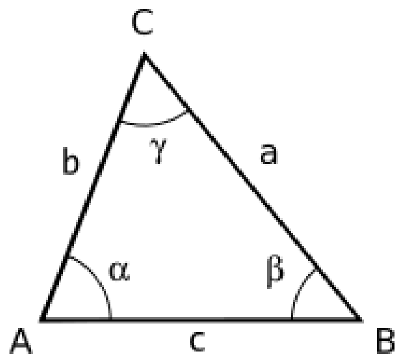
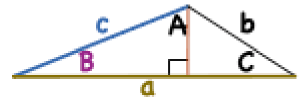
# Harold’s Triangles Cheat Sheet

18 January 2023

## Trig Laws and Formulas

| Law | Equation |
|---|---|
| **Reference Triangle** |  |
| **Law of Sines** | $\frac{a}{sin(A)} = \frac{b}{sin(B)} = \frac{c}{sin(C)}$ <br> $\frac{sin(A)}{a} = \frac{sin(B)}{b} = \frac{sin(C)}{c}$ |
| **Law of Cosines** | $a^2 = b^2 + c^2 - 2bc \cdot cos(A)$ <br> $b^2 = a^2 + c^2 - 2ac \cdot cos(B)$ <br> $c^2 = a^2 + b^2 - 2ab \cdot cos(C)$ |
| **Law of Tangents** | $\frac{a-b}{a+b} = \frac{tan\left[\frac{(A-B)}{2}\right]}{tan\left[\frac{(A+B)}{2}\right]}$ <br> $\frac{b-c}{b+c} = \frac{tan\left[\frac{(B-C)}{2}\right]}{tan\left[\frac{(B+C)}{2}\right]}$ <br> $\frac{a-c}{a+c} = \frac{tan\left[\frac{(A-C)}{2}\right]}{tan\left[\frac{(A+C)}{2}\right]}$ <br> $tan(A) \cdot tan(B) \cdot tan(C) = tan(A) + tan(B) + tan(C)$ |
| **Pythagorean Theorem** | If a right triangle, then <br> $c^2 = a^2 + b^2$ <br> **Special Case**: Same as Law of Cosines with angle C = 90°. <br> $c^2 = a^2 + b^2 - 2ab \cdot cos(90°)$ <br> $c^2 = a^2 + b^2 - 2ab \cdot 0 = a^2 + b^2$ |

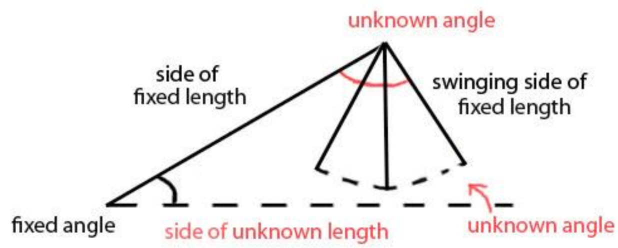
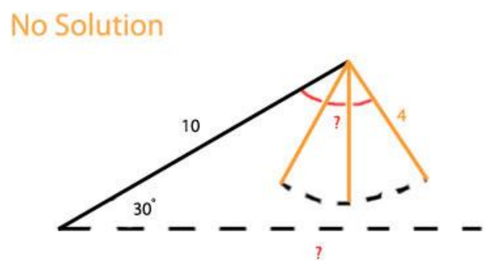
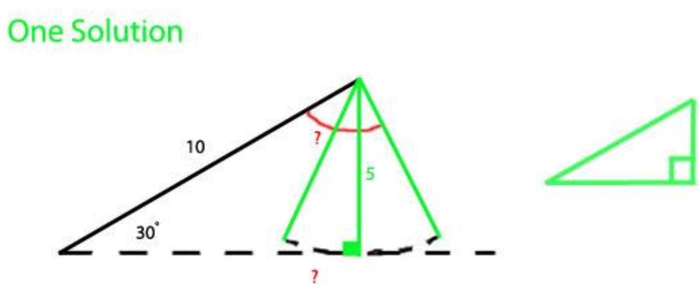
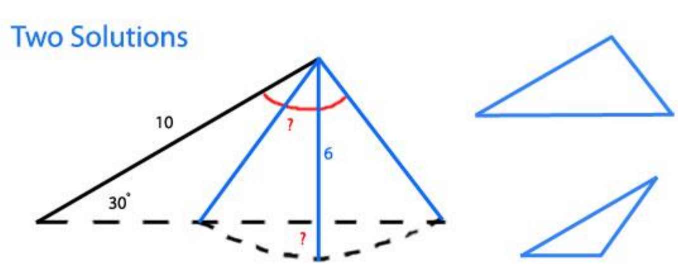
| | | |
|---|---|---|
| **Sum of Angles** | $A° + B° + C° = 180°$<br>$C° = 180° - (A° + B°)$ | $A + B + C = \pi$<br>$C = \pi - (A + B)$ |
| **Area Formula** | $A = \frac{1}{2}bh = \frac{1}{2}ab \cdot sin(C)$ |  |
| **Semi-Perimeter** | $s = \frac{a + b + c}{2}$ | |
| **Heron's Formula** | $A = \sqrt{s(s-a)(s-b)(s-c)}$ | |
| **Law of Cotangents** | $\frac{cot\left(\frac{A}{2}\right)}{s-a} = \frac{cot\left(\frac{B}{2}\right)}{s-b} = \frac{cot\left(\frac{C}{2}\right)}{s-c}$<br>$cot(A) \cdot cot(B) \cdot cot(C) = cot(A) + cot(B) + cot(C)$ | |
| **Mollweide's Formula** | $\frac{a + b}{c} = \frac{cos\left[\frac{(A-B)}{2}\right]}{sin\left(\frac{C}{2}\right)}$ | |

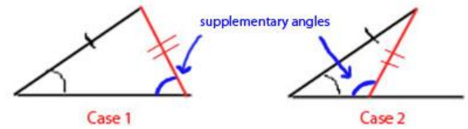
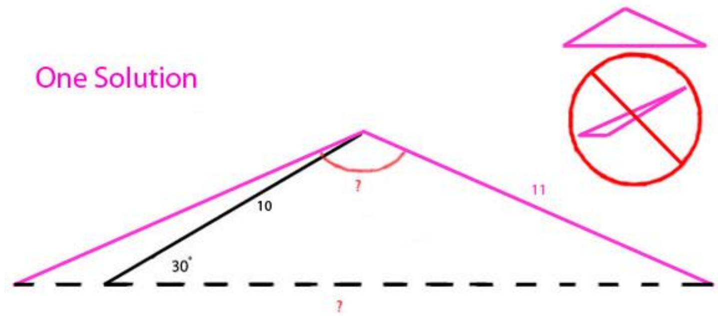
## Solving for Angles, Sides, and Area

| Order of Use | Comments |
|---|---|
| **1. Sum of Angles** | Easiest formula. |
| **2. Pythagorean Theorem** | Use if one of the angles is a right angle (90°). |
| **3. Law of Sines** | Least complex. Use before Law of Cosines, if possible. |
| **4. Law of Cosines** | Most complex. Use only once, then use Law of Sines. |
| **5. Heron's Formula** | Use for area if all three sides are known. |

| Given | Angles / Sides | Area |
|---|---|---|
| **SSS** | Law of Cosines | Heron's Formula |
| **SAS** | | $A = \frac{1}{2}ab \cdot sin(C)$ |
| **SSA** | Law of Sines | $A = \frac{1}{2}bh$ |
| **SAA** | | |
| **ASS** | | |
| **ASA** | | |
| **AAS** | | |
| **AAA** | Unsolvable. Not unique. One side needed. | |

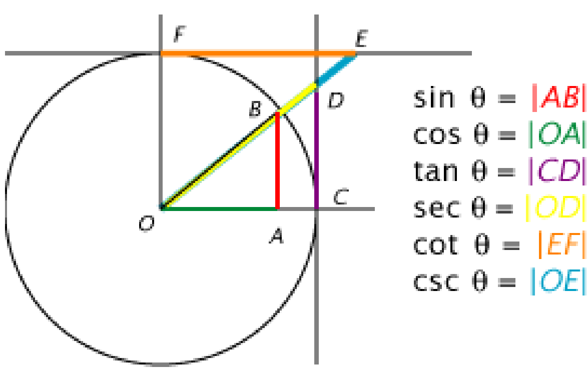
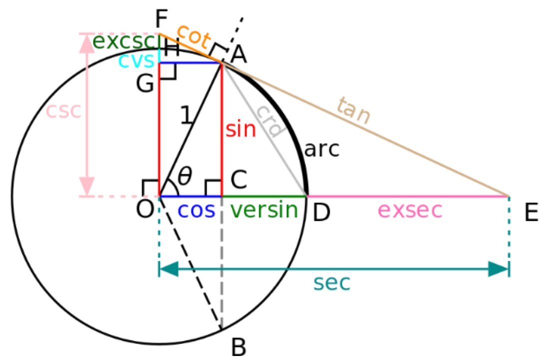
## Ambiguous Cases for SSA

| Scenario | # of Solutions | Illustrations |
|---|---|---|
| **SSA** | **0 – 2 Solutions** |   |
| **a < h** | **No Solution** |   |
| **a = h** | **One Solution** |   |
| **b > a > h** | **Two Solutions** |   |

| | | | |
|---|---|---|---|
| | | $\frac{b}{sin\,(B)} = \frac{b}{sin\,(180° - B)}$ |  |
| a ≥ b | **One Oblique Solution** |  | |
| | | When *a* = *b*, equilateral / isososocles<br>When *a* > *b*, obtuse | |

Source: https://mathimages.swarthmore.edu/index.php/Ambiguous_Case

## Interesting Trig Lengths on a Unit Circle

| Trig Graphically | |
|---|---|
|  |  |

## Important Trig Triangles

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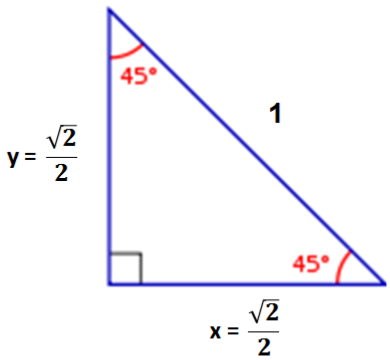
<tr><th>45-45-90 Triangle</th><th>30-60-90 Triangle</th></tr>
<tr><td>


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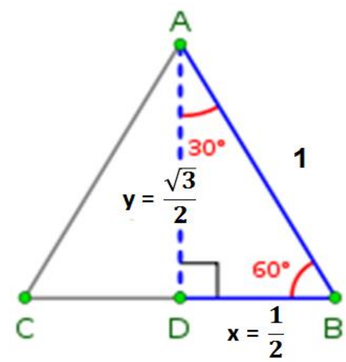

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**Proof:**

$a^2 + b^2 = c^2$

$x = y$

$x^2 + x^2 = 1^2$

$2x^2 = 1$

$x^2 = ½$

$\sqrt{x^2} = \sqrt{\frac{1}{2}}$

$x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$

</td><td>

**Proof:**

$a^2 + b^2 = c^2$

$y^2 + (½)^2 = 1^2$

$y^2 + ¼ = 1$

$y^2 = ¾$

$\sqrt{y^2} = \sqrt{\frac{3}{4}}$

$y = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

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<tr><th>Radians and Arc Length</th><th></th></tr>
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Radian = arc length (s) of a unit circle

$s = r\,\theta$

$C = \pi D = \pi (2r) = 2\pi r$

**Proof:**

If r = 1 (unit circle)

then $s = (1)\theta = \theta$ and $C = 2\pi\,(1) = 2\pi$

Therefore 360° = 2π radians

To convert degrees to radians:

$n° \times \left(\frac{\pi \text{ rad}}{180°}\right) = m$ radians

To convert radians to degrees:

$m \text{ rad} \times \left(\frac{180°}{\pi \text{ rad}}\right) = n°$

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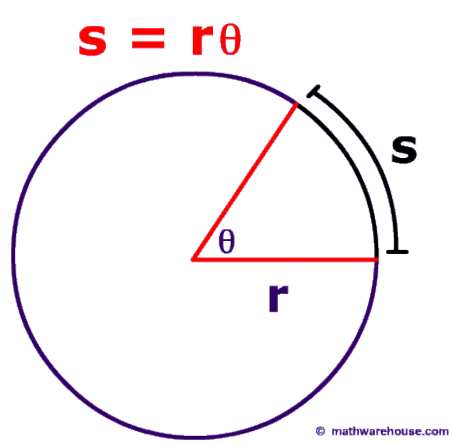

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