Harold's Series Cheat Sheet

11 November 2020

Operation	Arithmetic	Geometric
Summation Notation	$S_n = \sum_{k=1}^{n} a_k$	$S_n = \sum_{k=0}^{n-1} a_0 r^k = \sum_{k=1}^{n} a_0 r^{k-1}$
Summation Expanded	$S_n = a_1 + a_2 + \cdots + a_{n-1} + a_n$	$S_n = a_0 + a_0 r + a_0 r^2 + \cdots + a_0 r^{n-1}$
Sum of n Terms (finite series)	$S_n = n\left(\frac{a_1+a_n}{2}\right)$ or $S_n = \frac{n}{2}(2a_1 + (n-1)d)$	$S_n = a_0 \frac{(1-r^n)}{1-r}$
Sum of ∞ Terms (infinite series)	$S \to \infty$	$S = \frac{a_0}{1-r}$ *where* $\|r\| < 1$
Recursive n^th Term	$a_n = a_{n-1} + d$	$a_n = a_{n-1} r$
Explicit n^th Term	$a_n = a_1 + d(n-1)$	$a_n = a_0 r^{n-1}$

Operation	Formula		
Factorials	$n! = n \bullet (n-1) \bullet (n-2) \bullet \ldots \bullet 3 \bullet 2 \bullet 1$		
Permutations	${}_nP_r = \frac{n!}{(n-r)!}$		
Combinations	${}_nC_r = \frac{n!}{r!\,(n-r)!} = \binom{n}{r}$		
Pascal's Triangle	1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1	${}_0C_0$ ${}_1C_0$ ${}_1C_1$ ${}_2C_0$ ${}_2C_1$ ${}_2C_2$ ${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$ ${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$ ${}_5C_0$ ${}_5C_1$ ${}_5C_2$ ${}_5C_3$ ${}_5C_4$ ${}_5C_5$	$\binom{0}{0}$ $\binom{1}{0}$ $\binom{1}{1}$ $\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$ $\binom{3}{0}$ $\binom{3}{1}$ $\binom{3}{2}$ $\binom{3}{3}$
Binomial Theorem	$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k =$ $\binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \boldsymbol{\binom{n}{r} a^{n-r} b^r} + \cdots + \binom{n}{n-1} a b^{n-1} + \binom{n}{n} b^n$		
Example	$(a+b)^0 = 1$ $(a+b)^1 = a + b$ $(a+b)^2 = a^2 + 2ab + b^2$ $(a+b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$ $(a+b)^4 = a^4 + 4a^3 b + 6a^2 b^2 + 4ab^3 + b^4$ $(a+b)^5 = a^5 + 5a^4 b + 10a^3 b^2 + 10a^2 b^3 + 5ab^4 + b^5$		

Binomial Series	Expanded
$$(1+x)^r = \sum_{n=0}^{+\infty} \binom{r}{n} x^n$$	$$(1+x)^r = 1 + \sum_{n=1}^{+\infty} \frac{r(r-1)(r-2)\ldots(r-n+1)}{n!} x^n$$ $$= 1 + rx + \frac{r(r-1)}{2!} x^2 + \frac{r(r-1)(r-2)}{3!} x^3 + \cdots$$
Converges for $\lvert x \rvert < 1$ *and all complex* r *,* $r \neq 0$*, where* $$\binom{r}{n} = \prod_{k=1}^{n} \frac{r-k+1}{k}$$	$$\binom{r}{n} = \frac{r(r-1)(r-2)\ldots(r-n+1)}{n!}$$