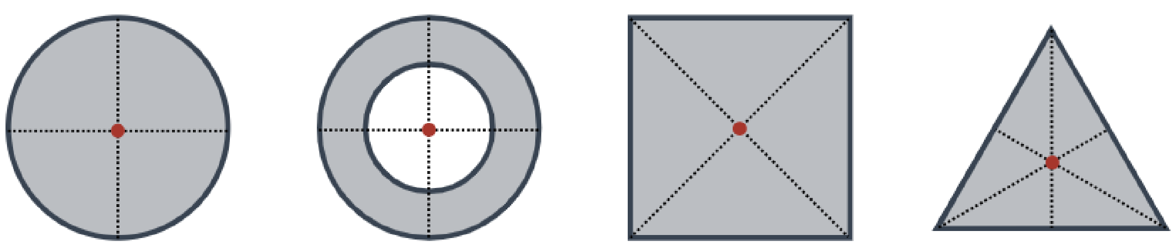
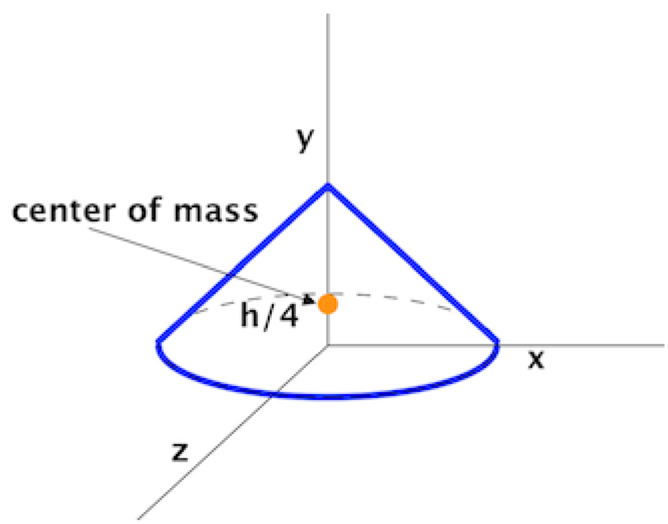
# Harold's Center of Mass
# Cheat Sheet
13 August 2020

## Terms

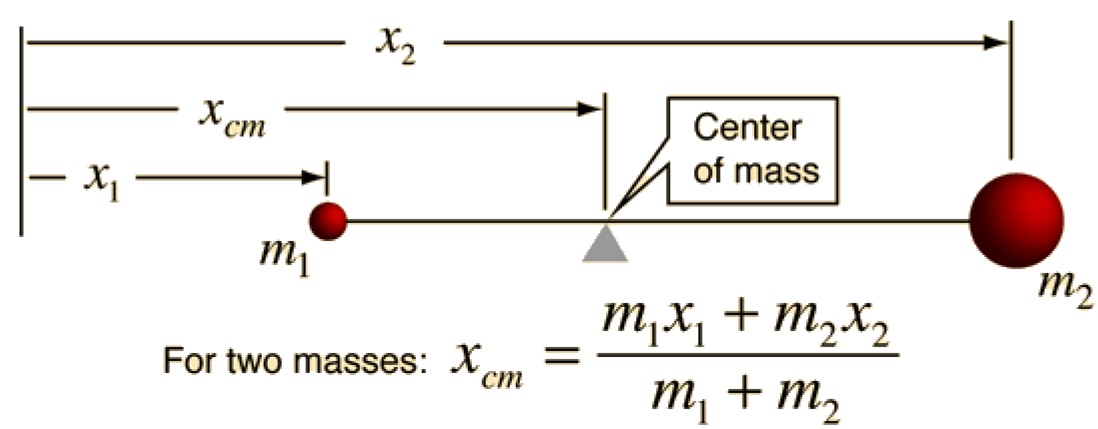
| Term | Description | Units |
|---|---|---|
| $\bar{x}$ | Center of Mass along the x-axis. Same as $x_{cm}$. | $m$ or $ft$ |
| $\bar{y}$ | Center of Mass along the y-axis. | $m$ or $ft$ |
| $\bar{z}$ | Center of Mass along the z-axis. | $m$ or $ft$ |
| $\bar{x}$<br>$(\bar{x}, \bar{y})$<br>$(\bar{x}, \bar{y}, \bar{z})$ | Center of Mass. The coordinates (point) where the object is perfectly balanced. (Centroid) | $m$ or $ft$ |
| $M$ | Total mass. How heavy the object is. Is equal to it's area or volume if uniform density. (similar to Weight) | $kg$ or $lb$ |
| $M_x$ and $M_y$ | Moment. The line or axis on which the object can spin perfectly balanced. | $kg$ or $lb$ |
| $M_x$ | Moment of Inertia about the x-axis | $kg$ or $lb$ |
| $M_y$ | Moment of Inertia about the y-axis | $kg$ or $lb$ |
| $I$ | Moment of Inertia, mass moment of Inertia, or rotational inertia of a body. $I_0$ = polar moment of inertia (about origin) | $kg\ m^2$ or $lb\ ft^2$ |
| $\rho$ | Greek symbol rho for density or mass / volume or mass / area. | $\rho = \frac{kg}{m^3}$ or $\frac{lb}{ft^3}$ |
| $A$ | Area of lamina or plate | $m^2$ or $ft^2$ |
| $V$ | Volume of a body or solid | $m^3$ or $ft^3$ |





## Discrete

| Term | 1-D | 2-D | 3-D |
| --- | --- | --- | --- |
| $\bar{x}$ | $\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$ | $\bar{x} = \frac{M_y}{M}$ | $\bar{x} = \frac{1}{M} \sum_{i=1}^{N} m_i\, x_i$ |
| $\bar{y}$ | NA | $\bar{y} = \frac{M_x}{M}$ | $\bar{y} = \frac{1}{M} \sum_{i=1}^{N} m_i\, y_i$ |
| $\bar{z}$ | NA | NA | $\bar{z} = \frac{1}{M} \sum_{i=1}^{N} m_i\, z_i$ |
| $M$ | $M = \sum_{i=1}^{N} m_i$ | | |
| $M_x$ | $M_x = \sum_{i=1}^{N} m_i\, y_i$ | | |
| $M_y$ | $M_y = \sum_{i=1}^{N} m_i\, x_i$ | | |
| $I$ | $I = m\, r^2 = \frac{L}{\omega}$ | $I = \sum_{i=1}^{N} m_i\, r_i^2$ | NA |
| $I_x$ | $I_x = \sum_{i=1}^{j} x_i^2 \frac{M}{L} \Delta x$ | $I_x = \sum_{i=1}^{k} \sum_{j=1}^{l} y^2\, \rho(x, y)\, \Delta A$ | $I_x = \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} (y^2 + z^2)\, \rho(x, y, z)\, \Delta V$ |
| $I_y$ | NA | $I_y = \sum_{i=1}^{k} \sum_{j=1}^{l} x^2\, \rho(x, y)\, \Delta A$ | $I_y = Above\ with\ (x^2 + z^2)$ |
| $I_z$ | NA | NA | $Iz_y = Above\ with\ (x^2 + y^2)$ |
| $I_0$ | $I_0 = I_x$ | $I_0 = I_x + I_y$ | $I_0 = I_x + I_y + I_z$ |



## Continuous

| Term | 2-D | 3-D |
|---|---|---|
| $\bar{x}$ | $\bar{x} = \frac{M_y}{M}$ | $\bar{x} = \frac{1}{M}\int_0^M x\,dm = \frac{M_{yz}}{M}$ |
| $\bar{y}$ | $\bar{y} = \frac{M_x}{M}$ | $\bar{y} = \frac{1}{M}\int_0^M y\,dm = \frac{M_{xz}}{M}$ |
| $\bar{z}$ | NA | $\bar{z} = \frac{1}{M}\int_0^M z\,dm = \frac{M_{xy}}{M}$<br>$\bar{z} = \frac{1}{V}\int_{z_{min}}^{z_{max}} z\,dV$ |
| $M$ | $M = \rho\,(Area)$<br>$M = \rho\int_a^b f(x)\,dx$<br>$M = \iint_R dm = \iint_R \rho(x,y)\,dA$<br>$dm = \rho(x,y)\,dy\,dx$ | $M = \rho\,(Volume)$<br>$M = \iiint_Q dm = \iiint_Q \rho(x,y,z)\,dV$<br>$dm = \rho(x,y,z)\,dz\,dy\,dx$ |
| $M_x$ | $M_x = \rho\iint_R y\,dA = \rho\int_a^b \frac{1}{2}([f(x)]^2)\,dx$ | $M_{xy} = \iiint_Q z\,\rho(x,y,z)\,dV$ |
| $M_y$ | $M_y = \rho\iint_R x\,dA = \rho\int_a^b x\,f(x)\,dx$ | $M_{yz} = \iiint_Q x\,\rho(x,y,z)\,dV$ |
| $M_z$ | NA | $M_{xz} = \iiint_Q y\,\rho(x,y,z)\,dV$ |
| $I$ | $I = \int_0^a m\,r^2\,dr$<br>$I = \iint_A \rho(\mathbf{r})\,d(\mathbf{r})^2\,dA(\mathbf{r})$ | $I = \iiint_V \rho(\mathbf{r})\,d(\mathbf{r})^2\,dV(\mathbf{r})$ |
| $I_x$ | $I_x = \iint_R y^2\,\rho(x,y)\,dA$ | $I_x = \iiint_Q (y^2+z^2)\,\rho(x,y,z)\,dV$ |
| $I_y$ | $I_y = \iint_R x^2\,\rho(x,y)\,dA$ | $I_y = \iiint_Q (x^2+z^2)\,\rho(x,y,z)\,dV$ |
| $I_z$ | NA | $I_z = \iiint_Q (x^2+y^2)\,\rho(x,y,z)\,dV$ |

$$x_{cm} = \frac{\int_0^L x\frac{M}{L}dx}{M} = \frac{1}{L}\frac{x^2}{2}\Bigg|_{x=0}^{x=L} = \frac{L}{2}$$