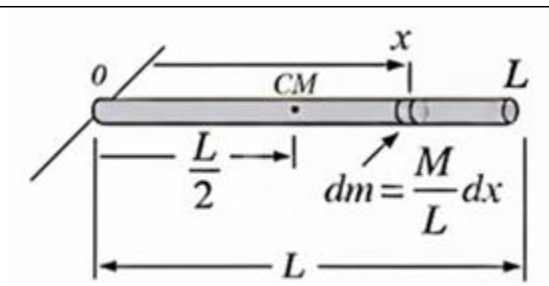
Harold's Moment of Inertia
Cheat Sheet
14 January 2026

Moment of Inertia Terms

Term	Description	Units
CoM vs. MoI	• Center of Mass (CoM) describes where mass is concentrated. Linear motion. • Moment of Inertia (MoI) describes how mass resists rotational motion. Rotational motion.	$m\ vs.\ kg \cdot m^2$
I	Moment of Inertia, mass moment of Inertia, or rotational inertia of a body. Inertia is the tendency of a body to resist changes in its velocity.	$kg \cdot m^2\ or\ lb \cdot ft^2$
I_x	Moment of Inertia about the x-axis	$kg \cdot m^2$
I_y	Moment of Inertia about the y-axis	$kg \cdot m^2$
I_z	Moment of Inertia about the z-axis	$kg \cdot m^2$
I_0	Moment of Inertia about the origin (Polar)	$kg \cdot m^2$
I_{xy}	Moment of Inertia about the x-axis & y-axis diagonal line	$kg \cdot m^2$
M	Total mass. How heavy the object is. Is equal to its area or volume if uniform density. (similar to Weight)	$kg\ or\ lb$
L	Length of the object	m
A	Area of the object	m^2
ρ	Greek symbol rho for density or mass / volume or mass / area.	$\rho = \frac{kg}{m^3}\ or\ \frac{lb}{ft^3}$
		

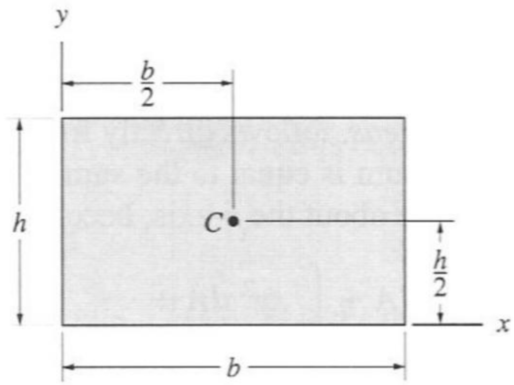
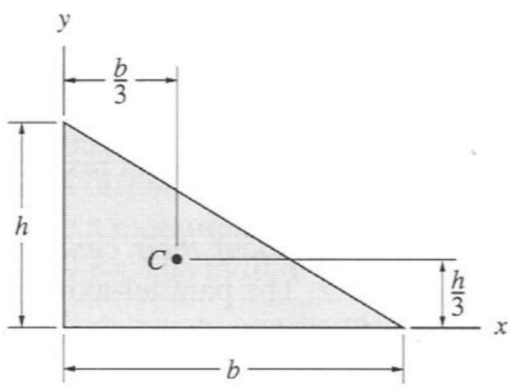
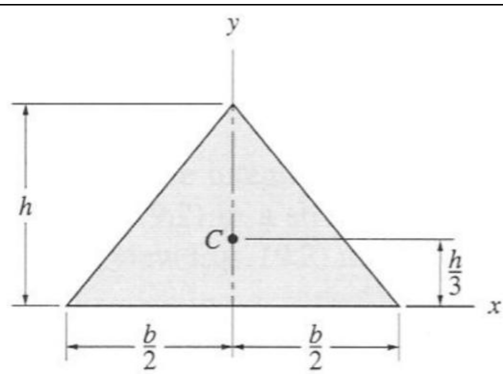
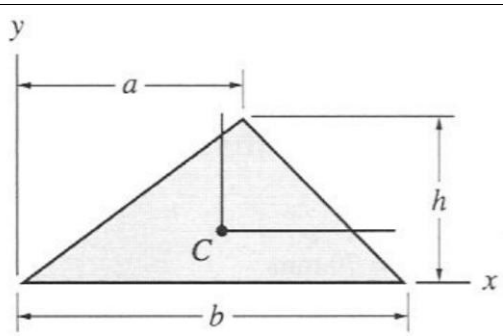
Moment of Inertia: Discrete or for Particles

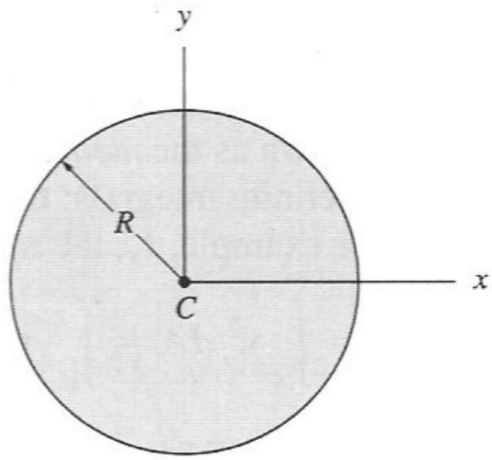
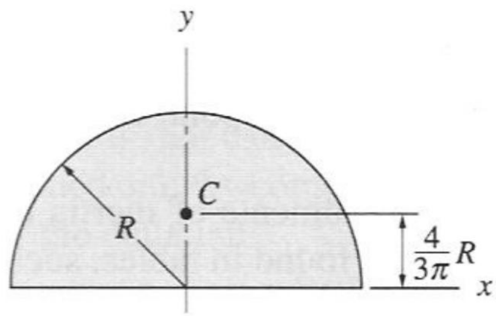
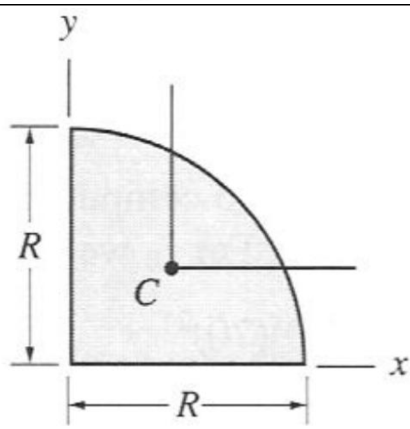
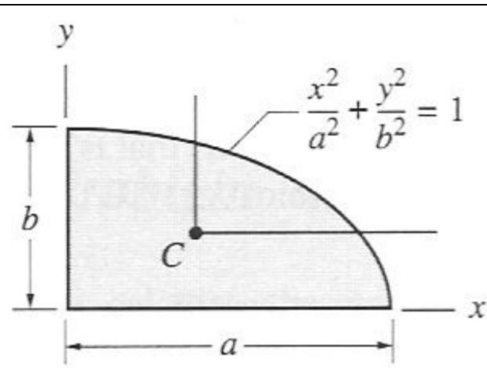
Term	1D	2D	3D
I	$I = m\,r^2 = \frac{L}{\omega}$	$I = \sum_{i=1}^{N} m_i\,r_i^2 = \int r^2\,dm$	NA
I_x	$I_x = \sum_{i=1}^{j} x_i^2 \frac{M}{L} \Delta x$	$I_x = \sum_{i=1}^{k}\sum_{j=1}^{l} y^2\,\rho(x,y)\,\Delta A$	$I_x = \sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} (y^2+z^2)\,\rho(x,y,z)\,\Delta V$
I_y	NA	$I_y = \sum_{i=1}^{k}\sum_{j=1}^{l} x^2\,\rho(x,y)\,\Delta A$	$I_y = Above\ with\ (x^2+z^2)$
I_z	NA	NA	$I_z = Above\ with\ (x^2+y^2)$
I_0	$I_0 = I_x$	$I_0 = I_x + I_y$	$I_0 = I_x + I_y + I_z$

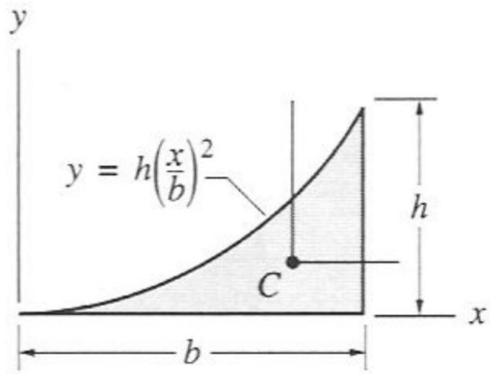
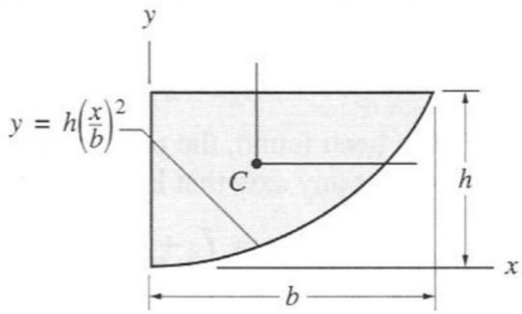
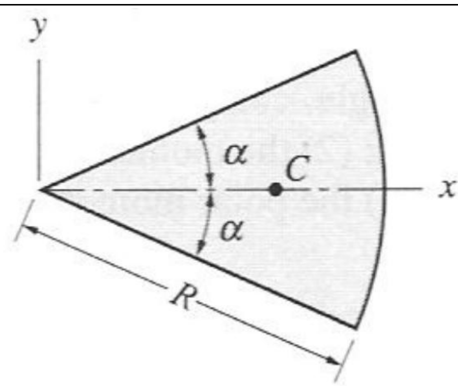
Moment of Inertia: Continuous

Term	2D	3D
I	$I = \int_0^a m\,r^2\,dr$ $I = \iint_A \rho(\boldsymbol{r})\,d(\boldsymbol{r})^2\,dA(\boldsymbol{r})$	$I = \iiint_V \rho(\boldsymbol{r})\,d(\boldsymbol{r})^2\,dV(\boldsymbol{r})$
I_x	$I_x = \iint_R y^2\,\rho(x,y)\,dA$	$I_x = \iiint_Q (y^2+z^2)\,\rho(x,y,z)\,dV$
I_y	$I_y = \iint_R x^2\,\rho(x,y)\,dA$	$I_y = \iiint_Q (x^2+z^2)\,\rho(x,y,z)\,dV$
I_z	NA	$I_z = \iiint_Q (x^2+y^2)\,\rho(x,y,z)\,dV$

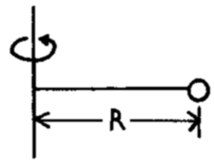
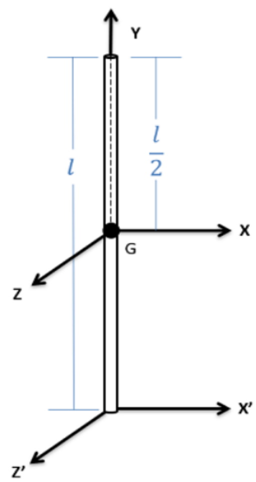
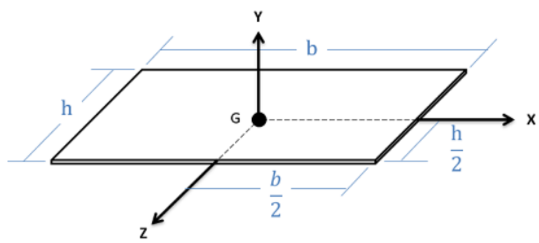
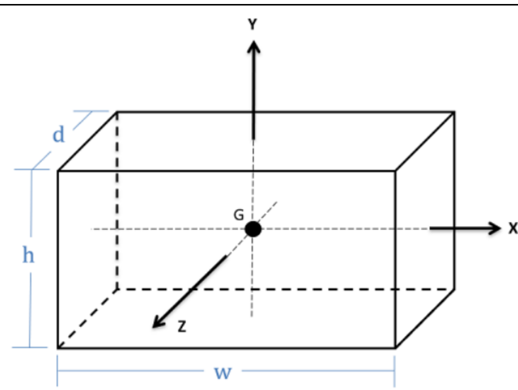
Moments of Inertia of Common 2D Plane Areas

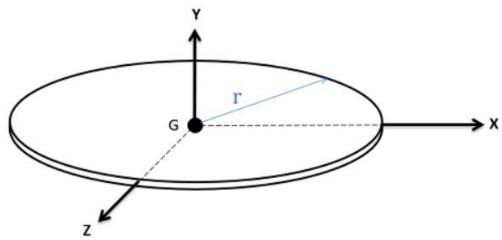
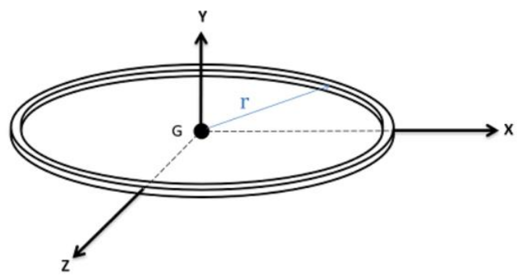
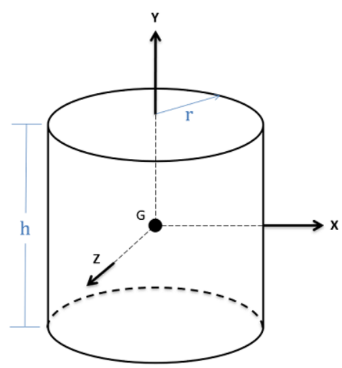
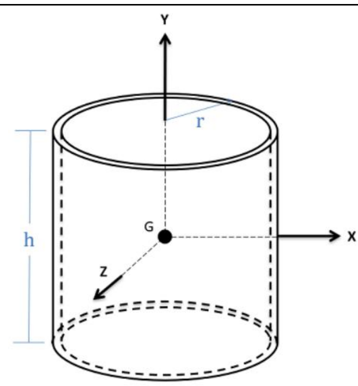
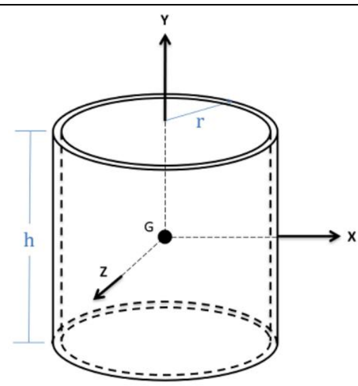
Shape	Diagram	Formulas
Rectangle		$I_x = \frac{1}{3}Ah^2 = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $I_{xy} = \frac{1}{4}b^2h^2$
Right Triangle		$I_x = \frac{1}{12}Ah^2 = \frac{1}{12}bh^3$ $I_y = \frac{1}{12}b^3h$ $I_{xy} = \frac{1}{24}b^2h^2$
Isosceles Triangle		$I_x = \frac{1}{6}Ah^2 = \frac{1}{12}bh^3$ $I_y = \frac{1}{16}b^3h$ $I_{xy} = 0$
Triangle		$I_x = \frac{1}{6}Ah^2 = \frac{1}{12}bh^3$ $I_y = \frac{1}{12}bh(a^2 + ab + b^2)$ $I_{xy} = \frac{1}{24}h^2(2a - b)$

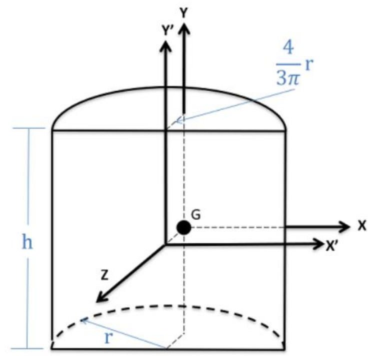
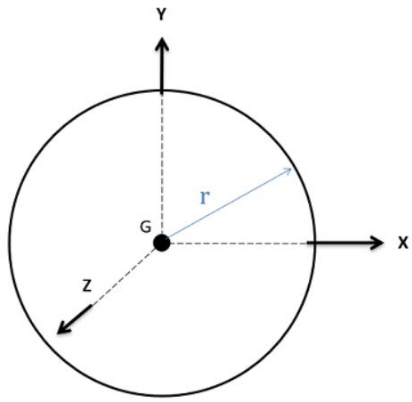
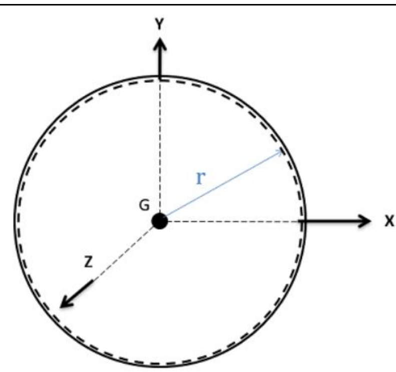
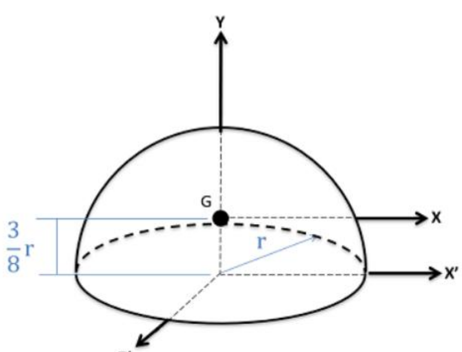
Circle		$I_x = \frac{1}{4}AR^2 = \frac{1}{4}\pi R^4$ $I_y = \frac{1}{4}AR^2 = \frac{1}{4}\pi R^4$ $I_{xy} = 0$
Semicircle		$I_x = \frac{1}{4}AR^2 = \frac{1}{8}\pi R^4$ $I_y = \frac{1}{4}AR^2 = \frac{1}{8}\pi R^4$ $I_{xy} = 0$
Quarter Circle		$I_x = \frac{1}{4}AR^2 = \frac{1}{16}\pi R^4$ $I_y = \frac{1}{4}AR^2 = \frac{1}{16}\pi R^4$ $I_{xy} = \frac{1}{16}R^4$
Quarter Ellipse		$I_x = \frac{1}{4}AR^2 = \frac{1}{16}\pi ab^3$ $I_y = \frac{1}{4}AR^2 = \frac{1}{16}\pi a^3 b$ $I_{xy} = \frac{1}{8}a^2b^2$

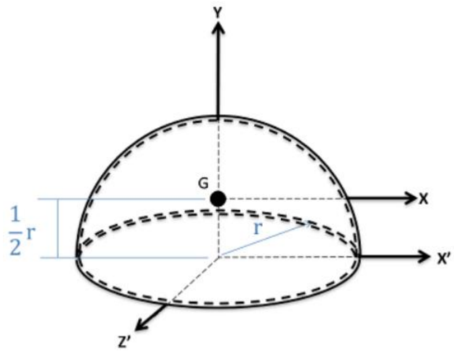
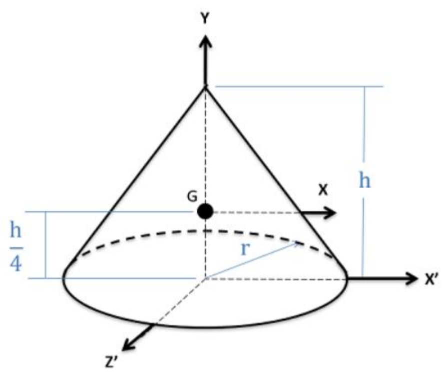
<table>
<tr><td>Half Parabola Complement</td><td>
</td><td>$I_x = \frac{1}{21} bh^3$
$I_y = \frac{1}{5} b^3 h$
$I_{xy} = \frac{1}{12} b^2 h^2$</td></tr>
<tr><td>Half Parabola</td><td>
</td><td>$I_x = \frac{2}{7} bh^3$
$I_y = \frac{2}{15} b^3 h$
$I_{xy} = \frac{1}{6} b^2 h^2$</td></tr>
<tr><td>Circular Sector</td><td>
</td><td>$I_x = \frac{1}{8} R^4 (2\alpha - \sin 2\alpha)$
$I_y = \frac{1}{8} R^4 (2\alpha + \sin 2\alpha)$
$I_{xy} = 0$</td></tr>
<tr><td colspan="3">Note: All formulas shown assume objects of uniform mass density.</td></tr>
</table>

Moments of Inertia of Common 3D Solids

Shape	Diagram	Formulas
Point Mass		$I_x = MR^2$
Slender Rod		$I_x = \frac{1}{12}M\ell^2$ $I_y = 0$ $I_z = \frac{1}{12}M\ell^2$ $I_{x'} = \frac{1}{3}M\ell^2$ $I_{z'} = \frac{1}{3}M\ell^2$
Flat Rectangular Plate		$I_x = \frac{1}{12}Mh^2 = \frac{1}{12}bh^3$ $I_y = \frac{1}{12}M(h^2 + b^2)$ $I_z = \frac{1}{12}\mathrm{M}b^2$
Rectangular Prism		$I_x = \frac{1}{12}M(h^2 + d^2)$ $I_y = \frac{1}{12}M(d^2 + w^2)$ $I_z = \frac{1}{12}M(h^2 + w^2)$ $V = dwh$

Flat Circular Plate		$I_x = \frac{1}{4}MR^2 = \frac{1}{4}\pi R^4$ $I_y = \frac{1}{2}MR^2 = \frac{1}{2}\pi R^4$ $I_z = \frac{1}{4}MR^2 = \frac{1}{4}\pi R^4$
Thin Circular Ring		$I_x = \frac{1}{2}MR^2$ $I_y = MR^2$ $I_z = \frac{1}{2}MR^2$
Solid Cylinder		$I_x = \frac{1}{4}MR^2 + \frac{1}{12}Mh^2$ $I_y = \frac{1}{2}MR^2$ $I_z = \frac{1}{4}MR^2 + \frac{1}{12}Mh^2$ $V = \pi r^2 h$
Thin Cylindrical Shell		$I_x = \frac{1}{6}M(3R^2 + h^2)$ $I_y = MR^2$ $I_z = \frac{1}{6}M(3R^2 + h^2)$
Thick Cylindrical Shell		$I_y = \frac{1}{2}M(R_2^2 + R_1^2)$

Half Cylinder		$I_x = \left(\frac{1}{4} - \frac{16}{9\pi^2}\right) MR^2 + \frac{1}{12} Mh^2$ $I_y = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right) MR^2$ $I_z = \left(\frac{1}{4} - \frac{16}{9\pi^2}\right) MR^2 + \frac{1}{12} Mh^2$ $V = \frac{1}{2}\pi r^2 h$
Solid Sphere		$I_x = \frac{2}{5} MR^2$ $I_y = \frac{2}{5} MR^2$ $I_z = \frac{2}{5} MR^2$ $V = \frac{4}{3}\pi r^3$
Spherical Shell		$I_x = \frac{2}{3} MR^2$ $I_y = \frac{2}{3} MR^2$ $I_z = \frac{2}{3} MR^2$
Solid Hemisphere		$I_x = \frac{83}{320} MR^2$ $I_y = \frac{2}{5} MR^2$ $I_z = \frac{83}{320} MR^2$ $I_{x'} = \frac{2}{5} MR^2$ $I_{z'} = \frac{2}{5} MR^2$

Hemispherical Shell		$I_x = \frac{5}{12}MR^2$ $I_y = \frac{2}{3}MR^2$ $I_z = \frac{5}{12}MR^2$ $I_{x'} = \frac{2}{3}MR^2$ $I_{z'} = \frac{2}{3}MR^2$
Right Circular Cone		$I_x = \frac{3}{80}M(4R^2 + h^2)$ $I_y = \frac{3}{10}MR^2$ $I_z = \frac{3}{80}M(4R^2 + h^2)$ $I_{x'} = \frac{1}{20}M(3R^2 + 2h^2)$ $I_{z'} = \frac{1}{20}M(3R^2 + 2h^2)$ $V = \frac{1}{3}\pi r^2 h$
Note: All formulas shown assume objects of uniform mass density.		

Sources

- Matchev, Konstantin (2026). University of Florida, Department of Physics, PHY4222, Table of Selected Moments of Inertia. https://www.phys.ufl.edu/~matchev/phy4222/momi_table.pdf
- Mechanics Map – Open Text Project (2026). Center of Mass and Mass Moments of Inertia for Homogeneous Bodies. https://mechanicsmap.psu.edu/websites/centroidtables/centroids3D/centroids3D.html
- University of Utah, Dept. of Mechanical Engineering (2026). ME 1300 - Statics & Strength of Materials, Selected Centroid and Moment of Inertia Shapes. https://my.mech.utah.edu/~me1300/Selected.pdf

See Also

- Harold's Physics Center of Mass Cheat Sheet
- Harold's Physics Moment of Inertia Cheat Sheet

