

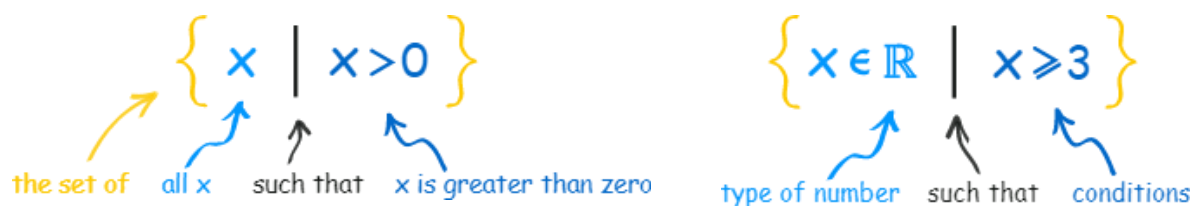
Harold's Sets Cheat Sheet

26 October 2022

Set Definitions

Term	Definition	Examples
Set	A well-defined collection of distinct mathematical objects	$C = \{2, 4, 5\}$ denotes a set of three numbers: 2, 4, and 5 $D = \{(2, 4), (-1, 5)\}$ denotes a set of two ordered pairs of numbers
Element	Objects, members	$a, 3, (x, y)$
Pair	Ordered pair. An element with two members. Order matters.	(x, y)
Tuple	Ordered tuple. A column of three mathematical objects. Order matters.	(a, b, c) or $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$
n-Tuple	Ordered n-tuple. \mathbb{Z}^3 is the set of all 3-tuples whose entries are integers. Order matters.	$\mathbb{Z}^3 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a, b, c \in \mathbb{Z} \right\}$
Set-Builder Notation	<i>Set Uppercase_letter = {number_type [: or]formula \wedge restrictions or conditions}</i>	$F = \{n \in \mathbb{Z} : n^3 \wedge 1 \leq n \leq 100\}$ The set of cubes of the first 100 positive integers.
Roster Notation	A list of the elements enclosed in curly braces with the individual elements separated by commas	$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Set-Builder Notation



Set-Builder Notation:

$$\{x \in \mathbb{R} \mid x \leq 2 \text{ or } x > 3\}$$

Number Line:



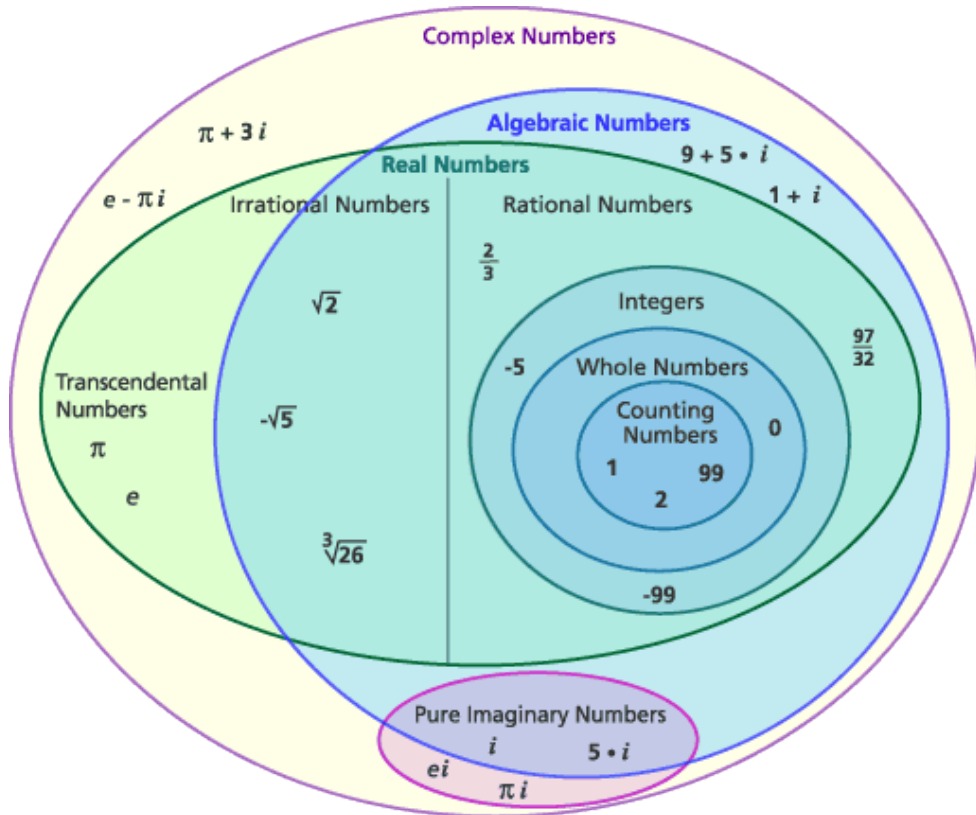
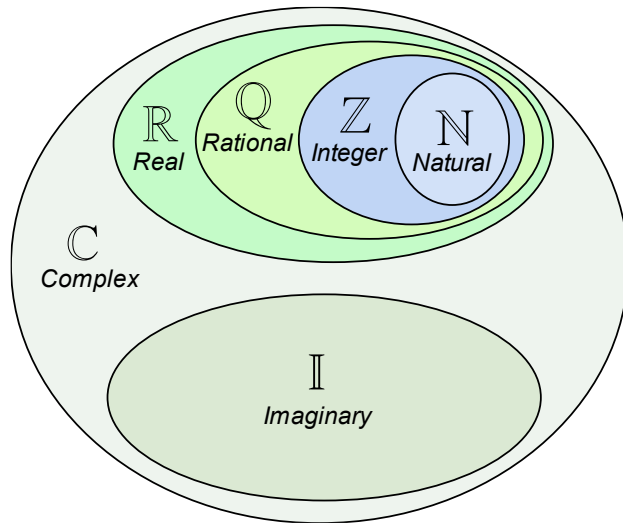
Interval Notation:

$$(-\infty, 2] \cup (3, +\infty)$$

Number Sets

Symbol	Definition	Set Notation	Examples	Equations
\emptyset	Empty or null set	$\{\}$	$\emptyset \in \{\emptyset\}$	$1 = 2$
\mathbb{N}	Natural numbers	$\{x \in \mathbb{Z} : x > 0\}$	$\{1, 2, 3, \dots\}$ or $\{0, 1, 2, 3, \dots\}$ (per ISO 80000-2 2-6.1)	$x - 3 = 0$
\mathbb{W}	Whole numbers	$\{x \in \mathbb{Z} : x \geq 0\}$	$\{0, 1, 2, 3, \dots\}$	$n \geq 0$
\mathbb{P}	Prime numbers	$\{a, b \in \mathbb{Z}^+ : (p \mid ab \rightarrow p \mid a \vee p \mid b)\}$	$\{2, 3, 5, 7, 11, 13, \dots\}$	unofficial
\mathbb{Z}	Integers	$\{x : x = \pm \mathbb{N} \vee x = 0\}$	$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$	$x + 7 = 0$
\mathbb{Q}	Rational numbers	$\{p/q : p, q \in \mathbb{Z} \wedge q \neq 0\}$	$\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$	$4x - 1 = 0$
\mathbb{I}	Irrational numbers	$\{x \in \mathbb{R} : x \notin \mathbb{Q}\}$	$\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$	$4x - 1 = 0$
\mathbb{A}	Algebraic numbers	$\{x \in \mathbb{R} : x = \text{root of a one variable polynomial } \wedge \text{ coefficients } \in \mathbb{Q}\}$	$\{5, -7, \frac{1}{2}, \sqrt{2}\}$	$2x^2 + 4x - 7 = 0$
\mathbb{T}	Transcendental numbers	$\{x \in \mathbb{R} : x \notin \mathbb{A}, x \notin \mathbb{Q}\}$	$\{\pi, e, e^\pi, \sin(x), \log_b a\}$	$\mathbb{T} = \mathbb{U} - \mathbb{A}$
\mathbb{R}	Real numbers	$\{x : x \text{ corresponds to a number on the number line}\}$	$\{\pi, 3.1415, -1, \frac{7}{8}, \sqrt{2}\}$	$x^2 - 2 = 0$
\mathbb{I}	Imaginary numbers	$\{b : bi \text{ where } i = \sqrt{-1}\}$	$\{2i, \sqrt{-1}\}$	$x^2 + 1 = 0$
\mathbb{C}	Complex numbers	$\{a, b \in \mathbb{R} : a + bi\}$	$\{1 + 2i, -3.4i, \frac{5}{8}\}$	$x^2 - 4x + 5 = 0$
\mathbb{U}	Universal set	all possible values in a particular context		
$\{0\}$	Zero integer	$\{x \in \mathbb{Z} : x = 0\}$	$\{0\}$	$n = 0$
$\mathbb{Z} - \{0\}$	Non-zero integers	$\{x \in \mathbb{Z} : x \neq 0\}$	$\{\dots, -3, -2, -1, 1, 2, 3, \dots\}$	$n \neq 0$
\mathbb{Z}^+	Positive integers	$\{x \in \mathbb{Z} : x > 0\}$	$\{1, 2, 3, \dots\}$	$n > 0$
\mathbb{Z}^-	Negative integers	$\{x \in \mathbb{Z} : x < 0\}$	$\{\dots, -3, -2, -1\}$	$n < 0$
$\mathbb{N} \cup \{0\}$	Non-negative integers	$\{x \in \mathbb{Z} : x \geq 0\}$	$\{0, 1, 2, 3, \dots\}$	$n \geq 0$
$\mathbb{Z}^- \cup \{0\}$	Non-positive integers	$\{x \in \mathbb{Z} : x \leq 0\}$	$\{\dots, -3, -2, -1, 0\}$	$n \geq 0$
$\{0\}, \mathbb{R}^\times$	Zero real	$\{x \in \mathbb{R} : x = 0\}$	$\{0, 0\}$	$x = 0$
$\mathbb{R} - \{0\}$ $\mathbb{R} \setminus \{0\}$	Non-zero real numbers	$\{x \in \mathbb{R} : x \neq 0\}$	$\{-0.001, 0.002\}$	$x \neq 0$
\mathbb{R}^+ $(0, \infty)$	Positive real numbers	$\{x \in \mathbb{R} : x > 0\}$	$\{0.0001, 0.0002, \dots\}$	$x > 0$

\mathbb{R}^- $(-\infty, 0)$	Negative real numbers	$\{x \in \mathbb{R} : x < 0\}$	$\{\dots, -0.0002, -0.0001\}$	$x < 0$
$[0, \infty)$	Non-negative real numbers	$\{x \in \mathbb{R} : x \geq 0\}$	$\{0, 0.0001, 0.0002, \dots\}$	$x \geq 0$
$(-\infty, 0]$	Non-positive real numbers	$\{x \in \mathbb{R} : x \leq 0\}$	$\{\dots, -0.0002, -0.0001, 0\}$	$x \leq 0$



Set Laws

Law	Union Example	Intersection Example
Idempotent Laws	$A \cup A = A$	$A \cap A = A$
Associative Laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity Laws	$A \cup \emptyset = A$	$A \cap \mathbb{U} = A$
Domination Laws	$A \cup \mathbb{U} = \mathbb{U}$	$A \cap \emptyset = \emptyset$
Double Complement Law	$(A^c)^c = A$	
Complement Laws	$A \cup A^c = \mathbb{U}$	$A \cap A^c = \emptyset$
Complements of \mathbb{U} and \emptyset	$\mathbb{U}^c = \emptyset$	$\emptyset^c = \mathbb{U}$
De Morgan's Laws	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$
Absorption Laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Set Difference Law		$A \setminus B = A \cap B^c$ $A - B = A \cap B^c$

Set Properties

Property	Description	Examples
Composition	Objects may be of various types. A set may contain elements of different varieties.	$A = \{2, \text{strewberry}, \text{monkey}\}$
Order	The order in which the elements are listed is unimportant	$A = \{10, 6, 4, 2\}$
Duplicates	Repeating an element does not change the set	$A = \{2, 2, 4, 6, 10\}$
Notation	Typically, capital letters will be used as variables denoting sets, and lower case letters will be used for elements in the set	$A = \{a, b\}$
Range	Every set A	$\emptyset \subseteq A \subseteq \mathbb{U}$
Empty Set	Set with no members.	\emptyset is a subset of every set.

Set Notation

Term	Definition	Examples
$\{\}$ $\{ \}$	Denotes a set	$A = \{a, e, i, o, u\}$
 :	'Such that' or 'for which'	$B = \{x \mid x \in \mathbb{N} \text{ and } x \leq 5\}$ $B = \{x: x \in \mathbb{N} \text{ and } x \leq 5\}$
\Rightarrow \equiv	Is equivalent or identical to	$(C \cap E) \Rightarrow (x \in C \wedge x \in E)$
$ A $ $n(A)$	Cardinality of A, the number of elements in set A	if $A = \{(1,2), (3,4), (5,6)\}$, then $ A = 3$
$A = B$	If and only if they have precisely the same elements. A is equal to B.	if $A = \{4, 9\}$ and $B = \{n^2 : n=2 \text{ or } n=3\}$, then $A = B$
$A \subseteq B$	If and only if every element of A is also an element of B. A is a subset of B.	$\{1, 8, 1107\} \subseteq \mathbb{N}$
$A \not\subseteq B$	A is not a subset of B. A is not contained in B.	$\{-1, -8, -1107\} \not\subseteq \mathbb{N}$
$A \subset B$	A is a proper subset of B. A is a subset of B that is not equal to B.	$\{1, 8, 1107\} \subset \mathbb{N}$
$A \not\subset B$	A is not a proper subset of B. A is not contained in B.	$\{-1, -8, -1107\} \not\subset \mathbb{N}$
$B \supseteq A$	If and only if every element of A is in B. B is a superset of A.	$\{1, 8, 1107\} \subseteq \mathbb{N}$
$a \in A$ $A \in B$ $a \in A$	A is a member of, an element of, or in A	$\frac{3}{4} \in \mathbb{Q}$
$a \notin A$	A is not a member of A, is not an element of A	$3.14 \notin \mathbb{Z}$
$A \cap B$ $A \cap B$ $A \cap B$	The set containing elements that are in both A and B. $A \cap B$ is the intersection of A and B.	if $A = \{1, 2\}$ and $B = \{2, 3\}$, then $A \cap B = \{2\}$
$A \cup B$ $A \cup B$ $A \cup B$	The set containing elements that are in either A or B or both. $A \cup B$ is the union of A and B.	if $A = \{1, 2\}$ and $B = \{2, 3\}$, then $A \cup B = \{1, 2, 3\}$
$A \setminus B$ $A - B$	Set difference. The set containing elements that are in A but not in B. $A \setminus B$ is "A drop B". $A - B$ is "A difference B".	if $A = \{1, 2\}$ and $B = \{2, 3\}$, then $A \setminus B = \{1\}$
$A \oplus B$	Symmetric difference is the set of elements that are a member of exactly one of A and B, but not both	$A \oplus B = (A - B) \cup (B - A)$
$A \cap B = \emptyset$	A and B are disjoint sets. No elements in common.	$A \cap B = \emptyset$
A^k	Cartisian product of a set A with itself	$A^k = A \times A \times \dots \times A$ k times

Logical Form of Set Notation

Set Notation	Logical Statement	Description
A	$x \in A$ $\forall x \{x \in A\}$	<ul style="list-style-type: none"> Is an element of
$\neg A$	$x \notin A$ $\forall x \{x \notin A\}$	<ul style="list-style-type: none"> Is not an element of
$A = B$ $A = B$	$A \leftrightarrow B$ $\forall x [(x \in A \rightarrow x \in B) \wedge (x \in B \rightarrow x \in A)]$ $A \subseteq B \wedge B \subseteq A$	<ul style="list-style-type: none"> Equal Equivalence Iff <u>def</u>
$A \neq B$ $A \neq B$	$\forall x (x \in A \wedge x \notin B)$	<ul style="list-style-type: none"> Not equal
$A \subseteq B$	$\forall x (x \in A \rightarrow x \in B)$ $\forall x \in A (x \in B)$ $x \notin A \setminus B$	<ul style="list-style-type: none"> Subset of $A \cap B = A \rightarrow A \subseteq B$
$A \not\subseteq B$	$\exists x (x \in A \wedge x \notin B)$	<ul style="list-style-type: none"> Not a subset of
$A \cap B$	$\forall x (x \in A \wedge x \in B)$	<ul style="list-style-type: none"> Intersection
$A \cup B$	$\forall x (x \in A \vee x \in B)$	<ul style="list-style-type: none"> Union
$A \setminus B$	$\forall x (x \in A \wedge x \notin B)$	<ul style="list-style-type: none"> Difference But Not
$A \oplus B$	$\forall x \{x \in A - B \vee x \in B - A\}$	<ul style="list-style-type: none"> Exactly one
$A \rightarrow B$	$\forall x (x \notin A \vee x \in B)$	<ul style="list-style-type: none"> If – Then
$A \cap B = \emptyset$	$\neg \exists x (x \in A \wedge x \in B)$ $\forall x \neg (x \in A \wedge x \in B)$ $\forall x (x \notin A \vee x \notin B)$ $\forall x (x \in A \rightarrow x \notin B)$	<ul style="list-style-type: none"> A and B are disjoint, having no elements in common
\mathcal{F}	$\{A_i \mid i \in I\}$	<ul style="list-style-type: none"> Family of sets
$x \in \bigcap \mathcal{F}$	$\{x \mid \forall A \in \mathcal{F} (x \in A)\}$ $\{x \mid \forall A (A \in \mathcal{F} \rightarrow x \in A)\}$	<ul style="list-style-type: none"> Intersection of family of sets
$x \in \bigcup \mathcal{F}$	$\{x \mid \exists A \in \mathcal{F} (x \in A)\}$ $\{x \mid \exists A (A \in \mathcal{F} \wedge x \in A)\}$	<ul style="list-style-type: none"> Union of family of sets
$\bigcap \mathcal{F}$	$\bigcap_{i \in I} A_i = \{x \mid \forall i \in I (x \in A_i)\}$ $\bigcap_{i \in I} A_i = A_1 \cap A_2 \cap A_3 \cap A_4 \cap \dots$	<ul style="list-style-type: none"> Intersection of an indexed family of sets
$\bigcup \mathcal{F}$	$\bigcup_{i \in I} A_i = \{x \mid \exists i \in I (x \in A_i)\}$ $\bigcup_{i \in I} A_i = \{x \in I \mid \exists i \in I A(i, x)\}$ $\bigcup_{i \in I} A_i = A_1 \cup A_2 \cup A_3 \cup A_4 \cup \dots$	<ul style="list-style-type: none"> Union of an indexed family of sets
$x \in \wp(A)$	$x \subseteq A$ $\forall y (y \in x \rightarrow y \in A)$	<ul style="list-style-type: none"> Power Set All subsets of set A, including \emptyset $\wp(A) = 2^{ A }$

Logical Form of Numbers

Definition	Logical Statement	Description
Even	$\exists k \in \mathbb{Z} (x = 2k)$ Set $E = \{2k : k \in \mathbb{Z}\}$ $2\mathbb{Z}$	<ul style="list-style-type: none"> Definition of Even
Odd	$\exists k \in \mathbb{Z} (x = 2k + 1)$ Set $O = \{2k + 1 : k \in \mathbb{Z}\}$	<ul style="list-style-type: none"> Definition of Odd
Prime	$\forall a, b \in \mathbb{Z}^+ (p \setminus ab \rightarrow p \setminus a \vee p \setminus b)$	<ul style="list-style-type: none"> A positive integer $p > 1$ that has no positive integer divisors other than 1 and p itself is prime. Here \setminus means "is a divisor of"
Not Prime	$\exists a, b \in \mathbb{Z}^+ (ab = n \wedge a < n \wedge b < n)$	<ul style="list-style-type: none"> a and b are factors of n, so not prime
Divides	$x y \leftrightarrow \exists k \in \mathbb{Z} (y = kx)$	<ul style="list-style-type: none"> Divisibility Divides Divides into x divides y evenly $x y$ to mean "x divides y," $x \nmid y$ means "x does not divide y"
Rational	$r \in \mathbb{R} \exists x, y \in \mathbb{Z} ((y \neq 0) \wedge (r = x/y)) \rightarrow r \in \mathbb{Q}$	<ul style="list-style-type: none"> Definition of Rational number A fraction composed of two integers, but no division by 0

Logical Form of Geometry

Definition	Logical Statement	Description
Line	$\{(x, y) \in \mathbb{R} \times \mathbb{R} y = mx + b\}$ $= \{(0, b), (1, m + b), (2, 2m + b), \dots\}$	<ul style="list-style-type: none"> You can think of the graph of the equation as a picture of its truth set!
Plane	$\mathbb{R} \times \mathbb{R} = \{(x, y) x \text{ and } y \text{ are real numbers}\}$	<ul style="list-style-type: none"> These are the coordinates of all the points in the plane $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$
3D Space	$\mathbb{R}^3 = \{(x, y, z) x, y \text{ and } z \text{ are real numbers}\}$	<ul style="list-style-type: none"> These are the coordinates of all the points in 3D space $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$
Spacetime	$\mathbb{R}^4 = \{(x, y, z, t) x, y, z \text{ and } t \text{ are real numbers}\}$	<ul style="list-style-type: none"> These are the coordinates of all the points in 3D space and 1D time $\mathbb{R}^4 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$

Logical Form of Functions

Definition	Logical Statement	Description
Function	$f: X \rightarrow Y$ $\forall x \in X \exists! y \in Y ((x, y) \in f)$ $f = \{(a, b) \in A \times B \mid b = f(a)\}$	<ul style="list-style-type: none"> Function f is a relation from A to B Example : $f = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = x^2\}$
Domain	$\text{Dom}(f)$ X	<ul style="list-style-type: none"> Domain of f
Range	$\text{Ran}(f)$ $\{f(a) \mid a \in A\}$ Y	<ul style="list-style-type: none"> Range \subseteq co-domain Co-domain Image of f (linear algebra term)
Surjection	$f = \forall y \in Y \{\exists \text{ at least one } x \in X \text{ such that } f(x) = y\}$ $\forall y \in Y, \exists x \in X \mid (f(x) = y)$ $\text{Ran}(f) = Y$	<ul style="list-style-type: none"> Onto Surjective f Every y is mapped to by at least one x No orphan y's <i>e.g., y is dating at least one x</i>
Injection	$f = \forall y \in Y \{\exists \text{ at most one } x \in X \text{ such that } f(x) = y\}$ $\neg \exists a_1 \in A \exists a_2 \in A (f(a_1) = f(a_2) \wedge a_1 \neq a_2)$ $\forall a_1, a_2 \in A \mid (f(a_1) = f(a_2) \rightarrow a_1 = a_2)$ $f(x) = f(y) \leftrightarrow x = y$ $f(x) \neq f(y) \leftrightarrow x \neq y$	<ul style="list-style-type: none"> One-to-one Injective f For any y there is at most one x Can have orphan y's <i>e.g., y is either married or single</i>
Bijection	$f = \text{iff } \forall y \in Y \{\exists \text{ a unique } x \in X \text{ such that } f(x) = y\}$ $f(Y) = y \leftrightarrow f^{-1}(y) = X$	<ul style="list-style-type: none"> Bijection = surjective and injective One-to-one correspondence Bijection f Invertible f Iff has a well-defined inverse (f^{-1}) Iff both surjective and injective One-to-one and onto <i>e.g., Everyone is married to a spouse</i>
Inverse	$f^{-1}: B \rightarrow A$ $\forall b \in B \exists! a \in A ((b, a) \in f^{-1})$ $f(g(x)) = x$ $f^{-1} \circ f = i_A$ and $f \circ f^{-1} = i_B$	<ul style="list-style-type: none"> Inverse f
k-to-1 Correspondence	<p>Let X and Y be finite sets. The function $f: X \rightarrow Y$ is a k-to-1 correspondence if for every $y \in Y$, there are exactly k different $x \in X$ such that $f(x) = y$.</p>	<ul style="list-style-type: none"> Bijection is $k = 1$

Cartesian Product

Set Notation	Logical Statement	Description
$A \times B$ $A \times B$ $A \times B$ $A \times B$	$\{(a, b) \mid a \in A \wedge b \in B\}$ $\{(a, b) \mid a \in A, b \in B\}$	<ul style="list-style-type: none"> • Cartesian product • Cross product • Set of all ordered pairs in which the first entry is in A and the second entry is in B

Properties of Cartesian Products

Law	Logical Statement	Description
Distributive	$A \times (B \cap C) = (A \times B) \cap (A \times C)$	• $\times \cap$
	$A \times (B \cup C) = (A \times B) \cup (A \times C)$	• $\times \cup$
Commutative	$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$	• $\times \cap \times$
	$(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$	• $\times \cup \times$
Domination	$A \times \emptyset = \emptyset$ $\emptyset \times A = \emptyset$	• $\times \emptyset$



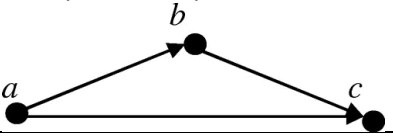
Relations

Property	i_A	Equivalence (=)	Partial Order (Poset)	Total Order (Linear)
Reflexive	✓	✓	✓	
Symmetric	✓	✓		
Anti-Symmetric			✓	✓
Asymmetric				
Transitive	✓	✓	✓	✓
Total				✓
Density				
Binary Relation		✓		

Set Relations (xRy)

Set Notation	Logical Statement	Description
Relation	$R \subseteq A \times B$ $\forall x (x \in R \rightarrow x \in A \times B)$ $R = \{(a, b) \in A \times B \mid \text{conditions}\}$ $xRy = (x, y) \in R$ <p>Example: $D_r = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x \text{ and } y \text{ differ by less than } r\} \Rightarrow x - y < r$</p>	<ul style="list-style-type: none"> Relation from A to B R is a subset of the cross product
Domain	$\text{Dom}(R)$ $\{a \in A \mid \exists b \in B ((a, b) \in R)\}$ $\text{Dom}(A) \subseteq A$	<ul style="list-style-type: none"> The domain of R is the set containing all the first coordinates of its ordered pairs
Codomain (Target)		<ul style="list-style-type: none"> All possible values in the range set Ran(R) is a subset of the Target The set of the <u>possible</u> output values of a function The definition of a function
Range (Image)	$\text{Ran}(R)$ $\{b \in B \mid \exists a \in A ((a, b) \in R)\}$ $\text{Ran}(B) \subseteq B$	<ul style="list-style-type: none"> The range of R is the set containing all the second coordinates of its ordered pairs The <u>actual</u> or most accurate output values of a function The image of a function
Inverse (R^{-1})	$\{(y, x) \in Y \times X \mid (x, y) \in R\}$ $(y, x) \in R^{-1} \leftrightarrow (x, y) \in R$ $(x, y) \in R^{-1} \rightarrow (y, x) \in R$	<ul style="list-style-type: none"> The inverse of R is the relation R^{-1} from B to A with the order of the coordinates of each pair reversed
Composition ($S \circ R$)	$S \circ R = \{(a, c) \in S \circ R \mid \exists b \mid (a, b) \in R \text{ and } (b, c) \in S\}$ $\{(a, c) \in A \times C \mid \exists b \in B ((a, b) \in R \text{ and } (b, c) \in S)\}$ $aRb \text{ and } bSc$ $\{(a, c) \in A \times C \mid \exists b \in B (aRb \wedge bSc)\}$	<ul style="list-style-type: none"> The composition of S and R is the relation $S \circ R$ from A to C aRb and bSc, meaning $R:a \rightarrow R:b \rightarrow S:b \rightarrow S:c$, so $(R:a, S:c)$ Ring operator
Identity (i_A)	$\{(x, y) \in A \times A \mid x = y\}$ $\{(x, x) \mid x \in A\}$	<ul style="list-style-type: none"> Identity relation

Order Properties of Binary Relations with Two Sets

Property	Logical Statement	Description
Reflexive	xRx $(x, x) \in R$ $\forall x \in A (xRx)$ $\forall x \in A ((x, x) \in R)$	<ul style="list-style-type: none"> $i_A \subseteq R$ where i_A is the identity relation of set A or $i_A = \{(x, x) \mid x \in A\}$ Directed graph: Loop 
Anti-Reflexive	$\neg (xRx)$ $\forall x \in A \neg (xRx)$	<ul style="list-style-type: none"> Directed graph: No loops
Symmetric	$xRy \rightarrow yRx$ $\forall x \in A \forall y \in A (xRy \rightarrow yRx)$	<ul style="list-style-type: none"> $R = R^{-1}$ Directed graph: 2-way arrow (edges come in pairs) or no arrows
Anti-Symmetric	$(xRy \wedge yRx) \rightarrow (x = y)$ $(x \neq y) \rightarrow \neg (xRy) \vee \neg (yRx)$ $\forall x \in A \forall y \in A ((xRy \wedge yRx) \rightarrow (x = y))$	<ul style="list-style-type: none"> Equivalence Directed graph: An arrow from x to y implies that there is no arrow from y to x <p>No: </p>
Asymmetric	$xRy \rightarrow \neg (yRx)$ $\forall x \in A \forall y \in A \forall z \in A (xRy \rightarrow \neg (yRx))$	<ul style="list-style-type: none"> Fails the vertical line test, so not a proper function, $f(x)$ Directed graph: 1-way arrow
Transitive	$(xRy \wedge yRz) \rightarrow xRz$ $\forall x \forall y \forall z ((xRy \wedge yRz) \rightarrow xRz)$ $\forall x \in A \forall y \in A \forall z \in A ((xRy \wedge yRz) \rightarrow xRz)$	<ul style="list-style-type: none"> $R \circ R \subseteq R$ Similar to $S \circ R$ Directed graph: Two routes from every vertex A to every vertex B, 1-hop and 2-hops 
Total	$xRy \vee yRx$ $\forall x \in A \forall y \in A (xRy \vee yRx)$	<ul style="list-style-type: none"> Either-or
Density	$xRy \rightarrow \exists z \mid xRz \wedge zRy$ $\forall x \in A \forall y (xRy) \rightarrow \exists z \mid xRz \wedge zRy$	<ul style="list-style-type: none"> A middle-man exists
Binary	$R^{-1} \circ R = \text{Relation on set A}$ $R \circ R^{-1} = \text{Relation on set C}$	<ul style="list-style-type: none"> Relation on set <set> Binary relation on set <set>
Identity	$i_A = \{(x, y) \in A \times A \mid x = y\}$ $i_A = \{(x, x) \mid x \in A\}$	<ul style="list-style-type: none"> Similar to a diagonal matrix

Mathematical Number Sets → Computer Science Data Types

Symbol	Definition	C Data Type	C++ Data Type
\emptyset	empty set, set with no members	void	
\mathbb{N}	natural numbers	enum unsigned unsigned char unsigned short unsigned int unsigned long unsigned long long	
\mathbb{Z}	integers	char short int long long long	
\mathbb{Q}	rational numbers	NA	std::ratio<1, 10>
\mathbb{R}	real numbers	float double long double	
\mathbb{I}	imaginary numbers	(see complex below) double complex z1; im = cimag(z1);	(see complex below) std::complex <double> z1; im = std::imag(z1);
\mathbb{C}	complex numbers	#include <complex.h> float complex double complex long double complex	#include <complex> std::complex<float> std::complex <double> std::complex <long double>

Sources:

- [SNHU MAT 470](#) - Real Analysis, [The Real Numbers and Real Analysis](#), Ethan D. Bloch, Springer New York, 2011.
- See also “Harold’s Logic Cheat Sheet”.
- <https://www.storyofmathematics.com/set-notation>
- <https://math24.net/set-identities.html>