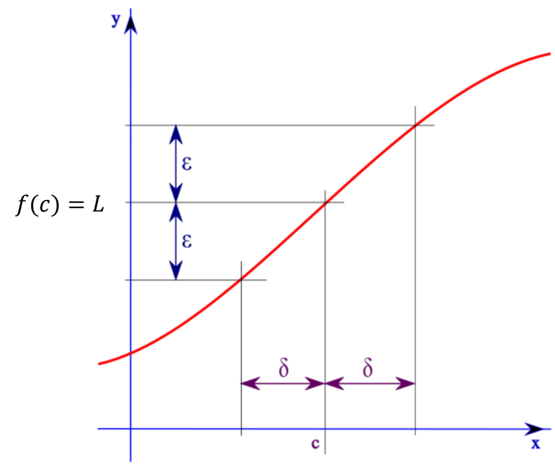
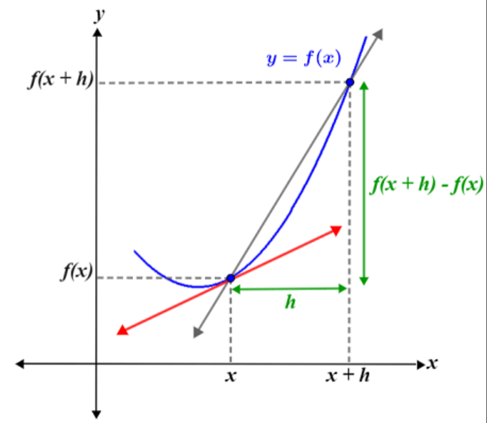
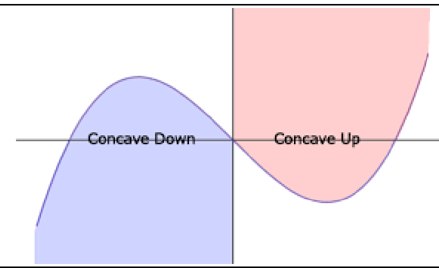
Harold's Calculus
Cheat Sheet
22 September 2025

Limits & Continuity	
Definition of Limit Let f be a function defined on an open interval containing c and let L be a real number. The statement: $\lim\limits_{x \to c} f(x) = L$ means that for each $\epsilon > 0$ there exists a $\delta > 0$ such that if $\lvert x - c \rvert < \delta$, then $\lvert f(x) - L \rvert < \epsilon$ **Tip:** Direct substitution: Plug in $f(c)$ and see if it provides a legal answer. If so, then $L = f(c)$.	
The Existence of a Limit The limit of $f(x)$ as x approaches c is L if and only if:	$\lim\limits_{x \to c^-} f(x) = L$ and $\lim\limits_{x \to c^+} f(x) = L$ (Bonus if $\lim\limits_{x = c} f(x) = f(c) = L$, but not required.)
Definition of Continuity A function $\boldsymbol{f}$ **is continuous** at c if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $\lvert x - c \rvert < \delta$ and $\lvert f(x) - f(c) \rvert < \varepsilon$. **Tip:** Rearrange $\lvert f(x) - f(c) \rvert$ to have $\lvert (x - c) \rvert$ as a factor. Since $\lvert x - c \rvert < \delta$ we can find an equation that relates both δ and ε together.	**<u>Prove</u> that $\boldsymbol{f(x) = x^2 - 1}$ is a continuous function.** $\lvert f(x) - f(c) \rvert < \epsilon$ $= \lvert (x^2 - 1) - (c^2 - 1) \rvert < \epsilon$ $= \lvert x^2 - 1 - c^2 + 1 \rvert < \epsilon$ $= \lvert x^2 - c^2 \rvert < \epsilon$ $= \lvert (x + c) \cdot (x - c) \rvert < \epsilon$ $= \lvert (x + c) \rvert \cdot \lvert (x - c) \rvert < \epsilon$ $= \lvert x + c \rvert \cdot \lvert x - c \rvert < \epsilon$ Since $\lvert x + c \rvert \le \lvert 2c \rvert$ (worst-case scenario) $= \lvert 2c \rvert \cdot \lvert x - c \rvert < \varepsilon$ $= \lvert 2c \rvert \, \delta < \varepsilon$ So, given $\varepsilon > 0$, we can **choose** $\boldsymbol{\delta = \left\lvert \frac{1}{2c} \right\rvert \varepsilon > 0}$ in the Definition of Continuity. So, substituting the chosen δ for $\lvert x - c \rvert$ we get: $\lvert f(x) - f(c) \rvert \le \lvert 2c \rvert \left(\left\lvert \frac{1}{2c} \right\rvert \varepsilon \right) = \varepsilon$ Since both conditions are met, $f(x)$ is continuous.
Two Special Trig Limits	$\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$ and $\lim\limits_{x \to 0} \frac{1 - \cos x}{x} = 0$

Derivative Notation				
Definitions of a Derivative of a Function (Slope Function / Difference Quotient)	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$ $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$			
Second Symmetric Derivative of a Function (Concavity Function)	$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$ $f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$			
First Derivative Notation	$y' = \frac{dy}{dx}$	$f'(x) = \frac{d}{dx} f(x)$	$\dot{x} = \frac{dx}{dt}$	$D_x[y]$
Second Derivative Notation	$y'' = \frac{d^2y}{dx^2}$	$f''(x) = \frac{d^2}{dx^2} f(x)$	$\ddot{x} = \frac{d^2x}{dt^2}$	$D_x^2 f(x)$
nth Derivative Notation	$y^{(n)} = \frac{d^ny}{dx^n}$	$f^{(n)}(x) = \frac{d^n}{dx^n} f(x)$	$\dddot{x} = \frac{d^3x}{dt^3}$	$D_x^n f(x)$

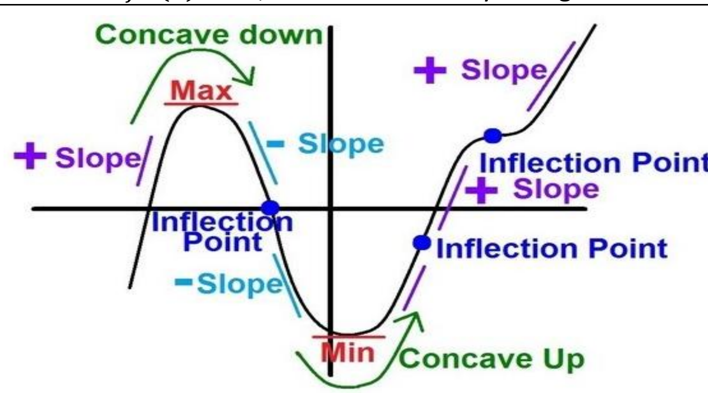
Common Derivatives	(See Cengage Learning 1-Page Calculus Formulas)
1. Constant Rule	$\frac{d}{dx}[c] = 0$
2. Constant Multiple Rule	$\frac{d}{dx}[cf(x)] = cf'(x)$
3. Sum and Difference Rule	$\frac{d}{dx}[f \pm g] = f' \pm g'$
4. Product Rule	$\frac{d}{dx}[fg] = fg' + g\,f'$
5. Quotient Rule	$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$
6. Chain Rule	$\frac{d}{dx}[(f \circ g)(x)] = \frac{d}{dx}\left[f\big(g(x)\big)\right] = f'\big(g(x)\big) \cdot g'(x)$ $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

7. Power Rule $P_n(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$	$\boldsymbol{\frac{d}{dx}[x^n] = nx^{n-1}}$	$\frac{d}{dx}[cx^n] = ncx^{n-1}$
	(Can be used for roots if *n* is a fraction.)	
8. Power Rule for x	$\frac{d}{dx}[x] = 1$ (Think $x = x^1$ and $x^0 = 1$)	
9. Power Rule w\Chain Rule	$\frac{d}{dx}[f(x)^n] = nf(x)^{n-1}\, f'(x)$	
10. Power Rule for Roots	$\frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}\left[x^{\frac{1}{2}}\right] = \frac{1}{2\sqrt{x}}$	
11. Power Rule for Roots w\Chain Rule	$\frac{d}{dx}\left[\sqrt[n]{f(x)}\right] = \frac{1}{n\left(\sqrt[n]{f(x)}\right)^{n-1}} f'(x)$	
12. Absolute Value	$\frac{d}{dx}[\lvert x\rvert] = \frac{x}{\lvert x\rvert}$	
13. Natural Exponent	$\frac{d}{dx}[e^x] = e^x$	
14. Natural Exponent w\Chain Rule	$\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} f'(x)$	
15. Exponential Rule	$\frac{d}{dx}[a^x] = (ln\, a)\, a^x$	
16. Exponential Rule w\Chain Rule	$\frac{d}{dx}\left[a^{f(x)}\right] = (ln\, a)\, a^{f(x)}\, f'(x)$	
17. Natural Logarithm	$\frac{d}{dx}[ln\, x] = \frac{1}{x}, x > 0$ $\frac{d}{dx}[ln\, \lvert x\rvert] = \frac{1}{x}, x \neq 0$	
18. Natural Logarithm w\Chain Rule	$\frac{d}{dx}[ln\, f(x)] = \frac{1}{f(x)} \cdot f'(x)$	
19. Logarithm	$\frac{d}{dx}[log_a\, x] = \frac{1}{x\, ln(a)}, x > 0$ $\frac{d}{dx}[log_a\, \lvert x\rvert] = \frac{1}{x\, ln(a)}, x \neq 0$	
20. Logarithm w\Chain Rule	$\frac{d}{dx}[log_a\, f(x)] = \frac{1}{ln\, x} \cdot \frac{f'(x)}{f(x)}$	
21. Sine	$\frac{d}{dx}[sin(x)] = cos(x)$	
22. Cosine	$\frac{d}{dx}[cos(x)] = -\, sin(x)$	
23. Tangent	$\frac{d}{dx}[tan(x)] = sec^2(x)$	
24. Cotangent	$\frac{d}{dx}[cot(x)] = -\, csc^2(x)$	
25. Secant	$\frac{d}{dx}[sec(x)] = sec(x)\, tan(x)$	
26. Cosecant	$\frac{d}{dx}[csc(x)] = -\, csc(x)\, cot(x)$	

27. Arcsine	$\frac{d}{dx}[sin^{-1}(x)] = \frac{1}{\sqrt{1-x^2}}$
28. Arccosine	$\frac{d}{dx}[cos^{-1}(x)] = \frac{-1}{\sqrt{1-x^2}}$
29. Arctangent	$\frac{d}{dx}[tan^{-1}(x)] = \frac{1}{1+x^2}$
30. Arccotangent	$\frac{d}{dx}[cot^{-1}(x)] = \frac{-1}{1+x^2}$
31. Arcsecant	$\frac{d}{dx}[sec^{-1}(x)] = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
32. Arccosecant	$\frac{d}{dx}[csc^{-1}(x)] = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$
33. Hyperbolic Sine $\left(\frac{e^x-e^{-x}}{2}\right)$	$\frac{d}{dx}[sinh(x)] = cosh(x)$
34. Hyperbolic Cosine $\left(\frac{e^x+e^{-x}}{2}\right)$	$\frac{d}{dx}[cosh(x)] = sinh(x)$
35. Hyperbolic Tangent	$\frac{d}{dx}[tanh(x)] = sech^2(x)$
36. Hyperbolic Cotangent	$\frac{d}{dx}[coth(x)] = -\,csch^2(x)$
37. Hyperbolic Secant	$\frac{d}{dx}[sech(x)] = -\,sech(x)\,tanh(x)$
38. Hyperbolic Cosecant	$\frac{d}{dx}[csch(x)] = -\,csch(x)\,coth(x)$
39. Hyperbolic Arcsine	$\frac{d}{dx}[sinh^{-1}(x)] = \frac{1}{\sqrt{x^2+1}}$
40. Hyperbolic Arccosine	$\frac{d}{dx}[cosh^{-1}(x)] = \frac{1}{\sqrt{x^2-1}}, x > 1$
41. Hyperbolic Arctangent	$\frac{d}{dx}[tanh^{-1}(x)] = \frac{1}{1-x^2}, \lvert x\rvert < 1$
42. Hyperbolic Arccotangent	$\frac{d}{dx}[coth^{-1}(x)] = \frac{1}{1-x^2}, \lvert x\rvert > 1$
43. Hyperbolic Arcsecant	$\frac{d}{dx}[sech^{-1}(x)] = \frac{-1}{x\sqrt{1-x^2}}$
44. Hyperbolic Arccosecant	$\frac{d}{dx}[csch^{-1}(x)] = \frac{-1}{\lvert x\rvert\sqrt{1+x^2}}$

Graphing with Derivatives

f	Gives Us	When set $= 0$	Graph Info	Physics
$f(x)$	Height, y	Roots	Sketch known functions	Position, $s(t)$
$f'(x)$	Slope, m	Critical Points, Local Extreme Values, Min/Max	Pick easy integer x values (-1, 0, 1) in between each critical point to determine where the slope is increasing/decreasing. –↘ ↗+	Velocity, $v(t)$
$f''(x)$	Concavity	Inflection Points	Concave up ∪ if + (min) Concave down ∩ if – (max)	Acceleration, $a(t)$

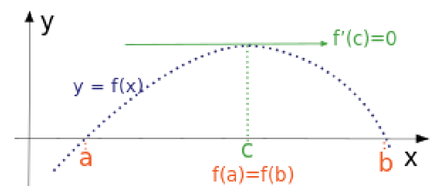
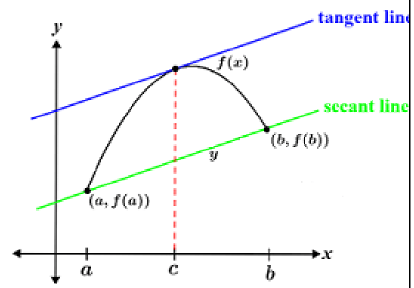
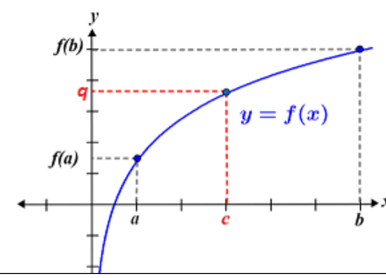
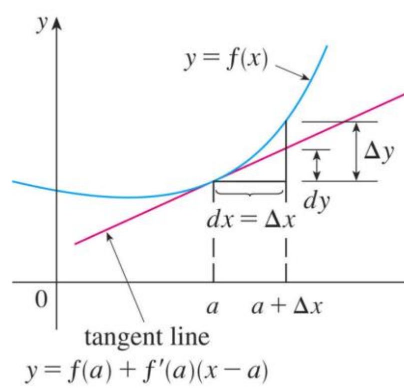
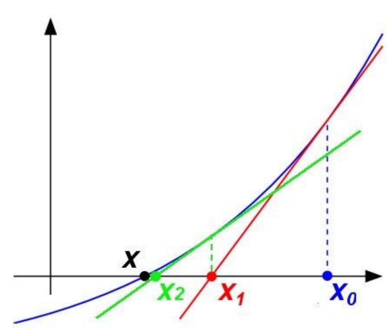
Analyzing the Graph of a Function	(See Harold's Graphing Rationals Cheat Sheet)
x-Intercepts (Zeros or Roots)	$f(x) = 0$
y-Intercept	$f(0) = y$
Domain	Valid x values
Range	Valid y values
Continuity	No division by 0, no negative square roots or logarithms
Vertical Asymptotes (VA)	x = division by 0 or undefined
Horizontal Asymptotes (HA)	$\lim_{x\to\infty^-} f(x) \to y$ and $\lim_{x\to\infty^+} f(x) \to y$
Infinite Limits at Infinity	$\lim_{x\to\infty^-} f(x) \to \infty$ and $\lim_{x\to\infty^+} f(x) \to \infty$
Differentiability	Limit from both directions arrives at the same slope
Relative Extrema	Create a table with domains: $f(x), f'(x), f''(x)$
Concavity	If $f''(x) \to +$, then **cup up** ∪ If $f''(x) \to -$, then **cup down** ∩
Points of Inflection	$f''(x) = 0$, then the concavity changes
Graph	

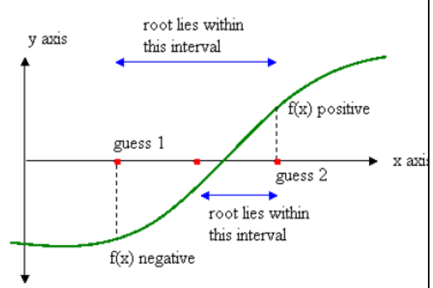
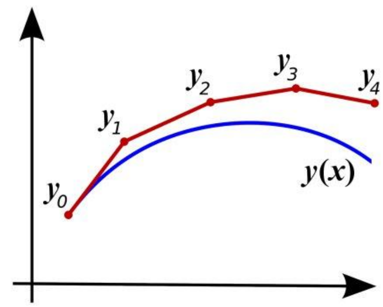
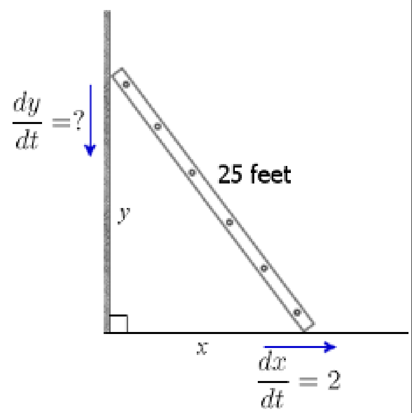
Derivative Tests	
Test for Increasing and Decreasing Functions	1. If $f'(x) \geq 0$, then f is **monotone increasing** (slope up) ↗ 2. If $f'(x) > 0$, then f is **strictly increasing** (slope up) ↗ 3. If $f'(x) \leq 0$, then f is **monotone decreasing** (slope down) ↘ 4. If $f'(x) < 0$, then f is **strictly decreasing** (slope down) ↘ 5. If $f'(x) = 0$, then f is **constant** (zero slope) → and is possibly a min/max
First Derivative Test	Critical points: 1. If $f'(x)$ changes from – to + at c, then f has a *relative* **minimum** at $(c, f(c))$ 2. If $f'(x)$ changes from + to – at c, then f has a *relative* **maximum** at $(c, f(c))$ 3. If $f'(x)$, is $+ c +$ or $- c -$, then $f(c)$ is neither Absolute minimum/maximum: 1. Test $f(x)$ at the domain boundaries $[a, b]$, meaning check $f(a)$ and $f(b)$ 2. Include $f(x)$ for all critical points as well 3. The largest/smallest wins
Second Derivative Test	Let $f'(c) = 0$, and $f''(x)$ exists, then 1. If $f''(x) > 0$, then f has a relative **minimum** at $(c, f(c))$ 2. If $f''(x) < 0$, then f has a relative **maximum** at $(c, f(c))$ 3. If $f''(x) = 0$, then the test fails
Test for Concavity	1. If $f''(x) > 0$ for all x, then the graph is concave up (**cup up** ∪) 2. If $f''(x) < 0$ for all x, then the graph is concave down (**cup down** ∩)
Inflection Points (Change in concavity)	If $(c, f(c))$ is a point of inflection of $f(x)$, then either 1. $f''(c) = 0$ or 2. $f''(x)$ does not exist at $x = c$

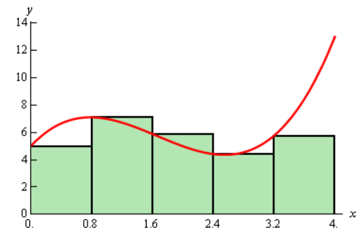
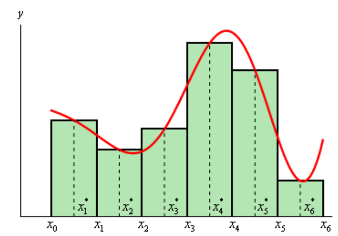
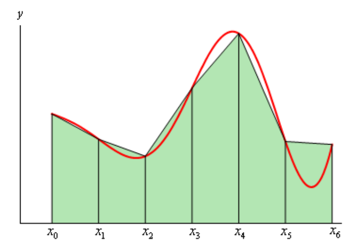
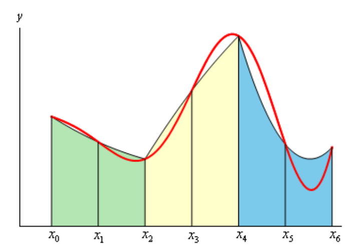
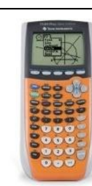
Extrema	
Local Maximum (Relative Max.)	A function $f(x)$ has a local max at $x = a$ if $f(a)$ is greater than or equal to (≥) the values of $f(x)$ in some interval around a. a is usually near the origin.
Local Minimum (Relative Min.)	A function $f(x)$ has a local minimum at $x = a$ if $f(a)$ is less than or equal to (≤) the values of $f(x)$ in some interval around a. a is usually near the origin.
Absolute Maximum (Global Max.)	A function $f(x)$ has an absolute maximum at $x = c$ if $f(c)$ is greater than or equal to (≥) $f(x)$ for all x in the domain of f.
Absolute Minimum (Global Min.)	A function $f(x)$ has an absolute minimum at $x = d$ if $f(d)$ is less than or equal to (≤) $f(x)$ for all x in the domain.
Critical Points	Find the derivative $f'(x)$ and set it to zero to find critical points. Critical points are where the derivative is zero or undefined.
Endpoints	For absolute extrema on a closed interval $[a, b]$, evaluate the function at the critical points and at the endpoints $f(a)$ and $f(b)$. The largest value among these will be the absolute maximum, and the smallest will be the absolute minimum.

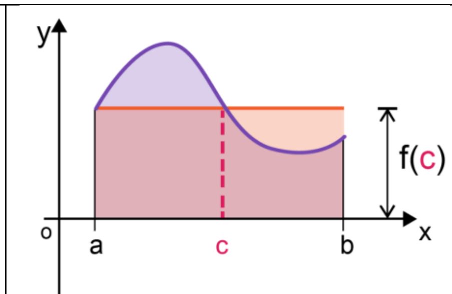
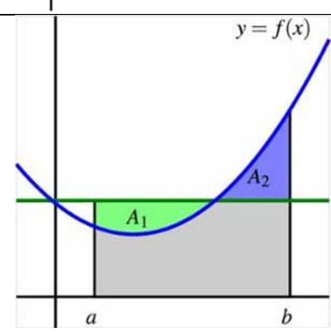
Equations of a Line	Used for Tangent Lines
Standard Form	$ax + by + c = 0$ where a is positive
Slope-Intercept Form	$y = mx + b$
Point-Slope Form	$y - y_0 = m(x - x_0)$ where $m = f'(x_0)$ at point (x_0, y_0)
Intercept Form	$\frac{x}{a} + \frac{y}{b} = 1$ where a is the $x - intercept$ and b is the $y - intercept$
Calculus Form	$f(x) = f'(c)x + f(0)$ $f(x) = f'(c)(x - c) + f(c)$
Slope	$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \to \frac{dy}{dx} = f'(x)$
Vertical Line	$x = a$
Horizontal Line	$y = b$

Physics	Translational Motion	
Position	1D $s(t) = \boldsymbol{s}_0 + \boldsymbol{v}_0 t + \frac{1}{2}\boldsymbol{a}t^2$	2D $x(t) = \boldsymbol{x}_0 + \boldsymbol{v}_{0,x} t + \frac{1}{2}\boldsymbol{a}_x t^2$ $y(t) = \boldsymbol{y}_0 + \boldsymbol{v}_{0,y} t + \frac{1}{2}\boldsymbol{g}t^2$
Velocity	$v(t) = s'(t)$	$v(t) = \boldsymbol{v}_0 + \boldsymbol{a}t$ $\boldsymbol{v}^2 = \boldsymbol{v}_0^2 + 2\boldsymbol{a}(x - x_0)$
Acceleration	$a(t) = v'(t) = s''(t)$	$a(t) = \boldsymbol{a}$
Jerk (Jolt)	$j(t) = a'(t) = v''(t) = s^{(3)}(t)$	When a car brakes sharply or accelerates quickly.
Gravitational Constant (*g*) (Planet Earth)	$\boldsymbol{g} \approx -9.81 \frac{m}{s^2}$	$\boldsymbol{g} \approx -32.2 \frac{ft}{s^2}$

Differentiation & Differentials		
Rolle's Theorem	Assume f is <u>continuous</u> on the closed interval $[a, b]$, and f is <u>differentiable</u> on the open interval (a, b). If $f(a) = f(b)$, then there exists at least one number c in (a, b) such that $f'(c) = 0$.	
Mean Value Theorem	If f meets the conditions of Rolle's Theorem, then you can find 'c'. $f'(c) = \dfrac{f(b) - f(a)}{b - a} = \dfrac{\Delta y}{\Delta x}$ $f(b) = f(a) + (b - a)f'(c)$	
Intermediate Value Theorem	Assume f is a <u>continuous</u> function with the interval $[a, b]$ as its domain. If f takes **values** $f(a)$ and $f(b)$ at each end of the interval, then it also takes any **value** between $f(a)$ and $f(b)$ at some point within the interval.	
Calculating Differentials (Tangent line approximation)	$f(x + \Delta x) \approx f(x) + \Delta y$ since $dy = f'(x)\,dx$ then $\Delta y \approx f'(x)\,\Delta x$ so $\boldsymbol{f(x + \Delta x) \approx f(x) + f'(x)\,\Delta x}$ Relative Error $= \dfrac{\Delta f}{f}$ in % Example: $\sqrt[4]{82}$ $f(x) = \sqrt[4]{x}$ $f(x + \Delta x) = f(81 + 1)$	
Newton-Raphson Method	Finds zeros of f, or finds c if $f(c) = 0$. $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ Example: $\sqrt[4]{82}$ $f(x) = x^4 - 82 = 0$ start with $x_0 = 3$	

Bisection Method	Finds zeros of f for a continuous function. 1. Find two points, a and b, where $f(a)\cdot f(b)<0$. 2. Calculate the midpoint, t, between a and b. 3. If $f(t)=0$, then t is the root of the function. 4. Divide the interval $[a,b]$ in half and repeat the process until step 3.	
Euler's Method	Approximates f given an initial value and the function's derivative (solves an ODE). Initial Condition: $y_0=y(x_0)$ Definition: $y_i=y(x_i)$ Derivative Function: $y'(x_i)=f(x_i,y_i)$ $y_{n+1}=y(x_n)+h\cdot y'(x_n)$ $y_{n+1}=y_n+h\cdot f(x_n,y_n)$	
Related Rates	Steps to solve: 1. Identify the known variables and rates of change. $x=15\ m$ $y=20\ m$ $x'=2\frac{m}{s}$ $y'=\_\_\_\_\frac{m}{s}$ 2. Construct an equation relating these quantities. $x^2+y^2=r^2$ (Typically, the Pythagorean Theorem, similar triangles, or volume formulas.) 3. Differentiate both sides of the equation. $2xx'+2yy'=0$ 4. Solve for the desired rate of change. $y'=-\frac{x}{y}x'$ 5. Substitute the known rates of change and quantities into the equation. $y'=-\frac{15}{20}\bullet 2=1.5\ \frac{m}{s}$	
L'Hôpital's Rule	If $\lim_{x\to c} f(x)=\lim_{x\to c}\frac{P(x)}{Q(x)}$ $=\left\{\frac{0}{0},\frac{\infty}{\infty},0\bullet\infty,\infty-\infty,0^0,1^\infty,\infty^0\right\}$, but not $\{0^\infty,\infty^\infty\}$, then $\lim_{x\to c}\frac{P(x)}{Q(x)}=\lim_{x\to c}\frac{P'(x)}{Q'(x)}=\lim_{x\to c}\frac{P''(x)}{Q''(x)}=\cdots$	

Numerical Methods		
Riemann Sum	$P_0(x) = \int_a^b f(x)\,dx = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i$ where $a = x_0 < x_1 < x_2 < \cdots < x_n = b$ and $\Delta x_i = x_i - x_{i-1}$ and $\|P\| = max\{\Delta x_i\}$ Types: • Left Sum (LHS) • Middle Sum (MHS) • Right Sum (RHS)	
Midpoint Rule (Middle Sum/MHS)	$P_0(x) = \int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(\bar{x}_i)\,\Delta x =$ $\Delta x\,[f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + \cdots + f(\bar{x}_n)]$ where $\Delta x = \frac{b-a}{n}$ and $\bar{x}_i = \frac{(x_{i-1}+x_i)}{2}$ = midpoint of $[x_{i-1}, x_i]$ Error Bounds: $\lvert E_M \rvert \le \frac{K(b-a)^3}{24n^2}$	
Trapezoidal Rule	$P_1(x) = \int_a^b f(x)\,dx \approx$ $\frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1})$ $+ f(x_n)]$ where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$ Error Bounds: $\lvert E_T \rvert \le \frac{K(b-a)^3}{12n^2}$	
Simpson's Rule	$P_2(x) = \int_a^b f(x)\,dx \approx$ $\frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots$ $+ 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$ Where n is even and $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$ Error Bounds: $\lvert E_S \rvert \le \frac{K(b-a)^5}{180n^4}$	
TI-84 Plus	[MATH] fnInt(f(x),x,a,b), [MATH] [1] [ENTER] Example: [MATH] fnInt(x^2,x,0,1) $\int_0^1 x^2\,dx = \frac{1}{3}$	
TI-Nspire CAS	[MENU] [4] Calculus [3] Integral [TAB] [TAB] [X] [^] [2] [TAB] [TAB] [X] [ENTER] Shortcut: [ALPHA] [WINDOWS] [4]	

Integration	(See Harold's Fundamental Theorem of Calculus Cheat Sheet)	
Basic Integration Rules (Integration is the "inverse" of differentiation, and vice versa.)	$\int f'(x)\,dx = f(x) + C$ $\frac{d}{dx}\int f(x)\,dx = f(x)$	**Tip**: Use the $f'(x)$ tables and integrate both sides to determine many common integrals.
Reimann Sum	$\sum_{i=1}^{n} f(c_i)\,\Delta x_i, \quad \text{where } x_{i-1} \le c_i \le x_i$	
Definition of a Definite Integral (Area under the curve)	$\lim_{\lVert\Delta\rVert\to 0} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$ $\text{where } \lVert\Delta\rVert = \Delta x = \frac{b-a}{n}$	
Swap Bounds	$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$	
Additive Interval Property	$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$	
First Fundamental Theorem of Calculus	$\int_a^b f(x)\,dx = F(b) - F(a)$	
Second Fundamental Theorem of Calculus	$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f\big(g(x)\big)g'(x)$ $\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f\big(h(x)\big)h'(x) - f\big(g(x)\big)g'(x)$	
Mean Value Theorem for Integrals	$\int_a^b f(x)\,dx = f(c)(b-a)$ Find 'c'.	
Average Value of a Function	Continuous: $\frac{1}{b-a}\int_a^b f(x)\,dx$ Discrete: $\text{Ave} = \frac{1}{n}\sum_{i=1}^{n} a_i$	

Common Anti-Derivatives	(See Cengage Learning 1-Page Calculus Formulas)
1. Zero Rule $f(x) = 0$	$\int 0\, dx = C$
2. Constant Rule $f(x) = k = kx^0$	$\int k\, dx = kx + C$
3. Constant Multiple Rule	$\int k\, f(x)\, dx = k \int f(x)\, dx$
4. Sum and Difference Rule	$\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$
5. Power Rule $P_n(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$	$\int \boldsymbol{x^n\, dx = \frac{x^{n+1}}{n+1} + C}$, where $n \neq -1$ $If\ n = -1$, then $\int x^{-1}\, dx = \ln\lvert x\rvert + C$
6. Power Rule w/Chain Rule	If $u = g(x)$, and $u' = \frac{d}{dx} g(x)$ then $\int u^n\, \boldsymbol{u'} dx = \frac{u^{n+1}}{n+1} + C, where\ n \neq -1$
7. Natural Exponent	$\int e^x\, dx = \mathrm{e}^{\mathrm{x}} + C$
8. Exponent	$\int a^x\, dx = \frac{1}{ln\, a} a^x + C$
9. Natural Logarithm	$\int \frac{1}{x}\, dx = \ln\lvert x\rvert + C$
10. Logarithm	$\int \frac{1}{x} dx = (ln\, a)\, log_a\, x + C$
11. Sine	$\int sin(x)\, dx = -cos(x) + C$
12. Cosine	$\int cos(x)\ \ dx = sin(x) + C$
13. Tangent	$\int \tan(x)\, dx = -\ln\lvert\cos\,(x)\rvert + C$
14. Cotangent	$\int \cot(x)\, dx = \ln\lvert sin\,(x)\rvert + C$
15. Secant	$\int \sec(x)\, dx = \ln\lvert\sec(x) + \tan(x)\rvert + C$
16. Cosecant	$\int \csc(x)\, dx = -\ln\lvert\csc(x) + cot(x)\rvert + C$
17. Secant²	$\int sec^2(x)\ \ dx = tan(x) + C$
18. Cosecant²	$\int csc^2(x)\, dx = -cot(x) + C$
19. Arcsine	$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = sin^{-1}\left(\frac{x}{a}\right) + C$
20. Arctangent	$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\, tan^{-1}\left(\frac{x}{a}\right) + C$
21. Arcsecant	$\int \frac{1}{x\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\, sec^{-1}\left(\frac{\lvert x\rvert}{a}\right) + C$

Integration Methods	
1. Memorized	See Cengage Learning 1-Page Calculus Formulas
2. U-Substitution	$\int f(g(x))g'(x)dx = F(g(x)) + C$ Set $u = g(x)$, then $du = g'(x)\ dx$ $\int f(u)\ du = F(u) + C$ $u =$ ____ , $du =$ ____ dx
3. Integration by Parts	$\int u\ dv = uv - \int v\ du$ $u =$ ____ , $v =$ ____ $du =$ ____ dx, $dv =$ ____ dx Pick 'u' using the **LIATE** Rule: **L – Logarithmic**: $\ln x, \log_b x$ **I – Inverse Trig.**: $\sin^{-1} x, \cos^{-1} x, \tan^{-1} x$ **A – Algebraic**: $x^2, 3x^{60}, \sqrt{x}, etc.$ **T – Trigonometric**: $\sin x, \cos x, \tan x$ **E – Exponential**: $e^x, 19^x$
4. Partial Fractions	$\int \frac{P(x)}{Q(x)}\ dx$ where $P(x)$ and $Q(x)$ are polynomials. **Case 1:** If the degree of $P(x) \geq Q(x)$ then do long division first. **Case 2:** If the degree of $P(x) < Q(x)$ then do partial fraction expansion.
5a. Trig Substitution for $\sqrt{a^2 - x^2}$	$\int \sqrt{a^2 - x^2}\ dx$ Substitution: $x = a \sin \theta$ Identity: $1 - \sin^2 \theta = \cos^2 \theta$
5b. Trig Substitution for $\sqrt{x^2 - a^2}$	$\int \sqrt{x^2 - a^2}\ dx$ Substitution: $x = a \sec \theta$ Identity: $\sec^2 \theta - 1 = \tan^2 \theta$
5c. Trig Substitution for $\sqrt{x^2 + a^2}$	$\int \sqrt{x^2 + a^2}\ dx$ Substitution: $x = a \tan \theta$ Identity: $\tan^2 \theta + 1 = \sec^2 \theta$
6. Computer Algebra System (CAS)	TI-Nspire CX CAS Graphing Calculator TI –Nspire CAS iPad app
7. Numerical Methods	Riemann Sum, Midpoint Rule, Trapezoidal Rule, Simpson's Rule, various quadrature rules, TI-84 Calculator, etc.
8. WolframAlpha	WolframAlpha is the Google of mathematics. Shows steps. Free.
9. AI Chatbot	OpenAI ChatGPT, Microsoft Copilot, Google Gemini, xAI Grok, etc.

Partial Fractions	(See Harold's Partial Fraction Decomposition Cheat Sheet)
Condition	$f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials and the degree of $P(x) < Q(x)$. If the degree of $P(x) \geq Q(x)$, then do long division first.
Example Expansion	$\frac{P(x)}{(ax+b)(cx+d)^2(ex^2+fx+g)}$ $= \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C}{(cx+d)^2} + \frac{Dx+E}{(ex^2+fx+g)}$
Typical Solution	$\int \frac{a}{x+b}\, dx = a\ ln\lvert x+b\rvert + C$

Sequences & Series	(See Harold's Series Cheat Sheet)
Sequence	$\lim_{n\to\infty} a_n = L$ (Limit) Example: $(a_n, a_{n+1}, a_{n+2}, \ldots)$
Geometric Series	$S = \lim_{n\to\infty} \frac{a(1-r^n)}{1-r} = \frac{a}{1-r}$ only if $\lvert r\rvert < 1$ where r is the **radius** of convergence and $(-r, r)$ is the **interval** of convergence

Convergence Tests	(See Harold's Series Convergence Tests Cheat Sheet)	
Series Convergence Tests	1. Divergence or n^{th} Term 2. Geometric Series 3. p-Series 4. Alternating Series 5. Integral	6. Ratio 7. Root 8. Direct Comparison 9. Limit Comparison 10. Telescoping Series

Taylor Series	(See Harold's Taylor Series Cheat Sheet) (See Harold's Infinite Series Cheat Sheet)
Taylor Series	$f(x) = P_n(x) + R_n(x)$ $= \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n + \frac{f^{(n+1)}(x^*)}{(n+1)!}(x-c)^{n+1}$ where $x \leq x^* \leq c$ (x^* is the worst-case scenario or max. value of $f(x)$ in the range.) and $\lim_{x\to+\infty} R_n(x) = 0$.