**Harold’s Counting**

**Cheat Sheet**

12 February 2025

(See also Harold’s Sets Cheat Sheet)

**Counting Rules**

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| **Rule** | **Description** | **Comments** |
| **Cardinality** | The cardinality of A is the number of elements in set A = | * if A = {(1,2), (3,4), (5,6)}, then |A| = 3 * Also denoted * Cardinality = Counting |
| **Product Rule** | Let be finite sets. Then, | * Counts sequences * Think Intersection (∩) |
| **Sum Rule** | Consider n sets, . If the sets are mutually disjoint (Ai ∩ Aj = ∅ for i ≠ j), then | * Counts sequences * Think Union (∪) |
| **Generalized Product Rule** | *n! = (n)(n-1)(n-2) … (2)(1)* | * In selecting an item from a set, if the number of choices at each step is independent, then the number of items in the set is the product of the number of choices in each step. |
| **Bijection Rule** | Let S and T be two finite sets.  If there is a bijection from S to T, then | * 1-to-1 Correspondence |
| **k-to-1 Rule** |  | * k-to-1 Correspondence |

**Counting Formulas & Techniques**

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| **Rule** | **Description** | **Comments** |
| **Factorial** |  | * The number of permutations of a finite set with n elements is P(n,n) |
| **Permutation** |  | * Order matters ( , ) * r-permutation * Counting sequences * Common application of the generalized product rule * Order can be fixed but arbitrary * Elements cannot be repeated * Use when elements are all different |
| **Grouping** | Password length is 18.  No character repeats.  Must contain: a, z, 1, and 9. | * Combines permutation with product rule |
| **Combination** |  | * Order does not matter { , } * Counting subsets * r-combination * n choose r * Counting the r-subsets * Combination = subset * Use when elements are all identical |
| Identity: | * An equation is called an **identity** if the equation holds for all values for which the expressions in the equation are well defined. |
| **Counting Subsets** | Bijection from:  5-bit strings with exactly 2 1's  To:  2-subsets of { 1, 2, 3, 4, 5 } = | * Binary example * Counting Strings by Counting Subsets |
|  | * Counting paths on a grid {N, E} * Binary example if m = #1s & n = #0s |

**Counting with Discrete Probability**

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| **Rule** | **Formula** | **Definition** |
| **Notation** | = Intersection or “”and”  = Union or “or”  Negation or “not” | * “and” implies multiplication. * “or” implies addition. * “not” implies negation. |
| **Independent** | If | * The occurrence of one event does not affect the probability of the other event, or vice versa. |
| **Mutually Independent** | No sets overlap.  Future outcomes are not impacted by previous outcomes. | * Applies to more than two events |
| **Dependent** | If | * The occurrence of one event affects the probability of the other event. |
| **Disjoint**  (“mutually exclusive”) | If , then | * The events can never occur together. |
| **Probability**  (“likelihood”) |  | * *S* = **S**ample space or entire set * *A, B, E* = **E**vent or subset |
| **Addition Rule**  (“or”) |  | **Inclusion-Exclusion Principle**   * Let A, B and C be three finite sets, then … * If sets overlap, then don’t double count * ”… in any of the 3.” * ”… divisible by 2, 3,or 5.” |
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| if mutually independent / disjoint: | * A collection of sets is **mutually disjoint** if the intersection of every pair of sets in the collection is empty. * Restatement of the Sum Rule |
| **Multiplication Rule**  (“and”) | if independent / disjoint:  if mutually independent / disjoint: |  |
| **Complement Rule / Subtraction Rule**  (“not”) |  | * S = entire set, E = subset * The complement of event *E* (denoted means “**not *E***”; * It consists of all simple outcomes that are not in *E*. * ”has at least one” so choose as “none” |
| **Union by Compliment** |  | * S = U = Universal set (all) * E.g., 104 - 94 |
| **Conditional Probability**  (“given that”) | if independent / disjoint: | * Means the probability of event A given that event B has already occurred. * Is a rephrasing of the Multiplication Rule. * P(A|B) is the proportion of elements in B that are ALSO in A. |
| **Total Probability Rule** |  | * To find the probability of event A, partition the sample space into several disjoint events. * A must occur along with one and only one of the disjoint events. |
| **Bayes’ Theorem** |  | * Allows P(A|B) to be calculated from P(B|A). * Meaning it allows us to reverse the order of a conditional probability statement, and is the only generally valid method! |

**Sources**:

* [SNHU MAT 230](https://www.snhu.edu/admission/academic-catalogs/coce-catalog#/courses/4kVhSZLtg) - Discrete Mathematics, zyBooks.