**Harold’s Triangles Cheat Sheet**

21 January 2024

**Trig Laws and Formulas**

|  |  |  |
| --- | --- | --- |
| **Law** | **Equation** | |
| **Reference Triangle** | Picture | |
| **Law of Sines** |  |  |
| **Law of Cosines** |  | |
| **Law of Tangents** |  | |
| **Law of Cotangents** |  | |
| **Pythagorean Theorem** | If a right triangle, then  **Special Case**: Same as Law of Cosines with angle C = 90°. | |
| **Sum of Angles** |  | *radians* |
| **Area Formula** |  |  |
| **Semi-Perimeter** |  | |
| **Heron’s Formula** |  | |
| **Mollweide’s Formula** |  | |

**Solving for Angles, Sides, and Area**

|  |  |
| --- | --- |
| **Order of Use** | **Comments** |
| **1. Sum of Angles** | Easiest formula. |
| **2. Pythagorean Theorem** | Use if one of the angles is a right angle (90°). |
| **3. Law of Sines** | Least complex. Use before Law of Cosines, if possible. |
| **4. Law of Cosines** | More complex. Use only once, then use Law of Sines. |
| **5. Heron’s Formula** | Use for area if all three sides are known. |
| **6. Law of Tangents** | Very complex and seldom used. |
| **7. Law of Cotangents** | Most complex and seldom used. |

|  |  |  |  |
| --- | --- | --- | --- |
| **Given** | **Find** | | |
| **Angle** | **Side** | **Area** |
| **SSS** | Law of Cosines | Given | Heron’s Formula |
| **SAS** | NA | Law of Cosines |  |
| **SSA** | Law of Sines | NA |  |
| **ASS** |
| **SAA** | Sum of Angles | Law of Sines |
| **ASA** |
| **AAS** |
| **AAA** | Given | Unsolvable. Not unique. One side needed. | |

**Ambiguous Cases for SSA**

|  |  |  |  |
| --- | --- | --- | --- |
| **Scenario** | **# of Solutions** | **Illustrations** | |
| **SSA** | **0 – 2 Solutions** |  | |
| **a < h** | **No Solution** |  | |
| **a = h** | **One Solution** |  | |
| **b > a > h** | **Two Solutions** |  | |
|  |  |
| **a ≥ b** | **One Oblique Solution** |  | |
| When *a = b*, equilateral / isososocles  When *a > b*, obtuse | |

Source: <https://mathimages.swarthmore.edu/index.php/Ambiguous_Case>

**Interesting Trig Lengths on a Unit Circle**

|  |  |
| --- | --- |
| **Trig Graphically** | |
| http://www.mrhosek.com/trigonometry/trig.gif | *10 Secret Trig Functions Your Math Teachers Never Taught You - Scientific  American Blog Network* |

**Fixed Angles Triangles**

|  |  |
| --- | --- |
| **45-45-90 Triangle** | **30-60-90 Triangle** |
|  | Diagram  Description automatically generated |
| **Proof:**  a² + b² = c²  x = y  x² + x² = 1²  2x² = 1  x² = ½ | **Proof:**  a² + b² = c²  y² + (½)² = 1²  y² + ¼ = 1  y² = ¾ |

**Fixed Sides Triangles**

|  |  |  |  |
| --- | --- | --- | --- |
| **Pythagorean Triples** | | | |
| A Pythagorean triple is a right triangle with only integer sides.  The examples on the right are expressed in the lowest form.  You can scale any of these with an integer (e.g., 2,3,4,5) to generate a family of similar triangles.  Find the Missing Pieces in Geometry - Ten Steps to Scoring Higher on the GRE - Crash Course for ... | 3 : 4 : 5  5 : 12 : 13  8 : 15 : 17  7 : 24 : 25  9 : 40 : 41  11 : 60 : 61  12 : 35 : 37  13 : 84 : 85  15 : 112 : 113  16 : 63 : 65  17 : 144 : 145  19 : 180 : 181  20 : 21 : 29  20 : 99 : 101  21 : 220 : 221  23 : 264 : 265 | 24 : 143 : 145  28 : 45 : 53  28 : 195 : 197  32 : 255 : 257  33 : 56 : 65  36 : 77 : 85  39 : 80 : 89  44 : 117 : 125  48 : 55 : 73  51 : 140 : 149  52 : 165 : 173  57 : 176 : 185  60 : 91 : 109  60 : 221 : 229  65 : 72 : 97  68 : 285 : 293 | 69 : 260 : 269  84 : 187 : 205  85 : 132 : 157  88 : 105 : 137  95 : 168 : 193  96 : 247 : 265  104 : 153 : 185  105 : 208 : 233  115 : 252 : 277  119 : 120 : 169  120 : 209 : 241  133 : 156 : 205  140 : 171 : 221  160 : 231 : 281  161 : 240 : 289  204 : 253 : 325 |

|  |  |
| --- | --- |
| **Radians and Arc Length** |  |
| Radian = arc length (s) of a unit circle  s = r  C = D = (2r) = 2r  **Proof:**  If r = 1 (unit circle)  then s = and C =  Therefore 360° = 2 radians  To convert degrees to radians:  n° x = m radians  To convert radians to degrees:  m rad x = n° | http://www.mathwarehouse.com/trigonometry/radians/images/picture-s=r-theta-circle.gif |