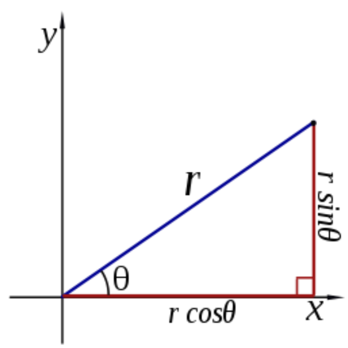
Harold’s AP Calculus BC
Cheat Sheet
24 January 2025

	Rectangular	Polar	Parametric
Point	$f(x) = y$ (x, y) (a, b)	$(r\,, \theta)$ *or* $r \angle \theta$ *Polar → Rect.* $x = r\cos\theta$ $y = r\,sin\,\theta$ $tan\,\theta = \frac{y}{x}$ *Rect. → Polar* $r^2 = x^2 + y^2$ $r = \sqrt{x^2 + y^2}$ $\theta = tan^{-1}\left(\frac{y}{x}\right)$	*Point* (a, b) *in Rectangular:* $x(t) = a$ $y(t) = b$ $< a, b >$ $t = 3^{rd}$*, variable, usually time, with 1 degree of freedom (df)*
Line	*Slope-Intercept Form:* $y = mx + b$ *Point-Slope Form:* $y - y_0 = m\,(x - x_0)$ *Intercept Form:* $\frac{x}{a} + \frac{y}{b} = 1$ *Normal Form:* $Ax + By + C = 0$ *Calculus Form:* $f(x) = f'(a)\,x + f(0)$		$< x, y > = < x_0, y_0 > + t < a, b >$ $< x, y > = < x_0 + at, y_0 + bt >$ *where* $< a, b > = < x_2 - x_1,\ y_2 - y_1 >$ $x(t) = x_0 + ta$ $y(t) = y_0 + tb$ $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{a}$

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Plane	*Dot Product of Point-Normal Form:* $n_x(x - x_0) + n_y(y - y_0) + n_z(z - z_0) = 0$ where: $\boldsymbol{n} = \langle n_x, n_y, n_z \rangle = \langle a, b, c \rangle$ *is orthogonal (perpendicular) to the plane* *General Form:* $Ax + By + Cz + D = 0$ *Intercept Form:* $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$	*Vector Form:* $\boldsymbol{n} \bullet (\boldsymbol{r} - \boldsymbol{r}_0) = 0$	$\boldsymbol{r} = \boldsymbol{r}_0 + s\boldsymbol{v} + t\boldsymbol{w}$ *Parametric Form:* $x = x_0 + su_1 + tv_1$ $y = y_0 + su_2 + tv_2$ $z = z_0 + su_3 + tv_3$ *where:* • (x_0, y_0, z_0) *is a point on the plane.* • $\langle u_1, u_2, u_3 \rangle$ *and* $\langle v_1, v_2, v_3 \rangle$ *are direction vectors on the plane.* • *s and t are parameters that vary over all real numbers.*

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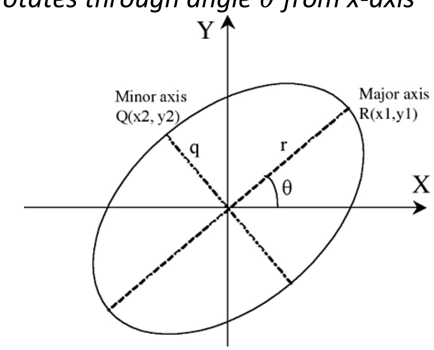
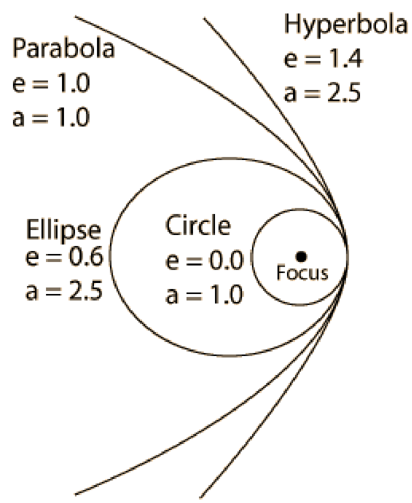
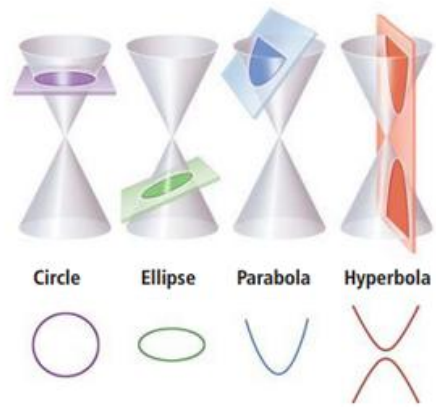
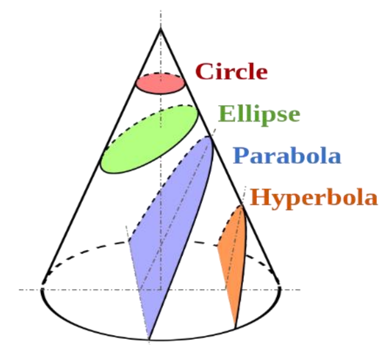
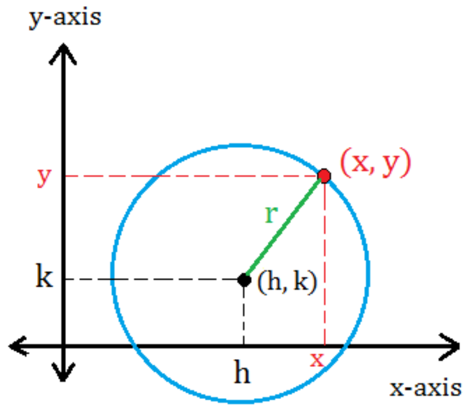
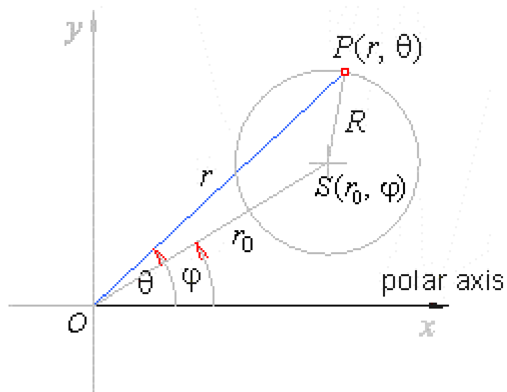
Conics	*General Equation for All Conics:* $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ *where* $Line$: $A = B = C = 0$ $Circle$: $A = C \ and \ B = 0$ $Ellipse$: $AC > 0$ $or \ B^2 - 4AC < 0$ $Parabola$: $AC = 0$ $or \ B^2 - 4AC = 0$ $Hyperbola$: $AC < 0$ $or \ B^2 - 4AC > 0$ *Note: If* $A + C = 0$*, square hyperbola* *Rotation:* *If B ≠ 0, then* [*rotate*]() *the coordinate system:* $\cot 2\theta = \frac{A - C}{B}$ $x = x' \cos\theta - y' \sin\theta$ $y = y' \cos\theta + x' \sin\theta$ *New = (x', y'), Old = (x, y)* *rotates through angle* θ *from x-axis*  	*General Equation for All Conics:* $r = \frac{p}{1 - e \cos\theta}$ $where \ p = \begin{cases} a\,(1 - e^2) \\ 2d \\ a\,(e^2 - 1) \end{cases} for \begin{cases} 0 \le e < 1 \\ e = 1 \\ e > 1 \end{cases}$ *p = semi-latus rectum* *or the line segment running from the focus to the curve in a direction parallel to the directrix* *Eccentricity:* $Circle$ $e = 0$ $Ellipse$ $0 < e < 1$ $Parabola$ $e = 1$ $Hyperbola$ $e > 1$  	 

	Rectangular	Polar	Parametric
Circle	$x^2 + y^2 = r^2$ $(x-h)^2 + (y-k)^2 = r^2$ *Center:* (h,k) *Vertices: NA* *Focus:* (h,k)  	*Centered at Origin:* *r = a (constant)* $\theta = \theta \;\; [0, 2\pi]$ *or* $[0, 360°]$ *Centered at* (r_0, ϕ)*:* $r^2 + r_0^2 - 2rr_0\cos(\theta - \phi) = R^2$ *Hint: Law of Cosines* *or* $r = r_0\cos(\theta - \phi) + \sqrt{a^2 - r_0^2\,sin^2(\theta - \phi)}$  	$x(t) = r\,cos(t) + h$ $y(t) = r\,sin(t) + k$ $[t_{min}, t_{max}] = [0, 2\pi]$ *Center:* (h,k) *Focus:* (h,k)

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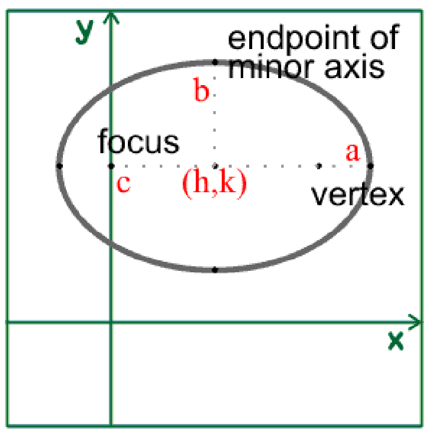
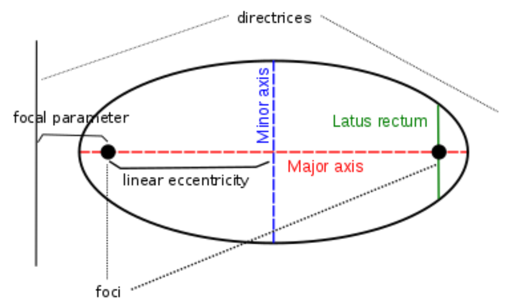
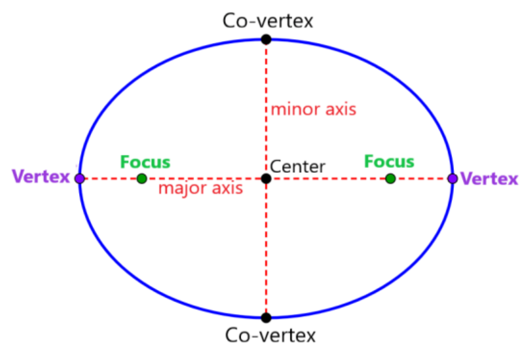
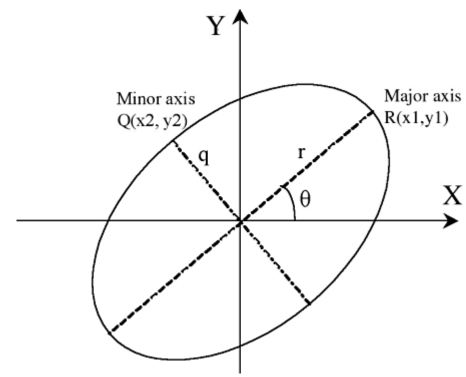
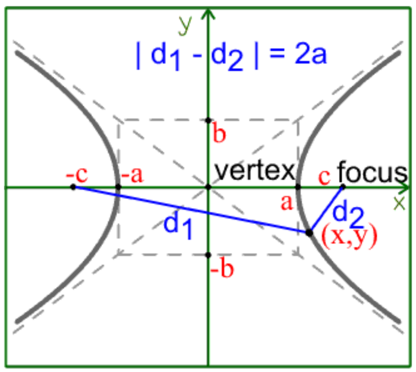
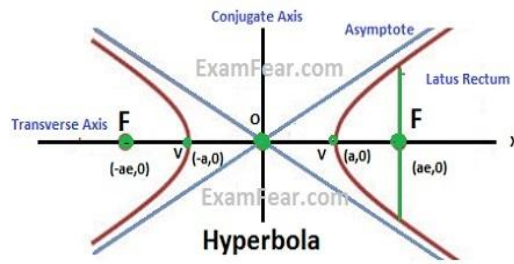
Ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ *Center:* (h,k) *Vertices:* $(h\pm a,k)$ *Co-Vertices:* $(h,k\pm b)$ *Foci:* $(h\pm c,k)$ *Focus length, c, from center:* $c^2 = a^2-b^2$ 	*Ellipse:* $r=\frac{a\,(1-e^2)}{1+e\cos\theta}\;\; for\; 0<e<1$ $where\; e=\frac{c}{a}=\frac{\sqrt{a^2-b^2}}{a}$ *relative to center* (h,k)   ***Interesting Note:*** *The <u>sum</u> of the distances from each focus to a point on the curve is constant.* $\lvert d_1+d_2\rvert=k$	$x(t)=a\cos(t)\;+h$ $y(t)=b\sin(t)+\;k$ $[t_{min},\;t_{max}]=[0,2\pi]$ *Center:* (h,k) *Rotated Ellipse:* $x(t)=a\cos t\cos\theta-b\sin t\sin\theta+h$ $y(t)=a\cos t\sin\theta+b\sin t\cos\theta+k$ θ *= the angle between the x-axis and the major axis of the ellipse* 

	Rectangular	Polar	Parametric
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ *Center:* (h, k) *Vertices:* $(h \pm a, k)$ *Foci:* $(h \pm c, k)$ *Focus length, c, from center:* $c^2 = a^2 + b^2$  ***Interesting Note:*** *The <u>difference</u> between the distances from each focus to a point on the curve is constant.* $\|d_1 - d_2\| = k$	*Vertical Axis of Symmetry:* $r = \frac{a\,(e^2-1)}{1 + e\cos\theta}\ \ for\ e > 1$ $Eccentricity: e > 1$ $where\ e = \frac{c}{a} = \frac{\sqrt{a^2+b^2}}{a} = \sec\theta > 1$ *relative to center* (h, k)  *p = semi-latus rectum* *or the line segment running from the focus to the curve in the directions* $\theta = \pm\frac{\pi}{2}$	*Left-Right Opening Hyperbola:* $x(t) = a\,sec(t) + h$ $y(t) = b\,tan(t) + k$ $[t_{min}, t_{max}] = [-c, c]$ *Vertex: (h, k)* *Up-Down Opening Hyperbola:* $x(t) = a\,tan(t) + h$ $y(t) = b\,sec(t) + k$ $[t_{min}, t_{max}] = [-c, c]$ *Vertex: (h, k)* *General Form:* $x(t) = At^2 + Bt + C$ $y(t) = Dt^2 + Et + F$ *where A and D have different signs*

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<td>Parabola</td>
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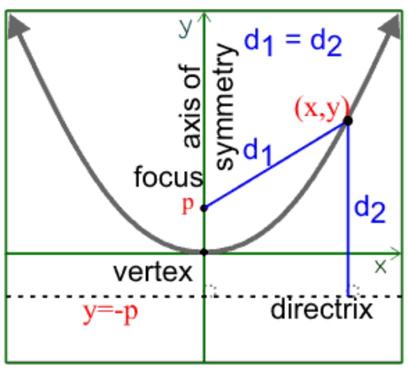
$y = ax^2 + bx + c$

$y = (x-h)^2 + k$

Vertical Axis of Symmetry:
$x^2 = 4\,py$
$(x-h)^2 = 4p(y-k)$
Vertex: (h,k)
Focus: $(h, k+p)$
Directrix: $y = k - p$

Horizontal Axis of Symmetry:
$y^2 = 4\,px$
$(y-k)^2 = 4p(x-h)$
Vertex: (h,k)
Focus: $(h+p, k)$
Directrix: $x = h - p$



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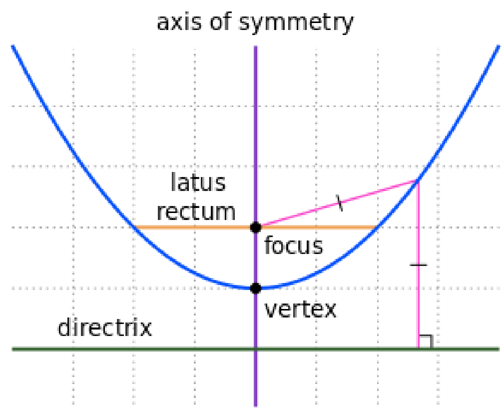
Vertical Axis of Symmetry:
$r = \dfrac{2d}{1 + e\cos\theta}$

$Eccentricity: e = 1$
and $d = 2p$

Trigonometric Form:
$y = x^2$
$r\sin\theta = r^2\ \cos^2\theta$
$r = \dfrac{\sin\theta}{\cos^2\theta} = \tan\theta\sec\theta$



Interesting Note:
The distances from a point on the curve to the focus is the <u>same</u> as to the directrix.

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Vertical Axis of Symmetry:
$x(t) = \ 2pt + \ h$
$y(t) = pt^2 + k$ *(opens upwards)*
$y(t) = -pt^2 - k$ *(opens downwards)*
$[t_{min},\ t_{max}] = [-c, c]$
Vertex: (h,k)

Horizontal Axis of Symmetry:
$y(t) = \ 2pt + \ k$
$x(t) = pt^2 + h$ *(opens to the right)*
$x(t) = -pt^2 - h$ *(opens to the left)*
$[t_{min},\ t_{max}] = [-c, c]$
Vertex: (h,k)

Projectile Motion:
$x(t) = x_0 + v_x t + \left(\frac{1}{2}\right) a_x t^2$
$y(t) = y_0 + v_{y0}t - 16t^2$ *feet*
$y(t) = y_0 + v_{y0}t - 4.9t^2$ *meters*
$v_x = v\cos\theta$
$v_y = v\sin\theta$

General Form:
$x = At^2 + \ Bt + C$
$y = Dt^2 + \ Et + F$
where A and D have the same sign

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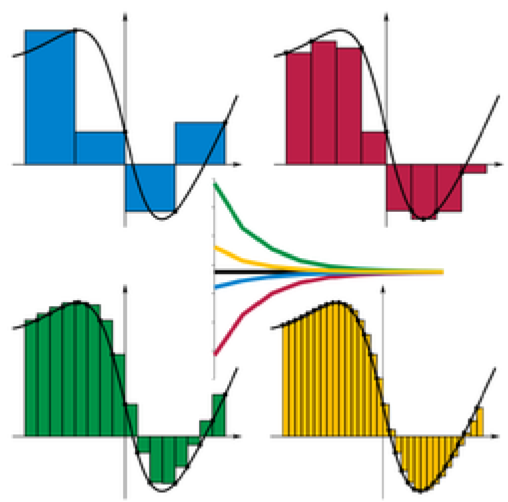
	Rectangular	Polar	Parametric
1st Derivative	$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(c) = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ $f'(x) = \frac{dy}{dx} = y' = D_x$	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$ *Hint: Use Product Rule for* $y = r\sin\theta$ $x = r\cos\theta$	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad provided \frac{dx}{dt} \neq 0$
2nd Derivative	$f''(x) = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = y'' = D_{xx}$	$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{d\theta}\left(\frac{dy}{dx}\right)}{\frac{dx}{d\theta}}$	$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)}{\frac{dx}{dt}}$
Integral	*Fundamental Theorem of Calculus:* $F(x) = \int_a^b f(x)\,dx = F(b) - F(a)$		*Riemann Sum:* $S = \sum_{i-1}^{n} f(y_i)(x_i - x_{i-1})$ *Left Sum:* $S = \left(\frac{1}{n}\right)\left[f(a) + f\left(a+\frac{1}{n}\right) + f\left(a+\frac{2}{n}\right) + \cdots + f(b-\frac{1}{n})\right]$ *Middle Sum:* $S = \left(\frac{1}{n}\right)\left[f\left(a+\frac{1}{2n}\right) + f\left(a+\frac{3}{2n}\right) + \cdots + f(b-\frac{1}{2n})\right]$ *Right Sum:* $S = \left(\frac{1}{n}\right)\left[f\left(a+\frac{1}{n}\right) + f\left(a+\frac{2}{n}\right) + \cdots + f(b)\right]$

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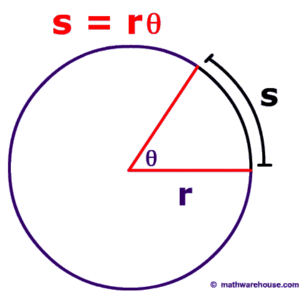
Inverse Functions	$f(f^{-1}(x)) = f^{-1}(f(x)) = x$ *Inverse Function Theorem:* $f^{-1}(f'(a)) = \dfrac{1}{f'(a)}$	$if\ \ y = sin\,\theta \quad then\ \theta = sin^{-1} y$ $if\ \ y = cos\,\theta \quad then\ \theta = cos^{-1} y$ $if\ \ y = tan\,\theta \quad then\ \theta = tan^{-1} y$ $if\ \ y = csc\,\theta \quad then\ \theta = csc^{-1} y$ $if\ \ y = sec\,\theta \quad then\ \theta = sec^{-1} y$ $if\ \ y = cot\,\theta \quad then\ \theta = cot^{-1} y$	$or\ \ \theta = \arcsin y$ $or\ \ \theta = \arccos\ y$ $or\ \ \theta = arctan\,y$ $or\ \ \theta = arccsc\,y$ $or\ \ \theta = arcsec\,y$ $or\ \ \theta = arccot\,y$
Arc Length	$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$ *Proof:* $\Delta s = \sqrt{(x - x_0)^2 + (y - y_0)^2}$ $\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ $ds = \sqrt{dx^2 + dy^2}$ $ds = \sqrt{dx^2 + dy^2 \left(\dfrac{dx^2}{dx^2}\right)}$ $ds = \sqrt{dx^2 + \left(\dfrac{dy}{dx}\right)^2 dx^2}$ $ds = \sqrt{dx^2 \left(1 + \left(\dfrac{dy}{dx}\right)^2\right)}$ $ds = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}\,dx$ $L = \int ds$	$L = \int \sqrt{r^2 + \left(\dfrac{dr}{d\theta}\right)^2}\,d\theta$ *Circle:* $L = s = r\theta$ *Proof:* $L = (fraction\ of\ circumference) \cdot \pi \cdot (diameter)$ $L = \left(\dfrac{\theta}{2\pi}\right)\pi\,(2r) = r\theta$  	$L = \int_\alpha^\beta \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\,dt$ $L = \int_\alpha^\beta \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2 + \left(\dfrac{dz}{dt}\right)^2}\,dt$ *Proof:* $ds = \sqrt{dx^2 + dy^2}$ $ds = \sqrt{dx^2\left(\dfrac{dt^2}{dt^2}\right) + dy^2\left(\dfrac{dt^2}{dt^2}\right)}$ $ds = \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2 dt^2}$ $ds = \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\,dt$ $L = \int ds$
Perimeter	*Square:* $P = 4s$ *Rectangle:* $P = 2l + 2w$ *Triangle:* $P = a + b + c$ *Circle:* $C = \pi d = 2\pi r$ *Ellipse:* $C \approx \pi(a + b)$	*Ellipse:* $C \approx 2\pi\sqrt{\dfrac{a^2+b^2}{2}}$ $C \approx \pi\left[3(a + b) - \sqrt{(3a + b)(a + 3b)}\right]$ $C \approx \pi\,(a + b)\left(1 + \dfrac{3h}{10 + \sqrt{4 - 3h}}\right)$	*Ellipse:* $C = 4a\int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 sin^2\theta}\,d\theta$ $h = \dfrac{(a - b)^2}{(a + b)^2}$ & $k^2 = \left(1 - \dfrac{b^2}{a^2}\right)$

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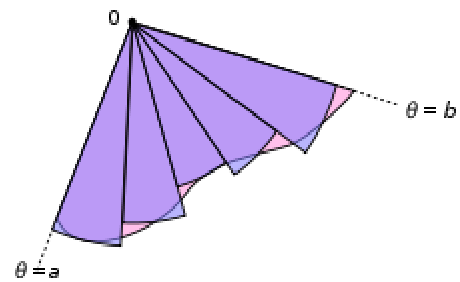
Area	*Square:* $A = s^2$ *Rectangle:* $A = lw$ *Rhombus:* $A = \frac{1}{2} ab$ *Parallelogram:* $A = Bh$ *Trapezoid:* $A = \frac{(B_1 + B_2)}{2} h$ *Kite:* $A = \frac{d_1 d_2}{2}$ *Triangle:* $A = \frac{1}{2} Bh$ *Triangle:* $A = \frac{1}{2} ab \sin(C)$ *Triangle using Heron's Formula:* $A = \sqrt{s(s-a)(s-b)(s-c)}$ $where\ s = \frac{a+b+c}{2}$ *Equilateral Triangle:* $A = \frac{1}{4}\sqrt{3}s^2$ *Frustum:* $A = \frac{1}{3}\left(\frac{B_1+B_2}{2}\right)h$ *Circle:* $A = \pi r^2$ *Circular Sector:* $A = \frac{1}{2} r^2\theta$ *Ellipse:* $A = \pi ab$	$A = \int_{\alpha}^{\beta} \frac{1}{2}[f(\theta)]^2 d\theta$ *where* $r = f(\theta)$  *Area of a sector where arc length* $s = r\theta$*:* $A = \int s\,dr = \int r\theta\,dr = \frac{1}{2}r^2\theta$	$A = \int_{\alpha}^{\beta} g(t)\, f'(t)\, dt$ *where* $f(t) = x$ *and* $g(t) = y$ *or* $x(t) = f(t)\ and\ y(t) = g(t)$ *Simplified:* $A = \int_{\alpha}^{\beta} y(t) \frac{dx(t)}{dt}\, dt$ *Proof:* $\int_{a}^{b} f(x)\ dx$ $y = f(x) = g(t)$ $dx = \frac{df(t)}{dt}dt = f'(t)\, dt$
Lateral Surface Area	*Cylinder:* $SA = 2\pi rh$ *Cone:* $SA = \pi rl$ $SA = 2\pi \int_{a}^{b} f(x) \sqrt{1 + [f'(x)]^2}\, dx$	*For rotation about the x-axis:* $SA = \int 2\pi y\ ds$ *For rotation about the y-axis:* $SA = \int 2\pi x\ ds$ $ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$ $r = f(\theta), \quad \alpha \le \theta \le \beta$	*For rotation about the x-axis:* $SA = \int 2\pi y\ ds$ *For rotation about the y-axis:* $SA = \int 2\pi x\ ds$ $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$ $if\ x = f(t), y = g(t), \alpha \le t \le \beta$

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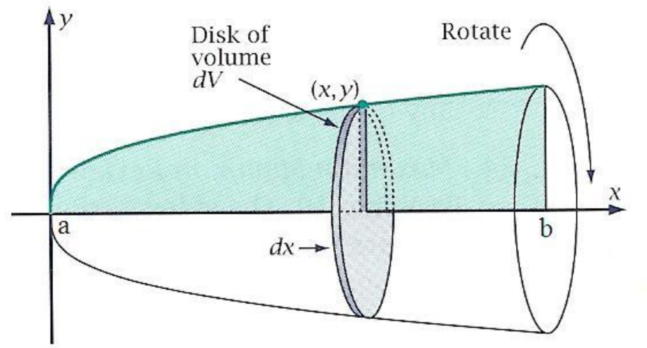
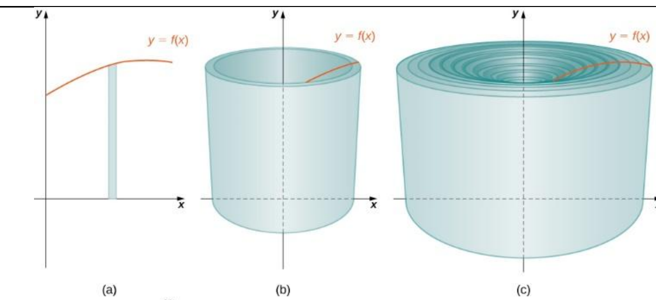
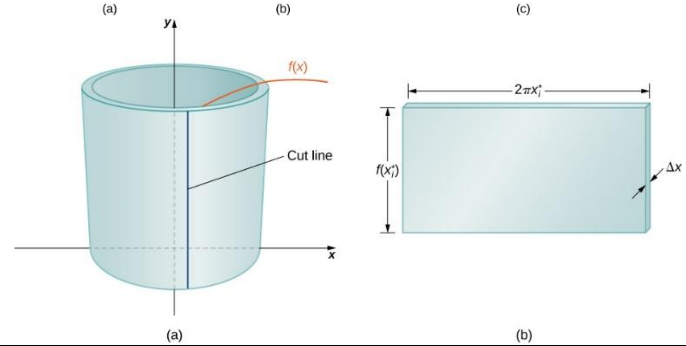
Total Surface Area	*Cube:* SA = $6s^2$ *Rectangular Box:* SA = $2lw + 2wh + 2hl$ *Regular Tetrahedron:* SA = $2bh$ *Cylinder:* SA = $2\pi r\,(r + h)$ *Cone:* SA = $\pi r^2 + \pi rl = \pi r\,(r + l)$ *Sphere:* SA = $4\pi r^2$ *Ellipsoid:* $SA \approx 4\pi\left(\frac{a^p b^p + a^p c^p + b^p c^p}{3}\right)^{1/p}$ *Where* $p \approx 1.6075$, $\lvert Relative\ Error\rvert \leq 1.061\%$ *(Knud Thomsen's Formula)*		
Surface of Revolution	*For revolution about the x-axis:* $A = 2\pi \int_a^b f(x)\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$ *For revolution about the y-axis:* $A = 2\pi \int_a^b x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$	*For revolution about the x-axis:* $A = 2\pi r \int_\alpha^\beta \cos\theta\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$ *For revolution about the y-axis:* $A = 2\pi r \int_\alpha^\beta \sin\theta\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$	*For revolution about the x-axis:* $A = 2\pi \int_a^b y(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ *For revolution about the y-axis:* $A = 2\pi \int_a^b x(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
Volume	*Cube:* $V = s^3$ *Rectangular Prism:* $V = lwh$ *Cylinder:* $V = \pi r^2 h$ *Triangular Prism:* $V = Bh$ *Tetrahedron:* $V = \frac{1}{3}Bh$ *Pyramid:* $V = \frac{1}{3}Bh = \frac{1}{3}lwh$ *Cone:* $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$ *Sphere:* $V = \frac{4}{3}\pi r^3$ *Ellipsoid:* $V = \frac{4}{3}\pi abc$		

	Rectangular	Polar	Parametric
Volume of Revolution	**Disk Method** $V = \int_a^b (area\ of\ circle)\ d(thickness)$ *Rotation about the x-axis:* $V = \int_a^b \pi\ [f(x)]^2 dx$ *Rotation about the y-axis:* $V = \int_c^d \pi x^2 dy$	 	
	Washer Method *Rotation about the x-axis:* $V = \int_a^b \pi\ \{[f(x)]^2 - [g(x)]^2\}\ dx$	$V = V_{Outer\ Disk} - V_{Inner\ Disk}$	
	Shell Method $V = \int_a^b (circumference)\ (hight)\ dx$ *Rotation about the y-axis:* $V = \int_a^b 2\pi x\ f(x)\ dx$ *Rotation about the x-axis:* $V = \int_c^d 2\pi y\ g(y)\ dy$	  	

	Rectangular	Polar	Parametric