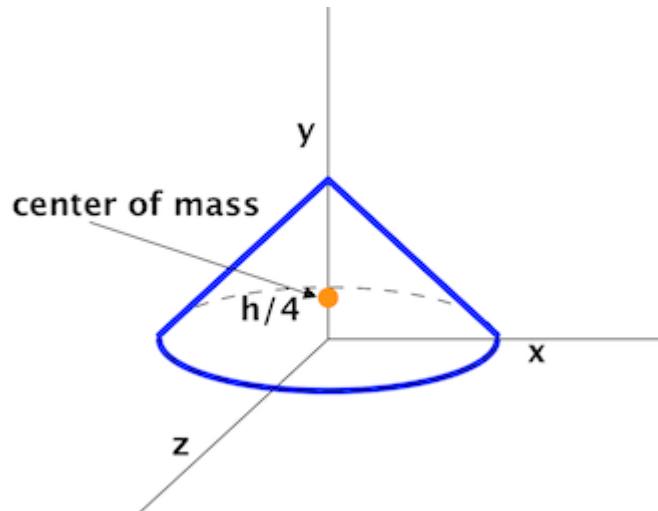
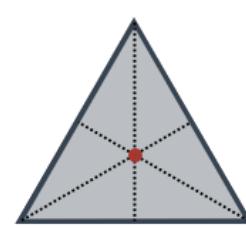
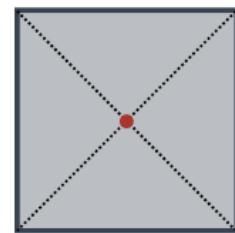
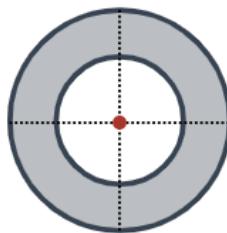
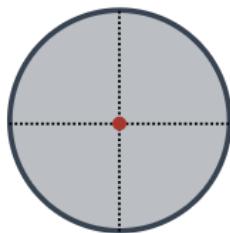


# Harold's Center of Mass Cheat Sheet

13 August 2020

## Terms

Term	Description	Units
$\bar{x}$	Center of Mass along the x-axis. Same as $x_{cm}$ .	$m$ or $ft$
$\bar{y}$	Center of Mass along the y-axis.	$m$ or $ft$
$\bar{z}$	Center of Mass along the z-axis.	$m$ or $ft$
$\bar{x}$ $(\bar{x}, \bar{y})$ $(\bar{x}, \bar{y}, \bar{z})$	Center of Mass. The coordinates (point) where the object is perfectly balanced. (Centroid)	$m$ or $ft$
$M$	Total mass. How heavy the object is. Is equal to it's area or volume if uniform density. (similar to Weight)	$kg$ or $lb$
$M_x$ and $M_y$	Moment. The line or axis on which the object can spin perfectly balanced.	$kg$ or $lb$
$M_x$	Moment of Inertia about the x-axis	$kg$ or $lb$
$M_y$	Moment of Inertia about the y-axis	$kg$ or $lb$
$I$	Moment of Inertia, mass moment of Inertia, or rotational inertia of a body. $I_0$ = polar moment of inertia (about origin)	$kg\ m^2$ or $lb\ ft^2$
$\rho$	Greek symbol rho for density or mass / volume or mass / area.	$\rho = \frac{kg}{m^3}$ or $\frac{lb}{ft^3}$
$A$	Area of lamina or plate	$m^2$ or $ft^2$
$V$	Volume of a body or solid	$m^3$ or $ft^3$



## Discrete

Term	1-D	2-D	3-D
$\bar{x}$	$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$	$\bar{x} = \frac{M_y}{M}$	$\bar{x} = \frac{1}{M} \sum_{i=1}^N m_i x_i$
$\bar{y}$	NA	$\bar{y} = \frac{M_x}{M}$	$\bar{y} = \frac{1}{M} \sum_{i=1}^N m_i y_i$
$\bar{z}$	NA	NA	$\bar{z} = \frac{1}{M} \sum_{i=1}^N m_i z_i$
$M$		$M = \sum_{i=1}^N m_i$	
$M_x$		$M_x = \sum_{i=1}^N m_i y_i$	
$M_y$		$M_y = \sum_{i=1}^N m_i x_i$	
$I$	$I = m r^2 = \frac{L}{\omega}$	$I = \sum_{i=1}^N m_i r_i^2$	NA
$I_x$	$I_x = \sum_{i=1}^j x_i^2 \frac{M}{L} \Delta x$	$I_x = \sum_{i=1}^k \sum_{j=1}^l y^2 \rho(x, y) \Delta A$	$I_x = \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n (y^2 + z^2) \rho(x, y, z) \Delta V$
$I_y$	NA	$I_y = \sum_{i=1}^k \sum_{j=1}^l x^2 \rho(x, y) \Delta A$	$I_y = \text{Above with } (x^2 + z^2)$
$I_z$	NA	NA	$I_{z_y} = \text{Above with } (x^2 + y^2)$
$I_0$	$I_0 = I_x$	$I_0 = I_x + I_y$	$I_0 = I_x + I_y + I_z$
<p>The diagram illustrates the calculation of the center of mass for two particles. Two red spheres represent masses <math>m_1</math> and <math>m_2</math>, located at positions <math>x_1</math> and <math>x_2</math> along a horizontal axis. A vertical dashed line represents the center of mass, which is marked with a grey triangle and labeled <math>x_{cm}</math>. The formula for the center of mass is shown below the diagram:</p> $\text{For two masses: } x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$			

## Continuous

Term	2-D	3-D
$\bar{x}$	$\bar{x} = \frac{M_y}{M}$	$\bar{x} = \frac{1}{M} \int_0^M x \ dm = \frac{M_{yz}}{M}$
$\bar{y}$	$\bar{y} = \frac{M_x}{M}$	$\bar{y} = \frac{1}{M} \int_0^M y \ dm = \frac{M_{xz}}{M}$
$\bar{z}$	NA	$\bar{z} = \frac{1}{M} \int_0^M z \ dm = \frac{M_{xy}}{M}$ $\bar{z} = \frac{1}{V} \int_{z_{min}}^{z_{max}} z \ dV$
$M$	$M = \rho \text{ (Area)}$ $M = \rho \int_a^b f(x) \ dx$ $M = \iint_R dm = \iint_R \rho(x, y) \ dA$ $dm = \rho(x, y) \ dy \ dx$	$M = \rho \text{ (Volume)}$ $M = \iiint_Q dm = \iiint_Q \rho(x, y, z) \ dV$ $dm = \rho(x, y, z) \ dz \ dy \ dx$
$M_x$	$M_x = \rho \iint_R y \ dA = \rho \int_a^b \frac{1}{2} ([f(x)]^2) \ dx$	$M_{xy} = \iiint_Q z \rho(x, y, z) \ dV$
$M_y$	$M_y = \rho \iint_R x \ dA = \rho \int_a^b x f(x) \ dx$	$M_{yz} = \iiint_Q x \rho(x, y, z) \ dV$
$M_z$	NA	$M_{xz} = \iiint_Q y \rho(x, y, z) \ dV$
$I$	$I = \int_0^a m r^2 \ dr$ $I = \iint_A \rho(\mathbf{r}) \ d(\mathbf{r})^2 \ dA(\mathbf{r})$	$I = \iiint_V \rho(\mathbf{r}) \ d(\mathbf{r})^2 \ dV(\mathbf{r})$
$I_x$	$I_x = \iint_R y^2 \rho(x, y) \ dA$	$I_x = \iiint_Q (y^2 + z^2) \rho(x, y, z) \ dV$
$I_y$	$I_y = \iint_R x^2 \rho(x, y) \ dA$	$I_y = \iiint_Q (x^2 + z^2) \rho(x, y, z) \ dV$
$I_z$	NA	$I_z = \iiint_Q (x^2 + y^2) \rho(x, y, z) \ dV$

$$x_{cm} = \frac{\int_0^L x \frac{M}{L} dx}{M} = \frac{1}{L} \frac{x^2}{2} \Big|_{x=0}^{x=L} = \frac{L}{2}$$

