Harold's Statistics
Hypothesis Testing
Cheat Sheet

4 Nov 2020

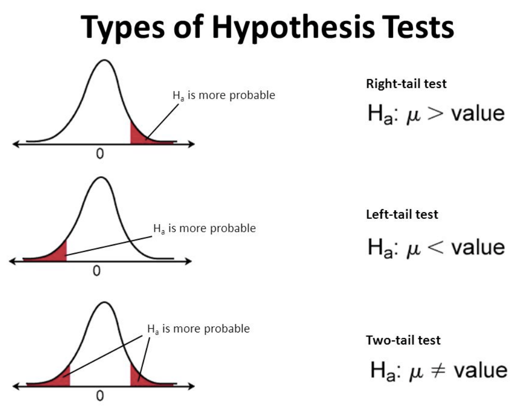
Hypothesis Terms	Definitions
Significance Level (α)	Defines the strength of evidence in probabilistic terms. Specifically, alpha represents the probability that tests will produce statistically significant results when the null hypothesis is correct.
Confidence Level (C)	The percentage of all possible samples that can be expected to include the true population parameter.
Critical Value (z_c)	z_c is the critical value of a standard normal distribution under H_0. Critical values divide the rejection and non-rejection regions. Set using p-values or to a threshold value of 0.05 (5%) or 0.01 (1%), but always ≤ 0.10 (10%).
Test Statistic (z_{data})	z_{data} is the test value of z of a standard normal distribution under H_0. If z_{data} is inside the rejection region, demarked by z_c, then we can reject the null hypothesis, H_0.
p-value	Probability of obtaining a sample "more extreme" than the ones observed in your data, assuming H_0 is true.
Hypothesis	A premise or claim that we want to test.
Null Hypothesis: H_0	Currently accepted value for a parameter. (middle of the distribution) Is assumed true for the purpose of carrying out the hypothesis test. • Always contains an "=" {=, ≤, ≥} • The null value implies a specific sampling distribution for the test statistic • $\boldsymbol{H_0}$ is the middle of the normal distribution curve at $z = 0$. • Can be rejected, or not rejected, **but NEVER supported**
Alternative Hypotheses: H_a	Also called Research Hypothesis or H_1. Is the opposite of H_0 and involves the claim to be tested. Is supported only by carrying out the test if the null hypothesis can be rejected. • Always contains ">" (right-tailed), "<" (left-tailed), or "≠" (two-tailed) [tail selection is important] • Can be supported (by rejecting the null), or not supported (by failing or rejecting the null), **but NEVER rejected**

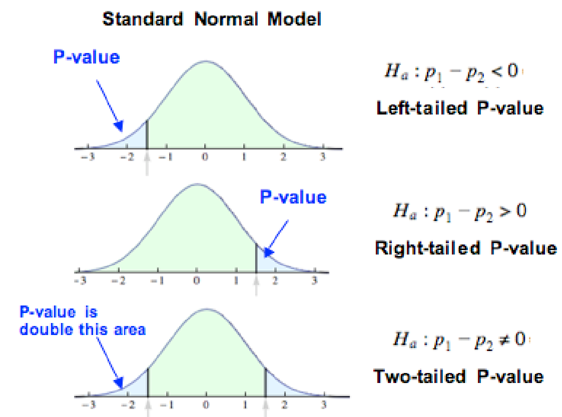
Hypothesis Testing	Steps
Hypothesis Testing	1) <u>Claim</u>: Formulate the **null** ($\boldsymbol{H_0}$) and the **alternative** ($\boldsymbol{H_a}$) hypothesis 2) <u>Graph</u>: Sketch and label critical value (left-tailed, right-tailed, two-tailed) 3) <u>Decision Rule</u>: Use significance level (α), confidence level (C), confidence Interval, or critical value (z_c). e.g. We will reject H_0 if $z_{data} > 1.645$ 4) <u>Critical Value</u>: Determine **critical values** (z_c) to mark the rejection regions 5) <u>Test Statistic:</u> Calculate the **test statistic** (z_{data}) from the sample data 6) <u>Conclusion</u>: Reject the null hypothesis (supporting the alternative hypothesis) otherwise fail to reject the null hypothesis, then state claim

1) Claim: Formulate Hypothesis

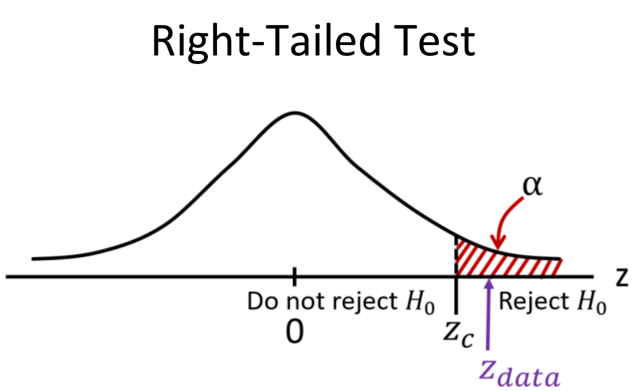
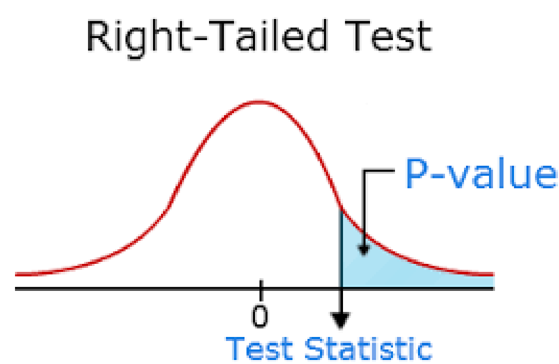
If claim consists of …	*then the hypothesis test is*	*and is represented by…*
"is equal to", "is exactly", "is the same as", "is between" "is at least" "is at most"	Two-tailed = Left-tailed ≤ Right-tailed ≥	H_0
"is not equal to", "is different from", "has changed from" "is less than", "is below", "is lower or smaller than" "is greater than", "is above", "is longer or bigger than"	Two-tailed ≠ Left-tailed < Right-tailed >	H_a

$Make\ sure\ H_0 + H_a = all\ possible\ outcomes$





2) Graph: Sketch and Label

Sketch and label critical value (z_c)	Look at the direction of the inequality symbol in H_a to determine where to shade.
	

3) Decision Rule

p-value	Use probability value to determine z_c in a Normal Distribution table
Significance level (α)	$\alpha = 1 - C$ Usually at a threshold value of 0.05 (5%) or 0.01 (1%), but always ≤ 0.10 (10%).
Confidence Level (C)	$C = 1 - \alpha$ With a confidence of 0.95 (95%) or 0.99 (99%), but always ≥ 0.90 (90%).

Confidence Interval (C) for μ	σ known, normal population or large sample $z\text{ - }interval = \bar{x} \pm E$ $= \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ $\frac{\alpha}{2} = \frac{1-C}{2}$ $z_{\alpha/2} = z - score\ for\ probabilities\ of\ {}^{\alpha}/_{2}$
Examples:	e.g. We will reject H_0 if significance level is less than 5% e.g. We will reject H_0 if confidence level is greater than 95% e.g. We will reject H_0 if confidence interval is between 5% and 95% ($e.g. \pm$5%) e.g. We will reject H_0 if $z_{data} > z_c$ in a right-tailed test

4) Determine Critical Values (z_c) / Rejection Region	
Critical Values (z_c)	Determine z_c by looking up α, C, or p-values in a standard normal distribution table. Two-tailed tests have two values for z_c .

5) Calculate Test Statistic (z_{data})		
Population/Sample Proportion (p, $\hat{p}$)	$z_{data} = \frac{\hat{p}-p}{SE(\hat{p})} = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}$	Assumes $np \geq 15$ ***and*** $nq \geq 15$.
Population/Sample Mean (μ, $\bar{x}$)	$z_{data} = \frac{\bar{x}-\mu}{SE(\bar{x})} = \frac{\bar{x}-\mu}{s/\sqrt{n}}$	Variance unknown. t Distribution, $df = n-1$ under H_0.
	$z_{data} = \frac{\bar{x}-\mu}{SE(x)} = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$	Variance known. Assumes data is normally distributed **or** $n \geq 30$ since t approaches standard normal Z if n is sufficiently large due to the CLT.

6) Conclusion	
Statistical Decision	Reject the null hypothesis (supporting the alternative hypothesis) using a test below
Conclusions of p-test	If p− value $\leq \alpha \Rightarrow$ Reject H_0 If p− value $> \alpha \Rightarrow$ Fail to Reject H_0
Conclusions of mean test	If significance level (α) is less than 5% ⇒ Reject H_0 If confidence level (C) is greater than 95% ⇒ Reject H_0 If test statistic is greater than the critical value, $z_{data} > z_c \Rightarrow$ Reject H_0
Conclusions of Confidence Interval (C) for μ / z-interval	Reject the null hypothesis if the test statistic falls in the **rejection region** otherwise, fail to reject the null hypothesis. If confidence interval is between 5% and 95% ($e.g. \pm$5%) ⇒ Reject H_0

Goodness-of-Fit Test – Chi-Square		
Expected Frequencies for a Chi-Square	$E = np$	$p = proportion$ $n = sample\ size$
Chi-Square Test Statistic	$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$ $\boldsymbol{\chi^2 = \sum \frac{(observed - expected)^2}{expected}}$	Large χ^2 values are evidence against the null hypothesis, which states that the percentages of observed and expected match (as in, any differences are attributed to chance).
Degrees of Freedom	$df = k - 1$	$k = number\ of\ possible\ values\ (categories)\ for\ the\ variable\ under\ consideration$

Independence Test – Chi-Square		
Expected Frequencies for a Chi-Square	$E = \frac{rc}{n}$	$r = \#\ of\ rows$ $c = \#\ of\ columns$
Chi-Square Test Statistic	$\chi^2 = \frac{(O-E)^2}{E}$	(see above)
Degrees of Freedom	$df = (r-1)(c-1)$	$r\ and\ c = number\ of\ possible\ values\ for\ the\ two\ variables\ under\ consideration$