

Maxwell's Equations

Gauss's Law for Electricity

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\int E \cdot dA = \frac{Q_{inside}}{\epsilon_0}$$

Gauss's Law for Magnetism

$$\nabla \cdot B = 0$$

$$\int B \cdot dA = \frac{d\Phi_B}{dt}$$

Faraday's Law of Induction

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

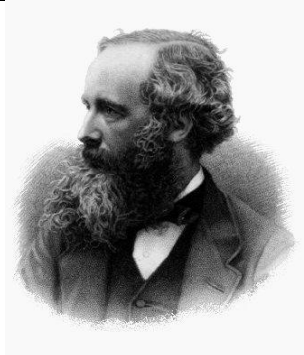
$$\int E \cdot dl = -\frac{d\Phi_B}{dt}$$

Ampère-Maxwell Law

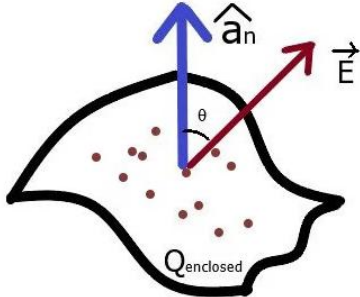
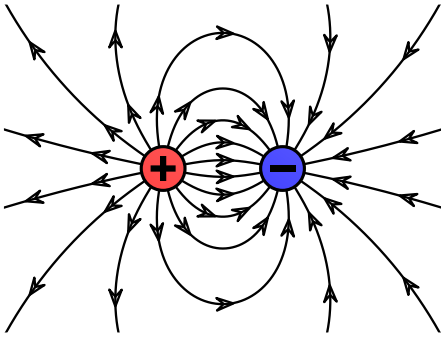
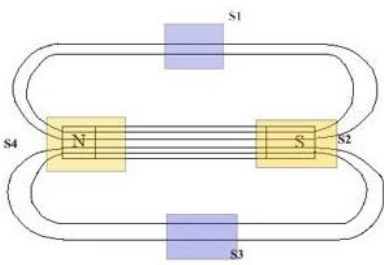
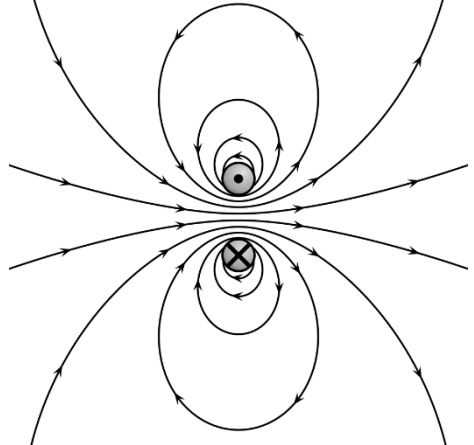
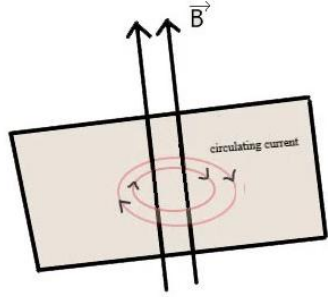
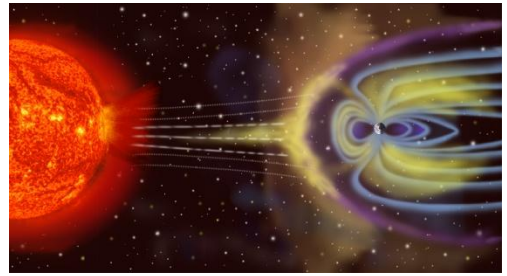
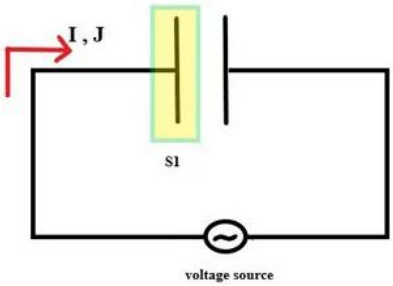
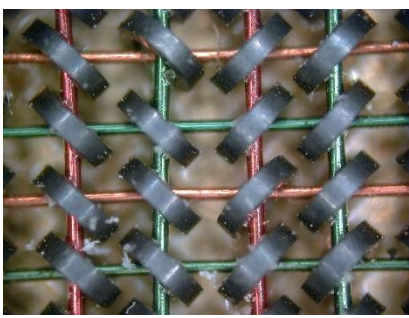
$$\nabla \times B = \mu_0 J + \mu_0 \frac{\partial E}{\partial t}$$

$$\int B \cdot dl = \mu_0 I + \frac{d\Phi_E}{dt}$$

Maxwell's Equations

Name	Differential Form	Integral Form
Maxwell's Equations in Electromagnetism	A set of four pivotal laws that describe the interactions between electric charges, magnetic fields, and electric currents.	 <p>James Clerk Maxwell</p>
1. Gauss' Law for Electricity	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$ $\nabla \cdot \mathbf{D} = \rho_f$	$\Phi_E = \oiint \mathbf{E} \cdot d\mathbf{S} = \frac{Q(V)}{\epsilon_0}$ $= \frac{1}{\epsilon_0} \iiint_{\Omega} \rho \, dV$ $\Phi_E = \oiint \mathbf{D} \cdot d\mathbf{S} = Q_f(V)$
2. Gauss' Law for Magnetism	$\nabla \cdot \mathbf{B} = 0$	$\Phi_B = \oiint \mathbf{B} \cdot d\mathbf{S} = 0$
3. Faraday's Law of Induction (Maxwell-Faraday Equation)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E} \cdot d\boldsymbol{\ell} = - \iint \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$ $= -\frac{d}{dx} \iint \mathbf{B} \cdot d\mathbf{S}$ $= -\frac{\Delta \Phi_B}{\Delta t}$
4. Ampère-Maxwell Law	$\nabla \times \mathbf{B} = \mu_0 \left(\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J} \right)$ $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}_f$	$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I + \mu_0 \epsilon_0 \iint \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{S}$ $= \mu_0 \iint_{\Sigma} \mathbf{J} \cdot d\mathbf{S} + \mu_0 \epsilon_0 \frac{d}{dx} \iint \mathbf{E} \cdot d\mathbf{S}$ $= \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ $\oint \mathbf{H} \cdot d\boldsymbol{\ell} = I_f + \mu_0 \epsilon_0 \iint \frac{\partial \mathbf{E}}{\partial t} \cdot d\mathbf{S}$ $= \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

Maxwell's Equations Diagrams

Name	Application	Diagram
1. Gauss' Law of Electricity	 <p>A diagram showing a closed surface with a normal vector \hat{a}_n and an electric field vector \vec{E} at an angle θ. The enclosed charge is labeled Q_{enclosed}.</p> $\nabla \cdot \vec{E} = \rho / \epsilon_0$	 <p>A diagram showing electric field lines radiating from a positive charge (red circle with a plus sign) and converging on a negative charge (blue circle with a minus sign).</p>
2. Gauss' Law of Magnetism	 <p>A diagram showing magnetic field lines forming closed loops around a bar magnet with North (N) and South (S) poles. The poles are labeled S1, S2, S3, and S4.</p> $\nabla \cdot \vec{B} = 0$	 <p>A diagram showing magnetic field lines forming closed loops around a current-carrying wire, with arrows indicating the direction of the field.</p>
3. Faraday's Law of Induction (Maxwell-Faraday Equation)	 <p>A diagram showing a magnetic field vector \vec{B} passing through a rectangular loop. A circulating current is induced in the loop.</p> $\nabla \times \vec{E} = - \partial \vec{B} / \partial t$	 <p>A diagram showing a solar flare or magnetic field interaction, with a bright orange sun on the left and a purple magnetic field structure on the right.</p>
4. Ampère-Maxwell Law	 <p>A diagram showing a circuit with a voltage source and a current-carrying wire. The current is labeled I, J and the wire is labeled S1.</p> $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \partial \vec{E} / \partial t$	 <p>A photograph of a woven mesh structure, likely a component of a microwave oven or a similar application.</p>

Symbols and Terms

Term Name	Variable	Value
Vector	<bold>	<i>Magnitude and direction</i>
Scalar	<italics>	Magnitude only
Electric Field	E	
Magnetic Field	B	
Displacement Field	D	
Magnetization Field	H	
Electric Charge Density	ρ	Total charge per unit volume
Electric Current Density	J	Total current per unit area
Permittivity of free space	ϵ_0	
Permeability of free space	μ_0	
Speed of Light	c	$c = (\epsilon_0 \mu_0)^{-\frac{1}{2}}$
Nabia Symbol	∇	Three-dimensional gradient operator, del.
Divergence Operator	$\nabla \cdot$	del dot
Curl Operator	$\nabla \times$	del cross
Surface Integral	\oiint	With a loop indicating the surface is closed
Volume Integral	\iiint	
Line Integral	\oint	With a loop indicating the curve is closed
Surface Integral	\iint	
Net Electric Flux	ϕ_E	$\phi_E = \oiint \mathbf{B} \cdot d\mathbf{S}$
Net Magnetic Flux	ϕ_B	$\phi_B = \oiint \mathbf{B} \cdot d\mathbf{S}$
Net Electric Current	I	$I = \iint_{\Sigma} \mathbf{J} \cdot d\mathbf{S}$
Surface Area	$d\mathbf{S}$	Differential vector element of surface area S , normal to surface Σ .
Time	t	seconds
Length	ℓ	meters
Free Charge	Q_f	Volts
Free Current	I_f	Amperes

Sources

- BlogSpot.com (17 February 2013). Laws of Electric/Electrical Engineering, Maxwell's Equations. <https://electrical-laws.blogspot.com/2013/02/maxwells-equations.html>
- Wikipedia (11 January 2026). Maxwell's Equations. https://en.wikipedia.org/wiki/Maxwell's_equations