**Harold’s Calculus**

**Cheat Sheet**

22 September 2025

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| **Limits & Continuity** |  |
| **Definition of Limit**  Let *f* be a function defined on an open interval containing and let be a real number. The statement:  means that for each there exists a such that  if  **Tip:**  Direct substitution: Plug in and see if it provides a legal answer. If so, then *.* |  |
| **The Existence of a Limit**  The limit of as approaches *c* is *L* if and only if: | and  (Bonus if , but not required.) |
| **Definition of Continuity**  A function  **is continuous** at if for every there exists a such that  and.  **Tip:**  Rearrange to have as a factor. Since we can find an equation that relates both and together. | **Prove that is a continuous function.**      Since (worst-case scenario)  So, given , we can **choose**  in the Definition of Continuity. So, substituting the chosen for we get:  Since both conditions are met, is continuous. |
| **Two Special Trig Limits** |  |

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| **Derivative Notation** |  | | | | |
| **Definitions of a Derivative of a Function**  (Slope Function / Difference Quotient) |  | | Derivative | | |
| **Second Symmetric Derivative of a Function**  (Concavity Function) |  | | Concave Up and Down Functions, and ... | | |
| **First Derivative Notation** |  |  | |  |  |
| **Second Derivative Notation** |  |  | |  |  |
| **nth Derivative Notation** |  |  | |  |  |

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| **Common Derivatives** | (See Cengage Learning [1-Page Calculus Formulas](https://www.toomey.org/tutor/harolds_cheat_sheets/1_Derivatives_and_Integrals_300dpi.pdf)) | |
| **1. Constant Rule** |  | |
| **2. Constant Multiple Rule** |  | |
| **3. Sum and Difference Rule** |  | |
| **4. Product Rule** |  | |
| **5. Quotient Rule** |  | |
| **6. Chain Rule** |  | |
| **7. Power Rule** |  |  |
| (Can be used for roots if *n* is a fraction.) | |
| **8. Power Rule for** |  | |
| **9. Power Rule w\Chain Rule** |  | |
| **10. Power Rule for Roots** |  | |
| **11. Power Rule for Roots w\Chain Rule** |  | |
| **12. Absolute Value** |  | |
| **13. Natural Exponent** |  | |
| **14. Natural Exponent w\Chain Rule** |  | |
| **15. Exponential Rule** |  | |
| **16. Exponential Rule w\Chain Rule** |  | |
| **17. Natural Logarithm** |  | |
| **18. Natural Logarithm w\Chain Rule** |  | |
| **19. Logarithm** |  | |
| **20. Logarithm w\Chain Rule** |  | |
| **21. Sine** |  | |
| **22. Cosine** |  | |
| **23. Tangent** |  | |
| **24. Cotangent** |  | |
| **25. Secant** |  | |
| **26. Cosecant** |  | |
| **27. Arcsine** |  | |
| **28. Arccosine** |  | |
| **29. Arctangent** |  | |
| **30. Arccotangent** |  | |
| **31. Arcsecant** |  | |
| **32. Arccosecant** |  | |
| **33. Hyperbolic Sine** |  | |
| **34. Hyperbolic Cosine** |  | |
| **35. Hyperbolic Tangent** |  | |
| **36. Hyperbolic Cotangent** |  | |
| **37. Hyperbolic Secant** |  | |
| **38. Hyperbolic Cosecant** |  | |
| **39. Hyperbolic Arcsine** |  | |
| **40. Hyperbolic Arccosine** |  | |
| **41. Hyperbolic Arctangent** |  | |
| **42. Hyperbolic Arccotangent** |  | |
| **43. Hyperbolic Arcsecant** |  | |
| **44. Hyperbolic Arccosecant** |  | |

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| **Graphing with Derivatives** | | | | |
|  | **Gives Us** | **When set** | **Graph Info** | **Physics** |
|  | Height, | Roots | Sketch known functions | Position, |
|  | Slope, | Critical Points,  Local Extreme Values,  Min/Max | Pick easy integer values (-1, 0, 1) in between each critical point to determine where the slope is increasing/decreasing.  **– ↘ ↗ +** | Velocity, |
|  | Concavity | Inflection Points | Concave up ⋃ if + (min)  Concave down ⋂ if – (max) | Acceleration, |

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| **Analyzing the Graph of a Function** | (See [Harold’s Graphing Rationals Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Graphing_Rationals_Cheat_Sheet.pdf)) |
| **x-Intercepts (Zeros or Roots)** |  |
| **y-Intercept** |  |
| **Domain** | Valid values |
| **Range** | Valid values |
| **Continuity** | No division by 0, no negative square roots or logarithms |
| **Vertical Asymptotes (VA)** | = division by 0 or undefined |
| **Horizontal Asymptotes (HA)** | and |
| **Infinite Limits at Infinity** | and |
| **Differentiability** | Limit from both directions arrives at the same slope |
| **Relative Extrema** | Create a table with domains: |
| **Concavity** | If , then **cup up**  If, then **cup down** |
| **Points of Inflection** | , then the concavity changes |
| **Graph** | A diagram of a slope  AI-generated content may be incorrect. |

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| **Derivative Tests** | |
| **Test for Increasing and Decreasing Functions** | 1. If , then *f* is **monotone increasing** (slope up) ↗  2. If , then *f* is **strictly increasing** (slope up) ↗  3. If , then *f* is **monotone decreasing** (slope down) ↘  4. If , then *f* is **strictly decreasing** (slope down) ↘  5. If , then *f* is **constant** (zero slope) → and is possibly a min/max |
| **First Derivative Test** | Critical points:  1. If changes from – to + at , then has a *relative* **minimum** at  2. If changes from + to – at , then has a *relative* **maximum** at  3. If , is + + or – –, then is neither  Absolute minimum/maximum:  1. Test at the domain boundaries , meaning check and  2. Include for all critical points as well  3. The largest/smallest wins |
| **Second Derivative Test** | Let , and exists, then  1. If , then *f* has a relative **minimum** at  2. If , then *f* has a relative **maximum** at  3. If , then the test fails |
| **Test for Concavity** | 1. If for all , then the graph is concave up (**cup** **up** ⋃)  2. If for all , then the graph is concave down (**cup** **down** ⋂) |
| **Inflection Points**  (Change in concavity) | If is a point of inflection of , then either  1. or  2. does not exist at |

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| **Extrema** | |
| **Local Maximum**  (Relative Max.) | A function  has a local max at  if  is greater than or equal to () the values of  in some interval around . is usually near the origin. |
| **Local Minimum**  (Relative Min.) | A function  has a local minimum at  if  is less than or equal to () the values of  in some interval around . is usually near the origin. |
| **Absolute Maximum**  (Global Max.) | A function  has an absolute maximum at  if  is greater than or equal to ()  for all  in the domain of . |
| **Absolute Minimum**  (Global Min.) | A function  has an absolute minimum at  if  is less than or equal to ()  for all  in the domain. |
| **Critical Points** | Find the derivative  and set it to zero to find critical points.  Critical points are where the derivative is zero or undefined. |
| **Endpoints** | For absolute extrema on a closed interval , evaluate the function at the critical points and at the endpoints  and . The largest value among these will be the absolute maximum, and the smallest will be the absolute minimum. |

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| **Equations of a Line** | **Used for Tangent Lines** |
| **Standard Form** | where is positive |
| **Slope-Intercept Form** |  |
| **Point-Slope Form** | where at point |
| **Intercept Form** | where is the  and is the |
| **Calculus Form** |  |
| **Slope** |  |
| **Vertical Line** |  |
| **Horizontal Line** |  |

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| **Physics** | **Translational Motion** | |
| **Position** | 1D | 2D |
| **Velocity** |  |  |
| **Acceleration** |  |  |
| **Jerk** (Jolt) |  | When a car brakes sharply or accelerates quickly. |
| **Gravitational Constant (*g*)**  (Planet Earth) |  |  |

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| **Differentiation & Differentials** | | | | | |  | | | |
| **Rolle’s Theorem** | Assume is continuous on the closed interval , and is differentiable on the open interval .  If , then there exists at least one number in such that . | | | | |  | | | |
| **Mean Value Theorem** | If meets the conditions of Rolle’s Theorem, then you can find ‘’. | | | | |  | | | |
| **Intermediate Value Theorem** | Assume is a continuous function with the interval as its domain.  If takes **values** and at each end of the interval, then it also takes any **value** between and at some point within the interval. | | | | |  | | | |
| **Calculating Differentials**  (Tangent line approximation) | so  Example: | | | | |  | | | |
| **Newton-Raphson Method** | Finds zeros of , or finds if .  Example: | | | | | Newton's Method in Calculus | Formula, Equation & Examples - Lesson |  Study.com | | | |
| **Bisection Method** | Finds zeros of for a continuous function.   1. Find two points, and , where . 2. Calculate the midpoint, , between and . 3. If , then is the root of the function. 4. Divide the interval in half and repeat the process until step 3. | | | | | Bisection method This Bisection method states that if f(x) is... | Download  Scientific Diagram | | | |
| **Euler’s Method** | Approximates given an initial value and the function’s derivative (solves an ODE).  Initial Condition:  Definition:  Derivative Function: | | | | | Euler's Method for Differential Equations | Overview & Formula | Study.com | | | |
| **Related Rates** | Steps to solve:   1. Identify the known variables and rates of change. 2. Construct an equation relating these quantities.   (Typically, the Pythagorean Theorem, similar triangles, or volume formulas.)   1. Differentiate both sides of the equation. 2. Solve for the desired rate of change. 3. Substitute the known rates of change and quantities into the equation. | | | | | http://i1104.photobucket.com/albums/h330/mathclassroom/Calculus/ladder.png | | | |
| **L’Hôpital’s Rule** |  | | | | | Guillaume de l'Hôpital (1661 - 1704) - Biography - MacTutor History of  Mathematics | | | |
| **Numerical Methods** | | | | | | | | | | |
| **Riemann Sum** | |  | where  and  and  Types:   * Left Sum (LHS) * Middle Sum (MHS) * Right Sum (RHS) | | | | | http://tutorial.math.lamar.edu/Classes/CalcI/AreaProblem_files/image010.gif | | |
| **Midpoint Rule**  (Middle Sum/MHS) | |  | where  and  Error Bounds: | | | | | ApproxDef_G1 | | |
| **Trapezoidal Rule** | |  | where  and  Error Bounds: | | | | | ApproxDef_G2 | | |
| **Simpson’s Rule** | |  | Where is even  and  and  Error Bounds: | | | | | ApproxDef_G3 | | |
| **TI-84 Plus** | |  | [MATH] fnInt(f(x),x,a,b), [MATH] [1] [ENTER]  Example: [MATH] fnInt(x^2,x,0,1) | | | | | http://www.gosale.com/product_images/4948000/texas-instruments-ti-84-plus-4948122.jpg | | |
| **TI-Nspire CAS** | |  | [MENU] [4] Calculus [3] Integral  [TAB] [TAB]  [X] [^] [2] [TAB]  [TAB] [X] [ENTER]  Shortcut: [ALPHA] [WINDOWS] [4] | | | | | http://i5.walmartimages.com/dfw/dce07b8c-6b5e/k2-_10014b04-7e9e-4157-a2ec-ab5b03fa1234.v1.jpg | | |
| **Integration** | | | | (See [Harold’s Fundamental Theorem of Calculus Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Fundamental_Theorem_of_Calculus_Cheat_Sheet.pdf)) | | | | |
| **Basic Integration Rules**  (Integration is the “inverse” of differentiation, and vice versa.) | | | |  | | | **Tip**: Use the tables and integrate both sides to determine many common integrals. | |
| **Reimann Sum** | | | |  | | | | |
| **Definition of a Definite Integral**  (Area under the curve) | | | |  | | | | |
| **Swap Bounds** | | | |  | | | | |
| **Additive Interval Property** | | | |  | | | | |
| **First Fundamental Theorem of Calculus** | | | |  | | | | |
| **Second Fundamental Theorem of Calculus** | | | |  | | | | |
| **Mean Value Theorem for Integrals** | | | |  |  | | | |
| **Average Value of a Function** | | | | Continuous:  Discrete: | . | | | |

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| **Common Anti-Derivatives** | | | (See Cengage Learning [1-Page Calculus Formulas](https://www.toomey.org/tutor/harolds_cheat_sheets/1_Derivatives_and_Integrals_300dpi.pdf)) |
| **1. Zero Rule** | | |  |
| **2. Constant Rule** | | |  |
| **3. Constant Multiple Rule** | | |  |
| **4. Sum and Difference Rule** | | |  |
| **5. Power Rule** | | |  |
| **6. Power Rule w/Chain Rule** | | | If then |
| **7. Natural Exponent** | | |  |
| **8. Exponent** | | |  |
| **9. Natural Logarithm** | | |  |
| **10. Logarithm** | | |  |
| **11. Sine** | | |  |
| **12. Cosine** | | |  |
| **13. Tangent** | | |  |
| **14. Cotangent** | | |  |
| **15. Secant** | | |  |
| **16. Cosecant** | | |  |
| **17. Secant2** | | |  |
| **18. Cosecant2** | | |  |
| **19. Arcsine** | | |  |
| **20. Arctangent** | | |  |
| **21. Arcsecant** | | |  |
| **Integration Methods** | |  | | |
| **1. Memorized** | | See Cengage Learning [1-Page Calculus Formulas](https://www.toomey.org/tutor/harolds_cheat_sheets/1_Derivatives_and_Integrals_300dpi.pdf)) | | |
| **2. U-Substitution** | | Set  \_\_\_\_\_ , \_\_\_\_\_ | | |
| **3. Integration by Parts** | | \_\_\_\_\_ , \_\_\_\_\_ .  \_\_\_\_ , \_\_\_\_  Pick ‘’ using the **LIATE** Rule:  **L –**[**Logarithmic**](http://en.wikipedia.org/wiki/Logarithmic_function):  **I –**[**Inverse Trig.**](http://en.wikipedia.org/wiki/Inverse_trigonometric_function):  **A –**[**Algebraic**](http://en.wikipedia.org/wiki/Polynomial):  **T –**[**Trigonometric**](http://en.wikipedia.org/wiki/Trigonometric_functions):  **E –**[**Exponential**](http://en.wikipedia.org/wiki/Exponential_function): | | |
| **4. Partial Fractions** | | where are polynomials.  **Case 1:** If the degree of  then do long division first.  **Case 2:** If the degree of  then do partial fraction expansion. | | |
| **5a. Trig Substitution for** | | Substitution:  Identity: | | |
| **5b. Trig Substitution for** | | Substitution:  Identity: | | |
| **5c. Trig Substitution for** | | Substitution:  Identity: | | |
| **6. Computer Algebra System (CAS)** | | [TI-Nspire CX CAS Graphing Calculator](http://education.ti.com/en/us/products/calculators/graphing-calculators/ti-nspire-cx-cas-handheld/tabs/overview)  [TI –Nspire CAS](https://itunes.apple.com/us/app/ti-nspire-cas/id545351700?mt=8) iPad app | | |
| **7. Numerical Methods** | | Riemann Sum, Midpoint Rule, Trapezoidal Rule, Simpson’s Rule, various quadrature rules, TI-84 Calculator, etc. | | |
| **8. WolframAlpha** | | [WolframAlpha](http://www.wolframalpha.com) is the Google of mathematics. Shows steps. Free. | | |
| **9. AI Chatbot** | | OpenAI [ChatGPT](https://openai.com/chatgpt), Microsoft [Copilot](https://copilot.microsoft.com/), Google [Gemini](https://gemini.google.com), xAI [Grok](https://x.ai/), [etc](https://tech.co/news/best-ai-chatbots). | | |
| **Partial Fractions** | (See [Harold’s Partial Fraction Decomposition Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Partial_Fraction_Decomposition_Calc_Cheat_Sheet.pdf)) | | | |
| **Condition** | where are polynomials  and the degree of .  If the degree of , then do long division first. | | | |
| **Example Expansion** |  | | | |
| **Typical Solution** |  | | | |

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| **Sequences & Series** | (See [Harold’s Series Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Series_Cheat_Sheet.pdf)) |
| **Sequence** | (Limit)  Example: () |
| **Geometric Series** | only if  where is the **radius** of convergence  and is the **interval** of convergence |

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| **Convergence Tests** | (See [Harold’s Series Convergence Tests Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Series_Convergence_Tests_Cheat_Sheetpdf)) | |
| **Series Convergence Tests** | 1. Divergence or Term 2. Geometric Series 3. p-Series 4. Alternating Series 5. Integral | 1. Ratio 2. Root 3. Direct Comparison 4. Limit Comparison 5. Telescoping Series |

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| **Taylor Series** | (See [Harold’s Taylor Series Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Taylor_Series_Cheat_Sheet.pdf))  (See [Harold’s Infinite Series Cheat Sheet](https://www.toomey.org/tutor/harolds_cheat_sheets/Harolds_Infinite_Series_Cheat_Sheet.pdf)) |
| **Taylor Series** | where  ( is the worst-case scenario or max. value of in the range.)  and |