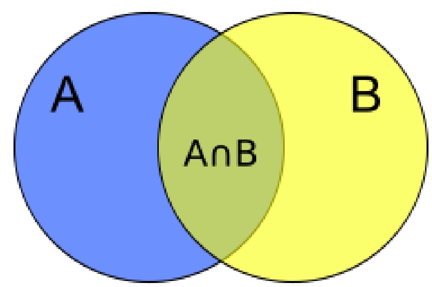
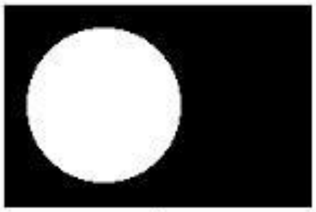
Harold's Probability
Cheat Sheet
30 May 2016

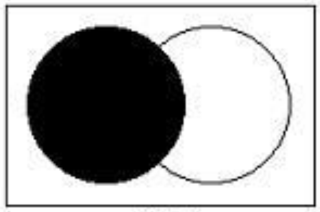
Probability

Rule	Formula	Definition
Notation	∩ = "and", Intersection, or "∧" ∪ = "or", Union, or "∨" $\overline{\square}$ = "not", negation, or "¬"	"And" implies multiplication. "Or" implies addition. "Not" implies negation.
Independent	If $P(A\|B) = P(A)$	The occurrence of one event does not affect the probability of the other, or vice versa.
Dependent	If $P(A \cap B) \neq 0$	The occurrence of one event affects the probability of the other event.
Disjoint/Mutually Exclusive	$P(A \cap B) = 0$ $P(A \cup B) = P(A) + P(B)$	The events can never occur together.
Probability	$P(A) = \dfrac{\#\ of\ Favorable\ Outcomes}{Total\ \#\ of\ Possible\ Outcomes}$ $where\ 0 \leq P(A) \leq 1$	
Addition Rule ("or")	$\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$ $P(A \cup B) = P(A) + P(B)$ *(if disjoint)*	
Multiplication Rule ("and")	$P(A \cap B) = P(A)\ P(B\|A)$ $P(A \cap B) = P(B)\ P(A\|B)$ $P(A \cap B) = P(B \cap A)$ $P(A \cap B) = P(A) - P\left(A \cap \overline{B}\right)$ $\boldsymbol{P(A \cap B) = P(A)\ P(B)}$ *(if independent)*	
Compliment Rule / Subtraction Rule ("not")	$P\left(A \cup \overline{A}\right) = P(A) + P\left(\overline{A}\right) = 1$ $P(A) = 1 - P\left(\overline{A}\right)$ $P(\overline{A}) = 1 - P(A)$	The compliemnt of event A (denoted $\overline{A}\ or\ A^c$) means "**not A**"; it consists of all simple outcomes that are not in A.
Conditional Probability ("given that")	$\boldsymbol{P(A\|B) = \dfrac{P(A \cap B)}{P(B)}}$ $P(A\|B) = P(A)$ *(if independent)* $P(B\|A) = P(B)$ *(if independent)*	Means the probability of A given B. Is a rephrasing of the Multiplication Rule. P(A|B) is the proportion of elements in B that are ALSO in A.
Total Probability Rule	$P(A) = P(A \cap B_1) + \cdots + P(A \cap B_n)$ $= P(B_1)\ P(A\|B_1) + \cdots + P(B_n)\ P(A\|B_n)$ $P(A) = P(A \cap B) + P(A \cap \bar{B})$ $= P(B)\ P(A\|B) + P(\bar{B})\ P(A\|\bar{B})$	To find the probability of event A, partition the sample space into several disjoint events. A must occur along with one and only one of the disjoint events.
Bayes' Theorem	$P(A\|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)\ P(B\|A)}{P(B)}$ $= \dfrac{P(A)\ P(B\|A)}{P(A)\ P(B\|A) + P(\bar{A})P\ (B\|\bar{A})}$	Allows us to reverse the order of a conditional probability statement, and is the only generally valid method!
De Morgan's Law	$\overline{P(A \cup B)} \equiv \overline{P(A)} \cap \overline{P(B)}$ $\overline{P(A \cap B)} \equiv \overline{P(A)} \cup \overline{P(B)}$	Uses negation to convert an "or" to an "and". Uses negation to convert an "and" to an "or".

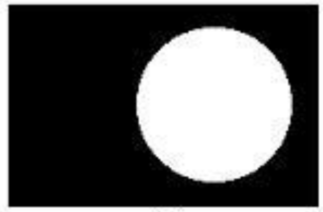
Venn Diagrams



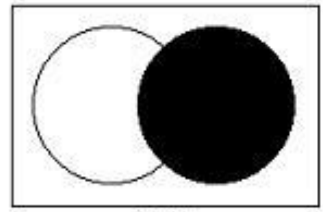
A



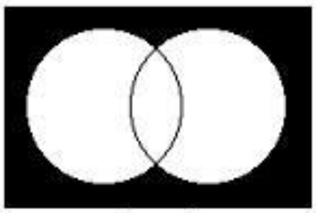
not A



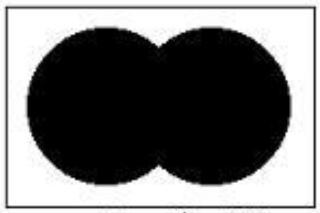
B



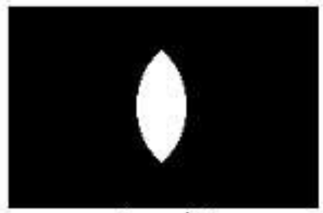
not B



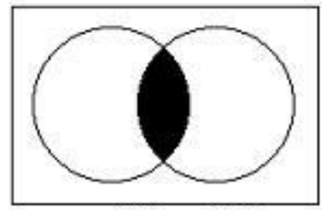
A or B



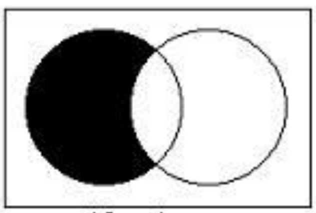
not A and not B



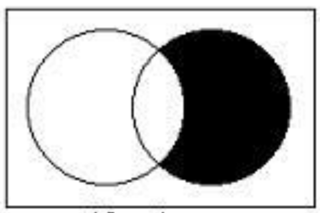
A and B



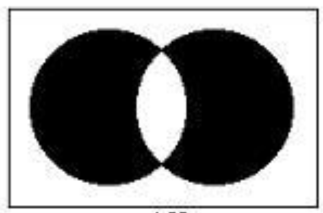
not (A and B)



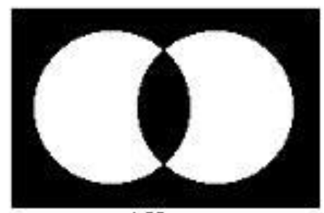
if A then B



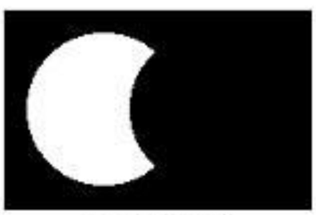
if B then A



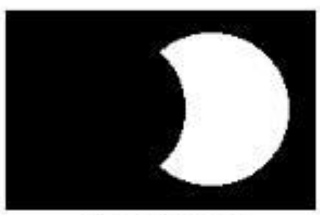
A iff B



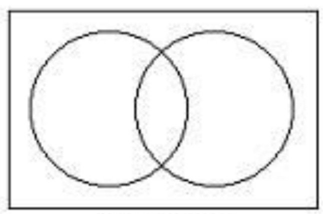
A iff not B



A and not B



B and not A



Tautology

Contradiction