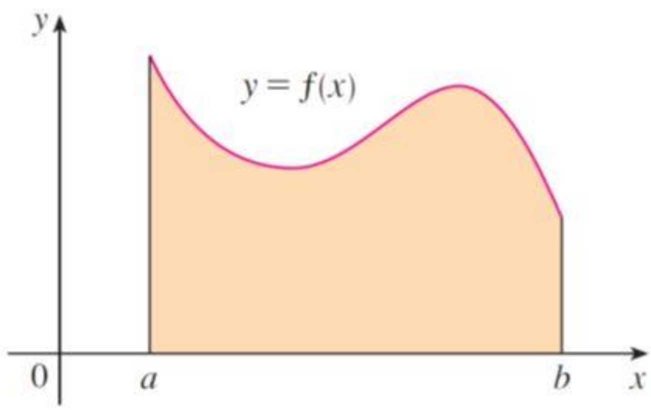
Harold's Fundamental Theorem of Calculus
Cheat Sheet
29 March 2024

The First Fundamental Theorem of Calculus: Integrating Derivatives

Formula	Example
Upper Bound Minus Lower Bound Formula $$\int_a^b f(x)\,dx = \int_a^b F'(x)\,dx = F(x)\big\vert_a^b = F(b) - F(a)$$ 	**Solve:** $$\int_0^4 e^x\,dx$$ 1st FToC Formula $$\int_a^b f(x)\,dx = F(b) - F(a)$$ Identify functions $$f(x) = F'(x) = e^x$$ $$F(x) = e^x$$ Plug into formula $$F(b) - F(a) = e^4 - e^0$$ $$= \boldsymbol{e^4 - 1}$$

The Second Fundamental Theorem of Calculus: Differentiating Integrals

Formula	Example
Simple Formula $$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = F'(x) = f(x)$$ General Formula $$\frac{d}{dx}\left[\int_{g(x)}^{h(x)} f(t)\,dt\right] = f(h(x))\,h'(x) - f(g(x))\,g'(x)$$	**Solve:** $$\frac{d}{dx}\int_{4x}^{x^2} e^t\,dt$$ 2[nd] FToC General Formula $$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))\,h'(x) - f(g(x))\,g'(x)$$ Identify functions $$f(t) = e^t$$ $$g(x) = 4x$$ $$h(x) = x^2$$ Substitute $$f(g(x)) = e^{4x}$$ $$f(h(x)) = e^{x^2}$$ Differentiate $$g'(x) = \frac{d}{dx}4x = 4$$ $$h'(x) = \frac{d}{dx}x^2 = 2x$$ Plug into formula $$= f(h(x))\,h'(x) - f(g(x))\,g'(x)$$ $$= e^{x^2}(2x) - e^{4x}(4)$$ $$= \mathbf{2xe^{x^2} - 4e^{4x}}$$
Proof: a) Apply the First Fundamantal Theorem of Calculus $$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = \frac{d}{dx}[F(t)]_{g(x)}^{h(x)}$$ $$= \frac{d}{dx}[F(h(x)) - F(g(x))]$$ b) Distribute $$= \frac{d}{dx}F(h(x)) - \frac{d}{dx}F(g(x))$$ c) Chain rule $$= F'(h(x))\,\frac{d}{dx}h(x) - F'(g(x))\,\frac{d}{dx}g(x)$$ d) Simplify $$= F'(h(x))\,h'(x) - F'(g(x))\,g'(x)$$ e) Substitute $$= f(h(x))\,h'(x) - f(g(x))\,g'(x)$$	