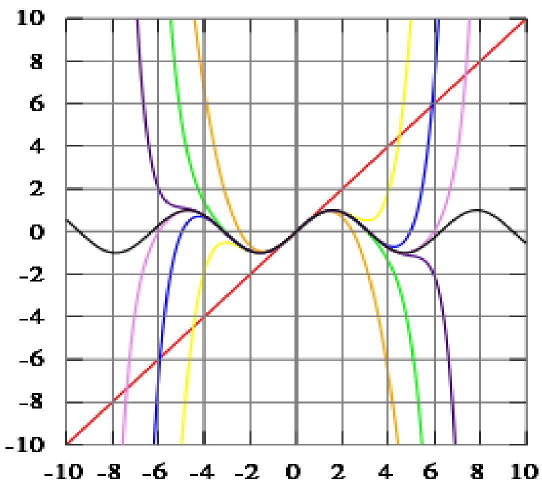
Harold's Taylor Series Cheat Sheet

20 April 2016

Power Series	
Power Series About Zero Geometric Series if $a_n = a$	$\sum_{n=0}^{+\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$
Power Series	$\sum_{n=0}^{+\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \cdots$

Approximation Polynomial	
	$\boldsymbol{f(x) = P_n(x) + R_n(x)}$ $P_n(x) = n^{th} degree\ polynomial\ approximation$ $R_n(x) = \pm\ Error$ $NOTE: P_n(x)\ is\ easy\ to\ integrate\ and\ differentiate$

Maclaurin Series	
Maclaurin Series Taylor Series centered about $x = 0$	$\boldsymbol{f(x) \approx P_n(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(0)}{n!} x^n}$
Maclaurin Series Remainder	$R_n(x) = \frac{f^{(n+1)}(x^*)}{(n+1)!} x^{n+1}$ where $x \le x^* \le max$ and $\lim_{x\to+\infty} R_n(x) = 0$

Taylor Series	
Taylor Series Maclaurin Series if $c = 0$	$f(x) \approx P_n(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(c)}{n!} (x-c)^n$
Taylor Series Remainder	$R_n(x) = \frac{f^{(n+1)}(x^*)}{(n+1)!} (x-c)^{n+1}$ where $x \le x^* \le c$ and $\lim_{x\to+\infty} R_n(x) = 0$

Series Examples	
Exponential Functions	
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad for\ all\ x$	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^8}{8!} + \cdots$
$a^x = e^{x \ln(a)} = \sum_{n=0}^{\infty} \frac{(x \ln(a))^n}{n!} \quad for\ all\ x$	$1 + x \ln(a) + \frac{[x \ln(a)]^2}{2!} + \frac{[x \ln(a)]^3}{3!} + \cdots$

Natural Logarithms	
$\ln(1-x) = \sum_{n=1}^{\infty} \frac{x^n}{n} \quad for\ \lvert x \rvert < 1$	$x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} + \frac{x^6}{6} + \frac{x^7}{7} + \frac{x^8}{8} + \cdots$
$\ln(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-1)^n}{n} \quad for\ \lvert x \rvert < 1$	$(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots$
$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n \quad for\ \lvert x \rvert < 1$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} - \frac{x^8}{8} + \cdots$
$\ln\left(\frac{1+x}{1-x}\right) = \sum_{n=1}^{\infty} \frac{2}{2n-1} x^{2n-1} \quad for\ \lvert x \rvert < 1$	$2x - \frac{2x^2}{3} + \frac{2x^3}{5} - \frac{2x^4}{7} + \frac{2x^5}{9} - \frac{2x^6}{11} + \frac{2x^7}{13} - \cdots$

Geometric Series	
$\frac{1}{x} = \sum_{n=0}^{\infty} (-1)^n (x-1)^n \quad for\ 0 < x < 2$	$1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 + \cdots$
$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n \quad for\ \lvert x \rvert < 1$	$1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 - \cdots$
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad for\ \lvert x \rvert < 1$	$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + \cdots$
$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \quad for\ \lvert x \rvert < 1$	$1 - x^2 + x^4 - x^6 + x^8 - x^{10} + x^{12} - x^{14} + \cdots$
$\frac{1}{1-x^2} = \sum_{n=0}^{\infty} x^{2n} \quad for\ \lvert x \rvert < 1$	$1 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + \cdots$
$\frac{1}{(1+x)^2} = \sum_{n=1}^{\infty} (-1)^{n-1} n x^{n-1} \quad for\ \lvert x \rvert < 1$	$1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5 + 7x^6 - \cdots$
$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1} \quad for\ \lvert x \rvert < 1$	$1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + \cdots$
$\frac{1}{(1+x)^3} = \sum_{n=2}^{\infty} \frac{(-1)^{n-1}(n-1)n}{2} x^{n-2}$ $for\ \lvert x \rvert < 1$	$1 - 3x + 6x^2 - 10x^3 + 15x^4 - 21x^5 + 28x^6 - \cdots$

$\frac{1}{(1-x)^3} = \sum_{n=2}^{\infty} \frac{(n-1)n}{2} x^{n-2} \quad for\ \lvert x \rvert < 1$	$1 + 3x + 6x^2 + 10x^3 + 15x^4 + 21x^5 + 28x^6 + \cdots$
$\sqrt{1+x} = \sum_{n=0}^{\infty} \frac{(-1)^n\,(2n)!}{4^n\,(n!)^2\,(1-2n)} x^n$ $for -1 < x \leq 1$	$1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{256}x^5 - \frac{21}{1{,}024}x^5 + \cdots$
$\sqrt{1-x} = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n\,(n!)^2\,(1-2n)} x^n$ $for -1 < x \leq 1$	$1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{5}{128}x^4 - \frac{7}{256}x^5 - \frac{21}{1{,}024}x^5 - \cdots$
$\sqrt{1+x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n\,(2n)!}{4^n\,(n!)^2\,(1-2n)} x^{2n}$ $for -1 < x \leq 1$	$1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{1}{16}x^6 - \frac{5}{128}x^8 + \frac{7}{256}x^{10} - \cdots$
$\sqrt{1-x^2} = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n\,(n!)^2\,(1-2n)} x^{2n}$ $for -1 < x \leq 1$	$1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6 - \frac{5}{128}x^8 - \frac{7}{256}x^{10} - \cdots$
Double Factorial (!!)	$(n)!! = n(n-2)(n-4)\ldots 6 \cdot 4 \cdot 2\ if\ even$ $(n)!! = n(n-2)(n-4)\ldots 5 \cdot 3 \cdot 1\ if\ odd$ $where\ 0!! = 1\ and\ {-1}!! = 1$
$\frac{1}{\sqrt{1+x}} = \sum_{n=0}^{\infty} \frac{(-1)^n\,(2n-1)!!}{(2n)!!} x^n$ $for -1 < x \leq 1$	$1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}x^4 - \cdots$
$\frac{1}{\sqrt{1-x}} = \sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n)!!} x^n$ $for -1 < x \leq 1$	$1 + \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}x^4 + \cdots$
$\frac{1}{\sqrt{1+x^2}} = \sum_{n=0}^{\infty} \frac{(-1)^n\,(2n-1)!!}{(2n)!!} x^{2n}$ $for -1 < x \leq 1$	$1 - \frac{1}{2}x^2 + \frac{1 \cdot 3}{2 \cdot 4}x^4 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^6 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}x^8 - \cdots$
$\frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n)!!} x^{2n}$ $for -1 < x \leq 1$	$1 + \frac{1}{2}x^2 + \frac{1 \cdot 3}{2 \cdot 4}x^4 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^6 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}x^8 + \cdots$
Binomial Series	
$(1+x)^r = \sum_{n=0}^{+\infty} \binom{r}{n} x^n$ $for\ \lvert x \rvert < 1\ and\ all\ complex\ r\ where$ $\binom{r}{n} = \prod_{k=1}^{n} \frac{r-k+1}{k}$ $= \frac{r(r-1)(r-2)\ldots(r-n+1)}{n!}$	$1 + rx + \frac{r(r-1)}{2!}x^2 + \frac{r(r-1)(r-2)}{3!}x^3 + \cdots$

Trigonometric Functions	
$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ *for all x*	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!} - \frac{x^{15}}{15!} + \cdots$
$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$ *for all x*	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!} - \frac{x^{14}}{14!} + \cdots$
$\tan(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}\, 2^{2n}\, (2^{2n}-1)\, B_{2n}}{(2n)!} x^{2n-1}$ $for\ \lvert x\rvert < \frac{\pi}{2}$	$x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2{,}835}x^9 + \frac{1{,}382}{155{,}925}x^{11} + \frac{21{,}844}{608{,}1075}x^{13} + \cdots$
$\sec(x) = \sum_{n=0}^{\infty} \frac{(-1)^n E_{2n}}{(2n)!} x^{2n}$ $for\ \lvert x\rvert < \frac{\pi}{2}$	$1 + \frac{x^2}{2!} + 5\frac{x^4}{4!} + 61\frac{x^6}{6!} + 1{,}385\frac{x^8}{8!} + 50{,}521\frac{x^{10}}{10!} + \cdots$
$\csc(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n-1}\, 2\, (2^{2n-1}-1)\, B_{2n}}{(2n)!} x^{2n-1}$ $for\ 0 < x < \pi$	$\frac{1}{x} + \frac{1}{6}x + \frac{7}{360}x^3 + \frac{31}{15{,}120}x^5 + \frac{127}{604{,}800}x^7 + \cdots$
$\cot(x) = \sum_{n=0}^{\infty} \frac{(-1)^n\, 2^{2n}\, B_{2n}}{(2n)!} x^{2n-1}$ $for\ 0 < x < \pi$	$\frac{1}{x} - \frac{1}{3}x - \frac{1}{45}x^3 - \frac{2}{189}x^5 - \frac{1}{4{,}725}x^7 - \frac{4}{2{,}835}x^9 - \cdots$

Inverse Trigonometric Functions	
$\sin^{-1}(x) = \sum_{n=0}^{\infty} \frac{(2n)!}{(2^n n!)^2 (2n+1)} x^{2n+1}$ $for\ \lvert x\rvert \le 1$ $\sin^{-1}(x) = \sum_{n=0}^{\infty} \frac{\Gamma\left(n+\frac{1}{2}\right)}{\sqrt{\pi}\,(2n+1)\, n!} x^{2n+1}$	$x + \frac{x^3}{2\cdot 3} + \frac{1\cdot 3x^5}{2\cdot 4\cdot 5} + \frac{1\cdot 3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7} + \frac{1\cdot 3\cdot 5\cdot 7x^9}{2\cdot 4\cdot 6\cdot 8\cdot 9} + \cdots$
$\cos^{-1}(x) = \frac{\pi}{2} - \sin^{-1}(x)$ $for\ \lvert x\rvert \le 1$	$\frac{\pi}{2} - x - \frac{x^3}{2\cdot 3} - \frac{1\cdot 3x^5}{2\cdot 4\cdot 5} - \frac{1\cdot 3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7} - \cdots$
$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} x^{2n+1}$ $for\ \lvert x\rvert < 1$	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots\ for\ -1 < x < 1$ $\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \frac{1}{7x^7} - \frac{1}{9x^9} + \cdots\ for\ x \ge 1$ $-\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \frac{1}{7x^7} - \frac{1}{9x^9} + \cdots\ for\ x < 1$
$\sec^{-1}(x) = -i\ln(x) + i\ln(2) - \frac{i}{4}\sum_{n=0}^{\infty} \frac{(2n+1)!\ x^{2n+2}}{4^n[(n+1)!]^2}$	$-i\ln(x) + i\ln(2) - \frac{i}{4}x^2 - \frac{3i}{32}x^4 - \frac{5i}{96}x^6 - \cdots$

$\csc^{-1}(x) = i\ln(x) - i\ln(2) + \frac{\pi}{2} + \frac{i}{4}\sum_{n=0}^{\infty}\frac{(2n+1)!\ x^{2n+2}}{4^n[(n+1)!]^2}$	$i\ln(x) - i\ln(2) + \frac{\pi}{2} + \frac{i}{4}x^2 + \frac{3i}{32}x^4 + \frac{5i}{96}x^6 + \cdots$
$\cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x)$ $for\ \lvert x\rvert < 1$	$\frac{\pi}{2} - x + \frac{x^3}{3} - \frac{x^5}{5} + \frac{x^7}{7} - \frac{x^9}{9} + \cdots\ for -1 < x < 1$ $\frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \frac{1}{7x^7} + \frac{1}{9x^9} - \cdots\ for\ x \geq 1$ $\pi + \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \frac{1}{7x^7} + \frac{1}{9x^9} - \cdots\ for\ x < 1$

Hyperbolic Functions	
$\sinh(x) = \frac{e^x - e^{-x}}{2} = \sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}\ for\ all\ x$	$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \frac{x^{11}}{11!} + \frac{x^{13}}{13!} + \frac{x^{15}}{15!} + \cdots$
$\cosh(x) = \frac{e^x + e^{-x}}{2} = \sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}\ for\ all\ x$	$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \frac{x^{10}}{10!} + \frac{x^{12}}{12!} + \frac{x^{14}}{14!} + \cdots$
$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $\tanh(x) = \sum_{n=1}^{\infty}\frac{2^{2n}(2^{2n}-1)\ B_{2n}}{(2n)!}x^{2n-1}$ $for\ \lvert x\rvert < \frac{\pi}{2}$	$x - 2\frac{x^3}{3!} + 16\frac{x^5}{5!} - 272\frac{x^7}{7!} + 7{,}936\frac{x^9}{9!} - 353{,}792\frac{x^{11}}{11!} + \cdots$ $x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{62}{2{,}835}x^9 - \frac{1{,}382}{155{,}925}x^{11} + \cdots$
$\operatorname{sech}(x) = \sum_{n=0}^{\infty}\frac{E_{2n}}{(2n)!}x^{2n}$ $for\ \lvert x\rvert < \frac{\pi}{2}$	$1 - \frac{x^2}{2!} + 5\frac{x^4}{4!} - 61\frac{x^6}{6!} + 1{,}385\frac{x^8}{8!} - 50{,}521\frac{x^{10}}{10!} + \cdots$
$\operatorname{csch}(x) = \frac{1}{x} + \sum_{n=1}^{\infty}\frac{(2-2^{2n})\ B_{2n}}{(2n)!}x^{2n-1}$ $for\ 0 < \lvert x\rvert < \pi$	$\frac{1}{x} - \frac{1}{6}x + \frac{7}{360}x^3 - \frac{31}{15{,}120}x^5 + \frac{127}{604{,}800}x^7 - \cdots$
$\coth(x) = \frac{1}{x} + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}\ 2^{2n}\ B_{2n}}{(2n)!}x^{2n-1}$ $for\ 0 < \lvert x\rvert < \pi$	$\frac{1}{x} + \frac{1}{3}x - \frac{1}{45}x^3 + \frac{2}{945}x^5 - \frac{1}{4{,}725}x^7 + \frac{4}{2{,}835}x^9 - \cdots$

Inverse Hyperbolic Functions	
$\sinh^{-1}(x) = \sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{(2^n n!)^2(2n+1)}x^{2n+1}$ $for\ \lvert x\rvert \leq 1$ $\sinh^{-1}(x) = \sum_{n=0}^{\infty}\frac{(-1)^n\ (2n-1)!!}{(2n+1)(2n)!!}x^{2n+1}$	$x - \frac{x^3}{2\cdot 3} + \frac{1\cdot 3x^5}{2\cdot 4\cdot 5} - \frac{1\cdot 3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7} + \frac{1\cdot 3\cdot 5\cdot 7x^9}{2\cdot 4\cdot 6\cdot 8\cdot 9} - \cdots$

$\cosh^{-1}(x) = \frac{\pi}{2} i - i \sum_{n=0}^{\infty} \frac{2^{-n}}{n!\,(2n+1)} x^{2n+1}$ $for\ \lvert x \rvert \le 1$	$\frac{\pi\, i}{2} - i\, x - \frac{i\, x^3}{2 \cdot 3} - \frac{i \cdot 1 \cdot 3x^5}{2 \cdot 4 \cdot 5} - \frac{i \cdot 1 \cdot 3 \cdot 5x^7}{2 \cdot 4 \cdot 6 \cdot 7} - \cdots$
$\tanh^{-1}(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$ $for\ \lvert x \rvert < 1, x \ne \pm 1$	$x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9} + \frac{x^{11}}{11} + \frac{x^{13}}{13} + \frac{x^{15}}{15} + \cdots$
$\operatorname{sech}^{-1}(x) = -\ln(x) + \ln(2) + \frac{1}{4} \sum_{n=0}^{\infty} \frac{(2n+1)!\ x^{2n+2}}{4^n [(n+1)!]^2}$	$-\ln(x) + \ln(2) + \frac{1}{4} x^2 + \frac{3i}{32} x^4 + \frac{5i}{96} x^6 + \cdots$
$\operatorname{csch}^{-1}(x) = -\ln(x) + \ln(2) + \frac{1}{4} \sum_{n=0}^{\infty} \frac{(-1)^n\ (2n+1)!\ x^{2n+2}}{4^n [(n+1)!]^2}$	$-\ln(x) + \ln(2) + \frac{1}{4} x^2 - \frac{3i}{32} x^4 + \frac{5i}{96} x^6 - \cdots$
$\coth^{-1}(x) = -\frac{i\pi}{2} + \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$	$= -\frac{i\pi}{2} + x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9} + \frac{x^{11}}{11} + \frac{x^{13}}{13} + \cdots$

Bernoulli Numbers	**Euler Numbers**	**Gamma Function**
$B_0 = 1$ $B_1 = -\frac{1}{2}$ $B_2 = \frac{1}{6}$ $B_4 = -\frac{1}{30}$ $B_6 = \frac{1}{42}$ $B_8 = -\frac{1}{30}$ $B_{10} = \frac{5}{66}$ $B_{12} = -\frac{691}{2{,}730}$ $B_{14} = \frac{7}{6}$ $B_{16} = -\frac{3{,}617}{510}$ $B_{18} = \frac{438{,}675}{798}$ $B_{20} = -\frac{174{,}611}{330}$ $B_{22} = \frac{854{,}513}{138}$	$E_0 = 1$ $E_1 = 0$ $E_2 = -1$ $E_3 = 0$ $E_4 = 5$ $E_5 = 0$ $E_6 = -61$ $E_8 = 1{,}385$ $E_{10} = -50{,}521$ $E_{12} = 2{,}702{,}765$ $E_{14} = -199{,}360{,}981$ $E_{16} = 19{,}391{,}512{,}145$ $E_{18} = -2{,}404{,}879{,}675{,}441$ $E_{20} = 370{,}371{,}188{,}237{,}525$ $E_{22} = -69{,}348{,}874{,}393{,}137{,}901$ $E_{2n+1} = 0$	$\Gamma_0 = \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ $\Gamma_1 = \Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$ $\Gamma_2 = \Gamma\left(\frac{5}{2}\right) = \frac{3\sqrt{\pi}}{4}$ $\Gamma_3 = \Gamma\left(\frac{7}{2}\right) = \frac{15\sqrt{\pi}}{8}$ $\Gamma_4 = \Gamma\left(\frac{9}{2}\right) = \frac{105\sqrt{\pi}}{16},$ $\Gamma_5 = \Gamma\left(\frac{11}{2}\right) = \frac{945\sqrt{\pi}}{32}$ $\Gamma_6 = \Gamma\left(\frac{13}{2}\right) = \frac{10{,}395\sqrt{\pi}}{64}$ $\Gamma_7 = \Gamma\left(\frac{15}{2}\right) = \frac{135{,}135\sqrt{\pi}}{128}$ $\Gamma_8 = \Gamma\left(\frac{17}{2}\right) = \frac{2{,}027{,}025\sqrt{\pi}}{256}$ $\Gamma_9 = \Gamma\left(\frac{19}{2}\right) = \frac{34{,}459{,}425\sqrt{\pi}}{512}$ $\Gamma_{10} = \Gamma\left(\frac{21}{2}\right) = \frac{654{,}729{,}075\sqrt{\pi}}{1{,}024}$
Generating Function		

$\frac{t}{e^t - 1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} t^n$	$\frac{1}{\cosh(t)} = \frac{2}{e^t + e^{-t}} = \sum_{n=0}^{\infty} \frac{E_n}{n!} t^n$	$\Gamma(t) = \int_0^{\infty} x^{t-1} e^{-x} \, dx$ $\Gamma\left(\frac{1}{2} + n\right) = \frac{(2n)!}{4^n \, n!} \sqrt{\pi}$ $\Gamma\left(\frac{n}{2}\right) = \frac{(n-2)!!}{2^{\frac{n-1}{2}}} \sqrt{\pi}$
Recursive Definition	**Iterated Sum**	**Recursive Definition**
$B_m(n)$ $= n^m - \sum_{k=0}^{m-1} \binom{m}{k} \frac{B_k(n)}{m-k+1}$ $B_0(n) = 1$	E_{2n} $= i \sum_{k=1}^{2n+1} \sum_{j=0}^{k} \binom{k}{j} \frac{(-1)^j (k-2j)^{2n+1}}{2^k \, i^k \, k}$	$\Gamma(n+1) = n \cdot \Gamma(n)$ $\Gamma\left(\frac{n}{2}\right) = \left(\frac{n-2}{2}\right) \cdot \Gamma\left(\frac{n-2}{2}\right)$ $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

References:

https://www.wolframalpha.com

https://en.wikipedia.org

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http://web.mit.edu/kenta/www/three/taylor.html