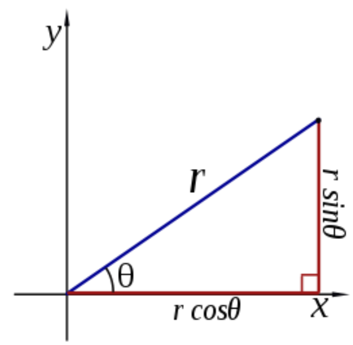
Harold's AP Calculus BC Formulas
(Rectangular – Polar – Parametric)
Cheat Sheet
6 November 2025

	Rectangular	Polar	Parametric
Point	$f(x) = y$ (x, y) (a, b)	$(r\,,\theta)$ *or* $r\,\angle\,\theta$ *Polar → Rect.* $x = r\cos\theta$ $y = r\sin\theta$ $tan\,\theta = \left(\frac{y}{x}\right)$ *Rect. → Polar* $r^2 = x^2 + y^2$ $r = \sqrt{x^2 + y^2}$ $\theta = tan^{-1}\left(\frac{y}{x}\right)$	*Point (a, b) in Rectangular:* $x(t) = a$ $y(t) = b$ $< a, b >$ $t = 3^{rd}$*, variable, usually time, with 1 degree of freedom (df)*
Line	*Slope-Intercept Form:* $y = mx + b$ *Point-Slope Form:* $y - y_0 = m\,(x - x_0)$ *Intercept Form:* $\frac{x}{a} + \frac{y}{b} = 1$ *Normal Form:* $Ax + By + C = 0$ *Calculus Form:* $f(x) = f'(a)\,x + f(0)$		$< x, y > = < x_0, y_0 > + t < a, b >$ $< x, y > = < x_0 + at, y_0 + bt >$ *where* $< a, b > = < x_2 - x_1,\, y_2 - y_1 >$ $x(t) = x_0 + ta$ $y(t) = y_0 + tb$ $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{a}$

	Rectangular	Polar	Parametric
Plane	*Dot Product of Point-Normal Form:* $n_x(x - x_0) + n_y(y - y_0) + n_z(z - z_0) = 0$ where: $\boldsymbol{n} = \langle n_x, n_y, n_z \rangle = \langle a, b, c \rangle$ *is orthogonal (perpendicular) to the plane* *General Form:* $Ax + By + Cz + D = 0$ *Intercept Form:* $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$	*Vector Form:* $\boldsymbol{n} \bullet (\boldsymbol{r} - \boldsymbol{r}_0) = 0$	$\boldsymbol{r} = \boldsymbol{r}_0 + s\boldsymbol{v} + t\boldsymbol{w}$ *Parametric Form:* $x = x_0 + su_1 + tv_1$ $y = y_0 + su_2 + tv_2$ $z = z_0 + su_3 + tv_3$ *where:* • (x_0, y_0, z_0) *is a point on the plane.* • $\langle u_1, u_2, u_3 \rangle$ *and* $\langle v_1, v_2, v_3 \rangle$ *are direction vectors on the plane.* • *s and t are parameters that vary over all real numbers.*

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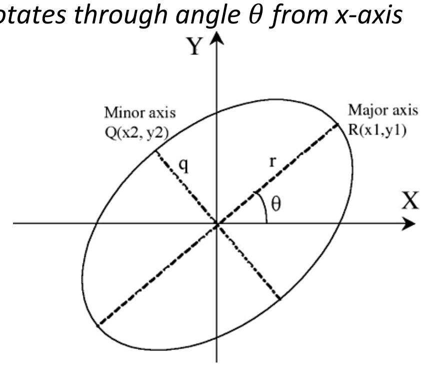
<tr><th></th><th>Rectangular</th><th>Polar</th><th>Parametric</th></tr>
<tr>
<td>Conics</td>
<td>General Equation for All Conics:
$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$
where
$Line: \quad A = B = C = 0$
$Circle: \quad A = C \ and \ B = 0$
$Ellipse: \quad AC > 0$
$or \ B^2 - 4AC < 0$
$Parabola: \quad AC = 0$
$or \ B^2 - 4AC = 0$
$Hyperbola: \quad AC < 0$
$or \ B^2 - 4AC > 0$
Note: If $A + C = 0$, square hyperbola

Rotation:
If B ≠ 0, then rotate the coordinate system:
$$\cot 2\theta = \frac{A - C}{B}$$
$$x = x' \cos\theta - y' \sin\theta$$
$$y = y' \cos\theta + x' \sin\theta$$
New = (x’, y’), Old = (x, y)
rotates through angle θ from x-axis

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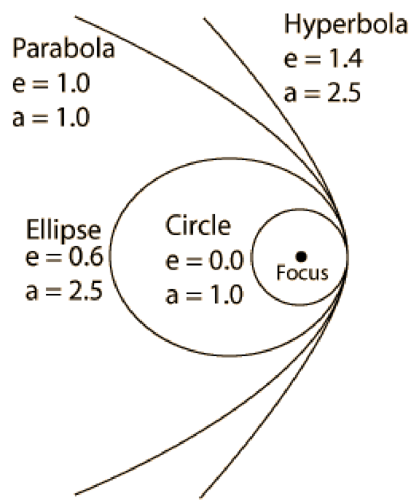
<td>General Equation for All Conics:
$$r = \frac{p}{1 - e \cos\theta}$$
$$where \ p = \begin{cases} a\,(1 - e^2) \\ 2d \\ a\,(e^2 - 1) \end{cases} for \begin{cases} 0 \le e < 1 \\ e = 1 \\ e > 1 \end{cases}$$
p = semi-latus rectum
or the line segment running from the focus to the curve in a direction parallel to the directrix

Eccentricity:
$Circle \quad e = 0$
$Ellipse \quad 0 < e < 1$
$Parabola \quad e = 1$
$Hyperbola \quad e > 1$

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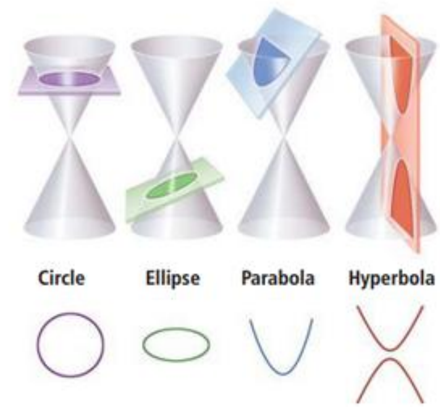
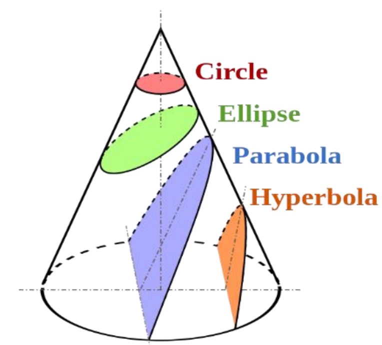
<td>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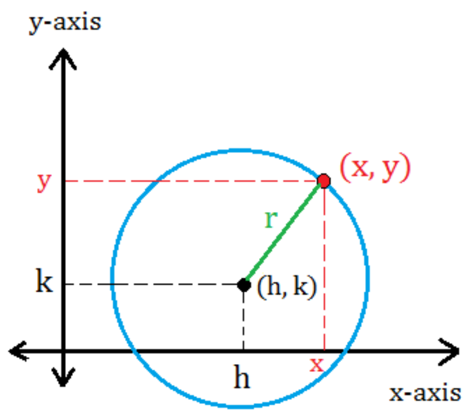
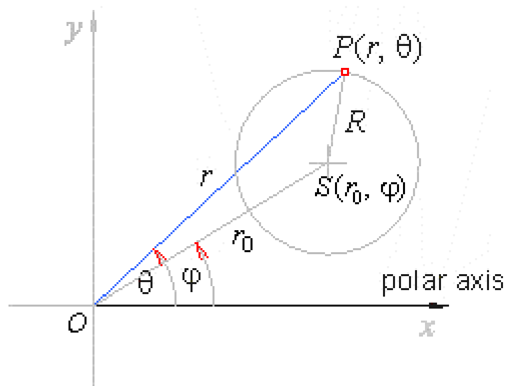
	Rectangular	Polar	Parametric
Circle	$x^2 + y^2 = r^2$ $(x-h)^2 + (y-k)^2 = r^2$ *Center:* (h, k) *Vertices: NA* *Focus:* (h, k)  	*Centered at Origin:* *r = a (constant)* $\theta = \theta \;\; [0, 2\pi]$ *or* $[0, 360°]$ *Centered at* (r_0, ϕ)*:* $r^2 + r_0^2 - 2rr_0\cos(\theta - \phi) = R^2$ *Hint: Law of Cosines* *or* $r = r_0\cos(\theta - \phi) + \sqrt{a^2 - r_0^2\, sin^2(\theta - \phi)}$  	$x(t) = r\,cos(t) + h$ $y(t) = r\,sin(t) + k$ $[t_{min}, t_{max}] = [0, 2\pi]$ *Center:* (h, k) *Focus:* (h, k)

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<td>Ellipse</td>
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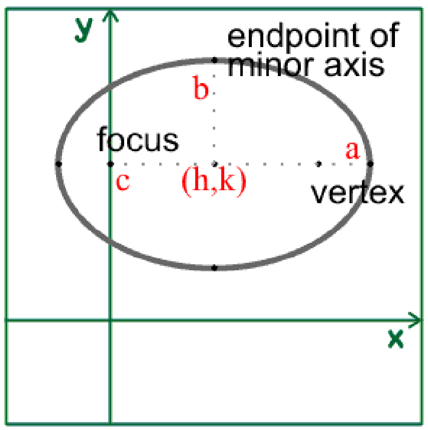
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Center: (h,k)
Vertices: $(h\pm a,k)$
Co-Vertices: $(h,k\pm b)$
Foci: $(h\pm c,k)$

Focus length, c, from center:
$c^2 = a^2-b^2$



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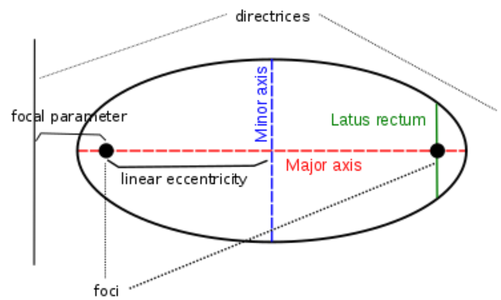
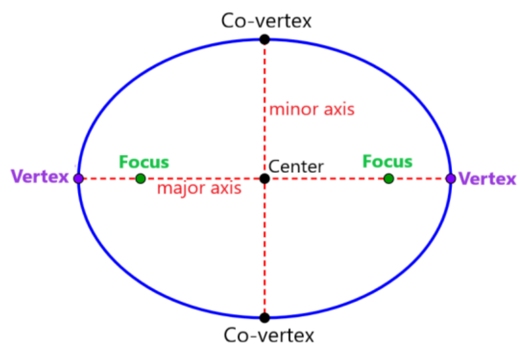
Ellipse:

$$r=\frac{a\,(1-e^2)}{1+e\cos\theta}\quad for\ 0<e<1$$

$$where\ e=\frac{c}{a}=\frac{\sqrt{a^2-b^2}}{a}$$

relative to center (h,k)





Interesting Note:
The <u>sum</u> of the distances from each focus to a point on the curve is constant.
$|d_1+d_2|=k$

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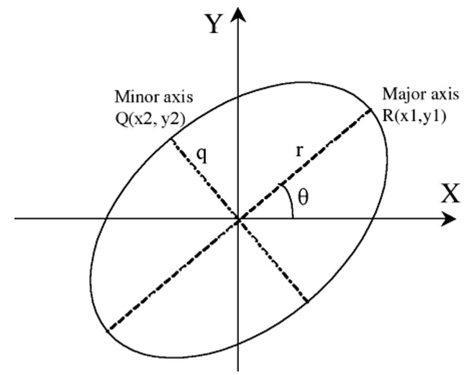
$x(t)=a\cos(t)\ +h$
$y(t)=b\sin(t)+\ k$
$[t_{min},\ t_{max}]=[0,2\pi]$

Center: (h,k)

Rotated Ellipse:
$x(t)=a\cos t\cos\theta-b\sin t\sin\theta+h$
$y(t)=a\cos t\sin\theta+b\sin t\cos\theta+k$

θ = the angle between the x-axis and the major axis of the ellipse



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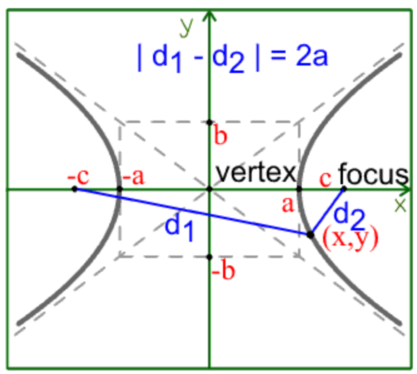
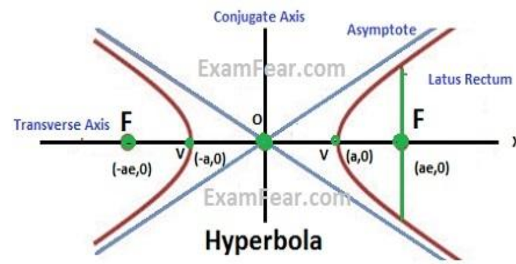
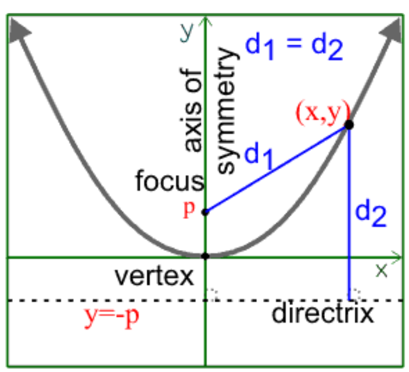
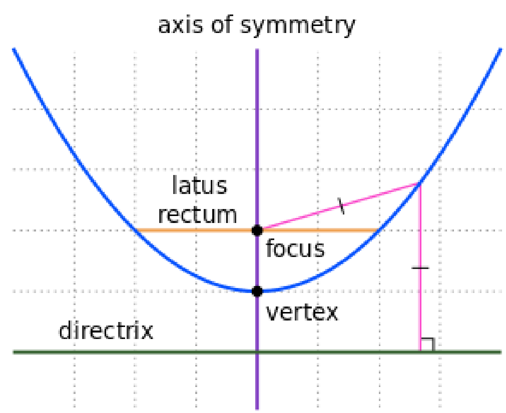
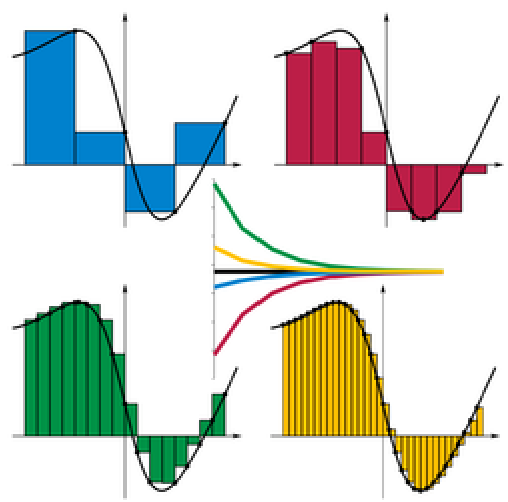
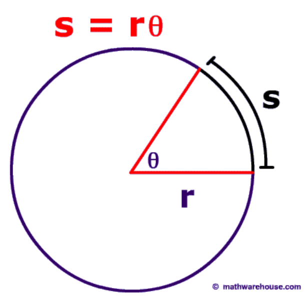
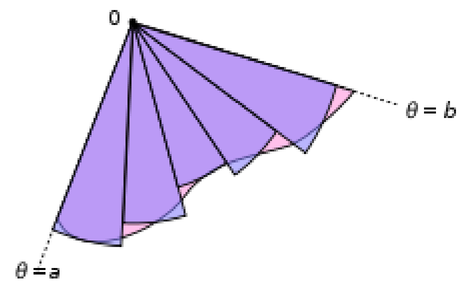
	Rectangular	Polar	Parametric
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ *Center:* (h, k) *Vertices:* $(h \pm a, k)$ *Foci:* $(h \pm c, k)$ *Focus length, c, from center:* $c^2 = a^2 + b^2$  ***Interesting Note:*** *The <u>difference</u> between the distances from each focus to a point on the curve is constant.* $\|d_1 - d_2\| = k$	*Vertical Axis of Symmetry:* $r = \frac{a\,(e^2 - 1)}{1 + e\cos\theta}\ \ for\ e > 1$ $Eccentricity: e > 1$ $where\ e = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a} = \sec\theta > 1$ *relative to center* (h, k)  *p = semi-latus rectum* *or the line segment running from the focus to the curve in the directions* $\theta = \pm\frac{\pi}{2}$	*Left-Right Opening Hyperbola:* $x(t) = a\,sec(t) + h$ $y(t) = b\,tan(t) + k$ $[t_{min}, t_{max}] = [-c, c]$ *Vertex: (h, k)* *Up-Down Opening Hyperbola:* $x(t) = a\,tan(t) + h$ $y(t) = b\,sec(t) + k$ $[t_{min}, t_{max}] = [-c, c]$ *Vertex: (h, k)* *General Form:* $x(t) = At^2 + Bt + C$ $y(t) = Dt^2 + Et + F$ *where A and D have different signs*

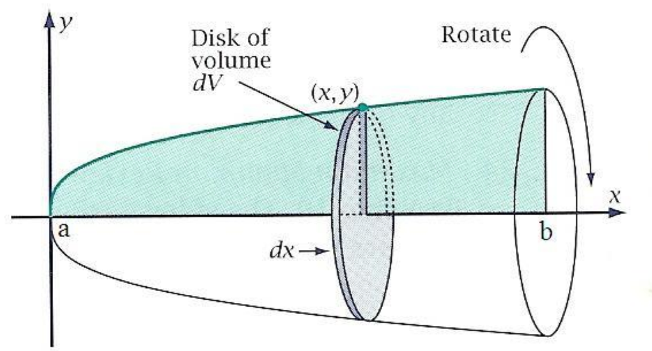
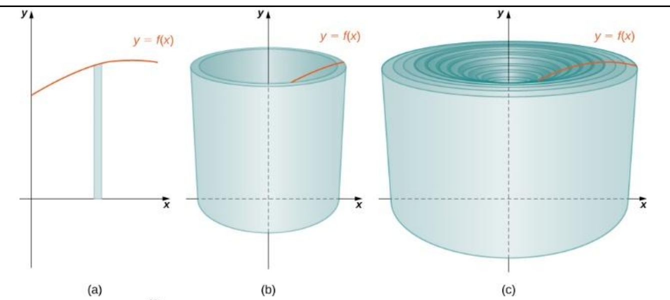
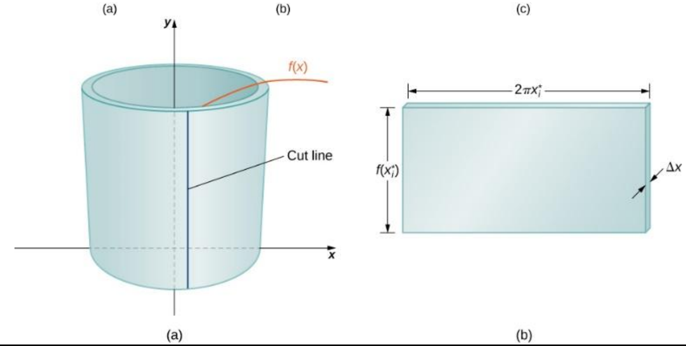
	Rectangular	Polar	Parametric
Parabola	$y = ax^2 + bx + c$ $y = (x-h)^2 + k$ *Vertical Axis of Symmetry:* $x^2 = 4\,py$ $(x-h)^2 = 4p(y-k)$ *Vertex:* (h, k) *Focus:* $(h, k+p)$ *Directrix:* $y = k - p$ *Horizontal Axis of Symmetry:* $y^2 = 4\,px$ $(y-k)^2 = 4p(x-h)$ *Vertex:* (h, k) *Focus:* $(h+p, k)$ *Directrix:* $x = h - p$  	*Vertical Axis of Symmetry:* $r = \dfrac{2d}{1 + e\cos\theta}$ $Eccentricity: e = 1$ *and* $d = 2p$ *Trigonometric Form:* $y = x^2$ $r\sin\theta = r^2\ \cos^2\theta$ $r = \dfrac{\sin\theta}{\cos^2\theta} = \tan\theta\sec\theta$  ***Interesting Note:*** *The distances from a point on the curve to the focus is the <u>same</u> as to the directrix.*	*Vertical Axis of Symmetry:* $x(t) = 2pt + h$ $y(t) = pt^2 + k$ *(opens upwards)* $y(t) = -pt^2 - k$ *(opens downwards)* $[t_{min}, t_{max}] = [-c, c]$ *Vertex:* (h, k) *Horizontal Axis of Symmetry:* $y(t) = 2pt + k$ $x(t) = pt^2 + h$ *(opens to the right)* $x(t) = -pt^2 - h$ *(opens to the left)* $[t_{min}, t_{max}] = [-c, c]$ *Vertex:* (h, k) *Projectile Motion:* $x(t) = x_0 + v_x t + \left(\frac{1}{2}\right) a_x t^2$ $y(t) = y_0 + v_{y0}t - 16t^2$ *feet* $y(t) = y_0 + v_{y0}t - 4.9t^2$ *meters* $v_x = v\cos\theta$ $v_y = v\sin\theta$ *General Form:* $x = At^2 + Bt + C$ $y = Dt^2 + Et + F$ *where A and D have the same sign*

	Rectangular	Polar	Parametric
1st Derivative	$f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(c)=\lim\limits_{x\to c}\frac{f(x)-f(c)}{x-c}$ $f'(x)=\frac{dy}{dx}=y'=D_x$	$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{\frac{dr}{d\theta}\sin\theta+r\cos\theta}{\frac{dr}{d\theta}\cos\theta-r\sin\theta}$ *Hint: Use Product Rule for* $y=r\sin\theta$ $x=r\cos\theta$	$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$, *provided* $\frac{dx}{dt}\neq 0$
2nd Derivative	$f''(x)=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}=y''=D_{xx}$	$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{\frac{d}{d\theta}\left(\frac{dy}{dx}\right)}{\frac{dx}{d\theta}}$	$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}=\frac{\frac{d}{dt}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}}$
Integral	*Fundamental Theorem of Calculus:* $F(x)=\int_a^b f(x)\,dx=F(b)-F(a)$		*Riemann Sum:* $S=\sum_{i-1}^{n} f(y_i)(x_i-x_{i-1})$ *Left Sum:* $S=\left(\frac{1}{n}\right)\left[f(a)+f\left(a+\frac{1}{n}\right)+f\left(a+\frac{2}{n}\right)+\cdots+f(b-\frac{1}{n})\right]$ *Middle Sum:* $S=\left(\frac{1}{n}\right)\left[f\left(a+\frac{1}{2n}\right)+f\left(a+\frac{3}{2n}\right)+\cdots+f(b-\frac{1}{2n})\right]$ *Right Sum:* $S=\left(\frac{1}{n}\right)\left[f\left(a+\frac{1}{n}\right)+f\left(a+\frac{2}{n}\right)+\cdots+f(b)\right]$

	Rectangular	Polar	Parametric
Inverse Functions	$f(f^{-1}(x)) = f^{-1}(f(x)) = x$ *Inverse Function Theorem:* $f^{-1}(f'(a)) = \frac{1}{f'(a)}$	$if\ y = sin\,\theta \quad then\ \theta = sin^{-1} y$ $if\ y = cos\,\theta \quad then\ \theta = cos^{-1} y$ $if\ y = tan\,\theta \quad then\ \theta = tan^{-1} y$ $if\ y = csc\,\theta \quad then\ \theta = csc^{-1} y$ $if\ y = sec\,\theta \quad then\ \theta = sec^{-1} y$ $if\ y = cot\,\theta \quad then\ \theta = cot^{-1} y$	$or\ \theta = \arcsin y$ $or\ \theta = \arccos y$ $or\ \theta = arctan\, y$ $or\ \theta = arccsc\, y$ $or\ \theta = arcsec\, y$ $or\ \theta = arccot\, y$
Arc Length	$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$ *Proof:* $\Delta s = \sqrt{(x - x_0)^2 + (y - y_0)^2}$ $\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ $ds = \sqrt{dx^2 + dy^2}$ $ds = \sqrt{dx^2 + dy^2 \left(\frac{dx^2}{dx^2}\right)}$ $ds = \sqrt{dx^2 + \left(\frac{dy}{dx}\right)^2 dx^2}$ $ds = \sqrt{dx^2 \left(1 + \left(\frac{dy}{dx}\right)^2\right)}$ $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ $L = \int ds$	$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ *Circle:* $L = s = r\theta$ *Proof:* $L = (fraction\ of\ circumference) \cdot \pi \cdot (diameter)$ $L = \left(\frac{\theta}{2\pi}\right)\pi\,(2r) = r\theta$  	$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ $L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$ *Proof:* $ds = \sqrt{dx^2 + dy^2}$ $ds = \sqrt{dx^2 \left(\frac{dt^2}{dt^2}\right) + dy^2 \left(\frac{dt^2}{dt^2}\right)}$ $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 dt^2}$ $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ $L = \int ds$
Perimeter	*Square:* $P = 4s$ *Rectangle:* $P = 2l + 2w$ *Triangle:* $P = a + b + c$ *Circle:* $C = \pi d = 2\pi r$ *Ellipse:* $C \approx \pi(a + b)$	*Ellipse:* $C \approx 2\pi\sqrt{\frac{a^2+b^2}{2}}$ $C \approx \pi\left[3(a + b) - \sqrt{(3a + b)(a + 3b)}\right]$ $C \approx \pi\,(a + b)\left(1 + \frac{3h}{10 + \sqrt{4 - 3h}}\right)$	*Ellipse:* $C = 4a\int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 sin^2\theta}\, d\theta$ $h = \frac{(a - b)^2}{(a + b)^2}$ & $k^2 = \left(1 - \frac{b^2}{a^2}\right)$

	Rectangular	Polar	Parametric
Area	*Square:* $A = s^2$ *Rectangle:* $A = lw$ *Rhombus:* $A = ½\, ab$ *Parallelogram:* $A = Bh$ *Trapezoid:* $A = \frac{(B_1 + B_2)}{2}\, h$ *Kite:* $A = \frac{d_1\, d_2}{2}$ *Triangle:* $A = ½\, Bh$ *Triangle:* $A = ½\, ab \sin(C)$ *Triangle using Heron's Formula:* $A = \sqrt{s(s-a)(s-b)(s-c)}$ $where\ s = \frac{a+b+c}{2}$ *Equilateral Triangle:* $A = ¼\sqrt{3}s^2$ *Frustum:* $A = \frac{1}{3}\left(\frac{B_1+B_2}{2}\right) h$ *Circle:* $A = \pi r^2$ *Circular Sector:* $A = ½\, r^2\theta$ *Ellipse:* $A = \pi ab$	$A = \int_{\alpha}^{\beta} \frac{1}{2}[f(\theta)]^2 d\theta$ $where\ \ r = f(\theta)$  *Area of a sector where arc length* $s = r\theta$*:* $A = \int s\ dr = \int r\theta\ dr = \frac{1}{2}r^2\theta$	$A = \int_{\alpha}^{\beta} g(t)\, f'(t)\, dt$ *where* $f(t) = x$ *and* $g(t) = y$ *or* $x(t) = f(t)\ and\ y(t) = g(t)$ *Simplified:* $A = \int_{\alpha}^{\beta} y(t)\, \frac{dx(t)}{dt}\, dt$ *Proof:* $\int_{a}^{b} f(x)\ dx$ $y = f(x) = g(t)$ $dx = \frac{df(t)}{dt} dt = f'(t)\, dt$
Lateral Surface Area	*Cylinder:* $SA = 2\pi rh$ *Cone:* $SA = \pi rl$ $SA = 2\pi \int_{a}^{b} f(x)\sqrt{1 + [f'(x)]^2}\, dx$	*For rotation about the x-axis:* $SA = \int 2\pi y\ \ ds$ *For rotation about the y-axis:* $SA = \int 2\pi x\ \ ds$ $ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ \ d\theta$ $r = f(\theta), \quad \alpha \le \theta \le \beta$	*For rotation about the x-axis:* $SA = \int 2\pi y\ \ ds$ *For rotation about the y-axis:* $SA = \int 2\pi x\ \ ds$ $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ \ dt$ $if\ x = f(t), y = g(t), \alpha \le t \le \beta$

	Rectangular	Polar	Parametric
Total Surface Area	*Cube:* $SA = 6s^2$ *Rectangular Box:* $SA = 2lw + 2wh + 2hl$ *Regular Tetrahedron:* $SA = 2bh$ *Cylinder:* $SA = 2\pi r\,(r + h)$ *Cone:* $SA = \pi r^2 + \pi rl = \pi r\,(r + l)$ *Sphere:* $SA = 4\pi r^2$ *Ellipsoid:* $SA \approx 4\pi \left(\frac{a^p b^p + a^p c^p + b^p c^p}{3}\right)^{1/p}$ *Where* $p \approx 1.6075,\ \lvert Relative\ Error \rvert \leq 1.061\%$ *(Knud Thomsen's Formula)*		
Surface of Revolution	*For revolution about the x-axis:* $A = 2\pi \int_a^b f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$ *For revolution about the y-axis:* $A = 2\pi \int_a^b x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$	*For revolution about the x-axis:* $A = 2\pi r \int_\alpha^\beta \cos\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$ *For revolution about the y-axis:* $A = 2\pi r \int_\alpha^\beta \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$	*For revolution about the x-axis:* $A = 2\pi \int_a^b y(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$ *For revolution about the y-axis:* $A = 2\pi \int_a^b x(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$
Volume	*Cube:* $V = s^3$ *Rectangular Prism:* $V = lwh$ *Cylinder:* $V = \pi r^2 h$ *Triangular Prism:* $V = Bh$ *Tetrahedron:* $V = ⅓\,Bh$ *Pyramid:* $V = ⅓\,Bh = ⅓\,lwh$ *Cone:* $V = ⅓\,Bh = ⅓\,\pi r^2 h$ *Sphere:* $V = \frac{4}{3}\pi r^3$ *Ellipsoid:* $V = \frac{4}{3}\,\pi abc$		

	Rectangular	Polar	Parametric
Volume of Revolution	**Disk Method** $V = \int_a^b (area\ of\ circle)\ d(thickness)$ *Rotation about the x-axis:* $V = \int_a^b \pi\, [f(x)]^2 dx$ *Rotation about the y-axis:* $V = \int_c^d \pi x^2 dy$	 	
	Washer Method *Rotation about the x-axis:* $V = \int_a^b \pi\, \{[f(x)]^2 - [g(x)]^2\}\, dx$	$V = V_{Outer\ Disk} - V_{Inner\ Disk}$	
	Shell Method $V = \int_a^b (circumference)\ (hight)\ dx$ *Rotation about the y-axis:* $V = \int_a^b 2\pi x\, f(x)\, dx$ *Rotation about the x-axis:* $V = \int_c^d 2\pi y\, g(y)\, dy$	  	

	Rectangular	Polar	Parametric

