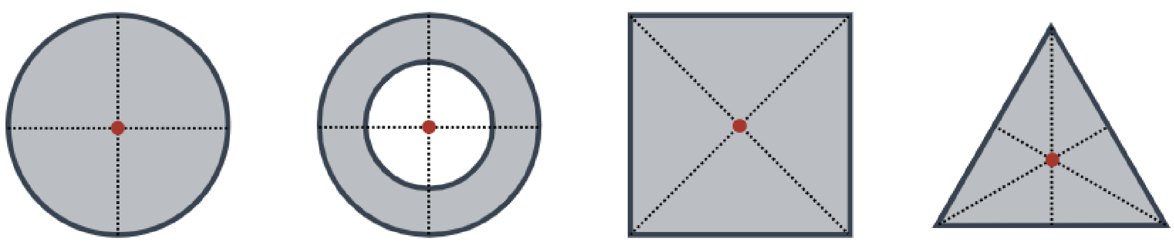
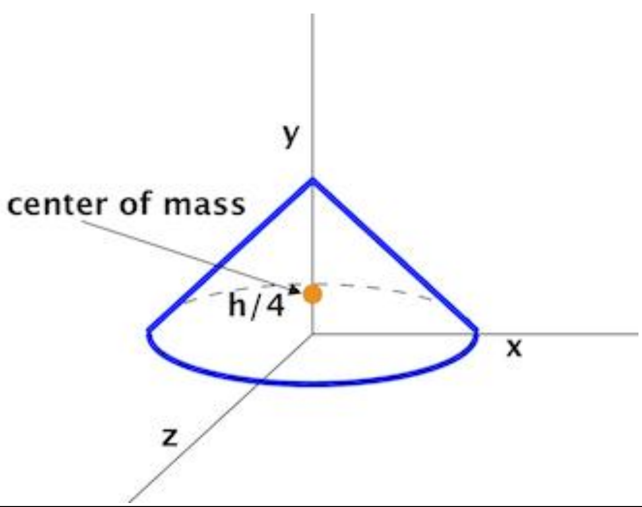
Harold's Center of Mass
Cheat Sheet

14 January 2026

Center of Mass Terms

Term	Description	Units
$\bar{x}$ $(\bar{x}, \bar{y})$ $(\bar{x}, \bar{y}, \bar{z})$	<u>Center of Mass (CoM)</u>. The coordinates (point) where the object is perfectly balanced. (Centroid)	$m \text{ or } ft$
$\bar{x}$	Center of Mass along the x-axis. Same as x_{cm}.	$m \text{ or } ft$
$\bar{y}$	Center of Mass along the y-axis.	$m \text{ or } ft$
$\bar{z}$	Center of Mass along the z-axis.	$m \text{ or } ft$
M	Total mass. How heavy the object is. Is equal to its area or volume if uniform density. (similar to Weight)	$kg \text{ or } lb$
$M_x \text{ and } M_y$	A <u>moment</u> is the line or axis on which the object can spin perfectly balanced.	$kg \text{ or } lb$
M_x	Moment about the x-axis	$kg \text{ or } lb$
M_y	Moment about the y-axis	$kg \text{ or } lb$
ρ	Greek symbol rho for density or mass/volume or mass/area.	$\rho = \frac{kg}{m^3} \text{ or } \frac{lb}{ft^3}$
A	Area of lamina or plate	$m^2 \text{ or } ft^2$
V	Volume of a body or solid	$m^3 \text{ or } ft^3$

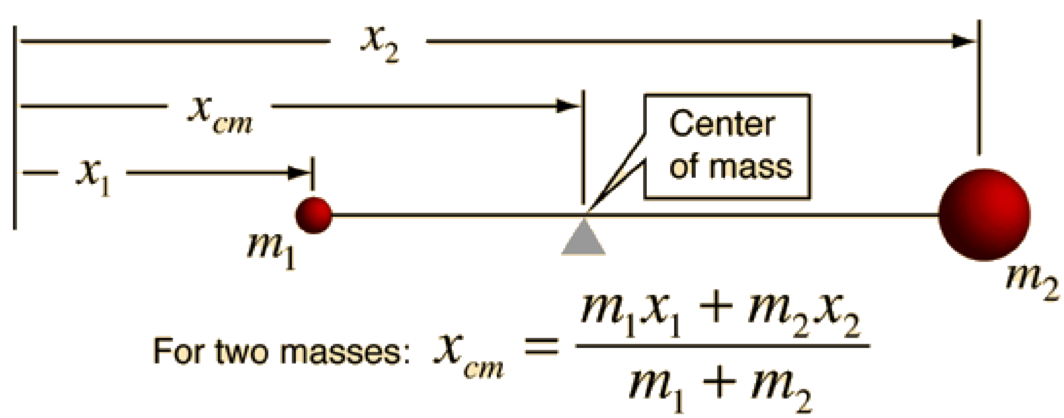




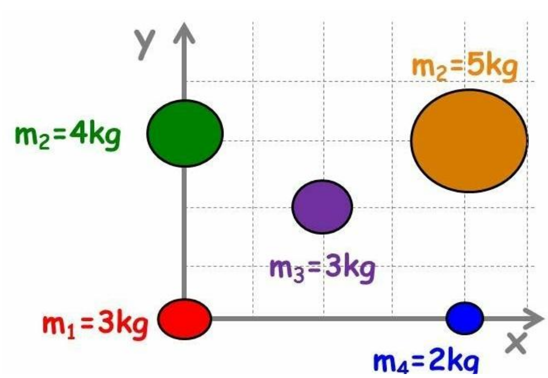
Center of Mass: Discrete or for Particles

Term	1D	2D	3D
$\bar{x}$	$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$	$\bar{x} = \frac{M_y}{M}$	$\bar{x} = \frac{1}{M} \sum_{i=1}^{N} m_i\, x_i$
$\bar{y}$	$\bar{y} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$	$\bar{y} = \frac{M_x}{M}$	$\bar{y} = \frac{1}{M} \sum_{i=1}^{N} m_i\, y_i$
$\bar{z}$	$\bar{z} = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$	NA	$\bar{z} = \frac{1}{M} \sum_{i=1}^{N} m_i\, z_i$
M	$M = \sum_{i=1}^{N} m_i = \int dm$		
M_x	$M_x = \sum_{i=1}^{N} m_i\, y_i = \int y\, dm$		
M_y	$M_y = \sum_{i=1}^{N} m_i\, x_i = \int x\, dm$		

1D

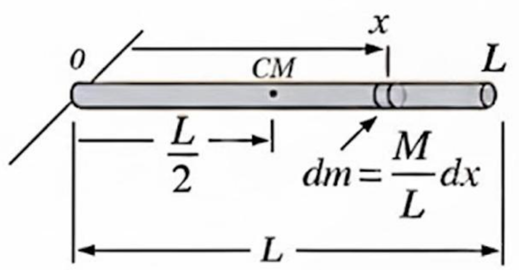


2D

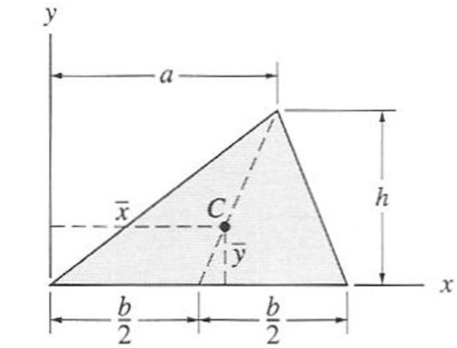
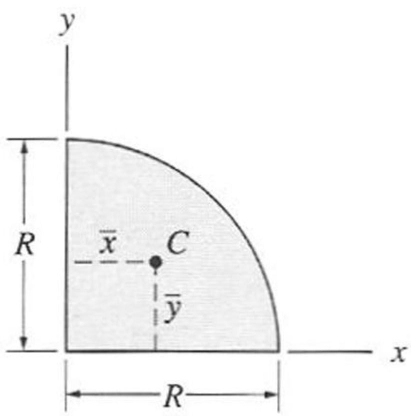
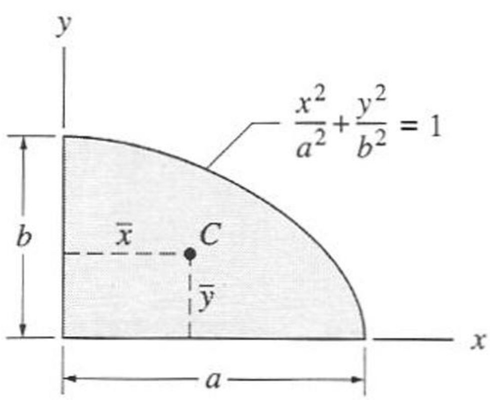
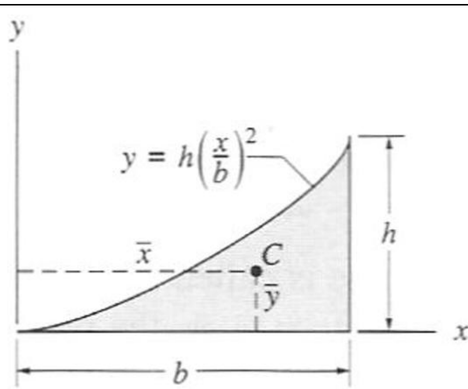


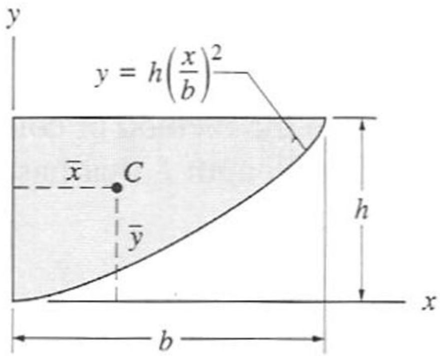
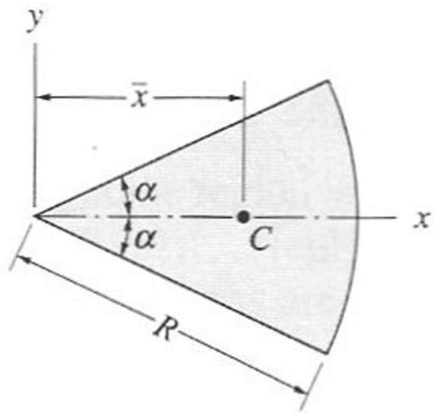
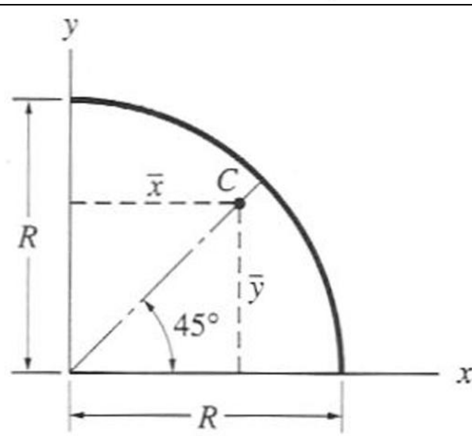
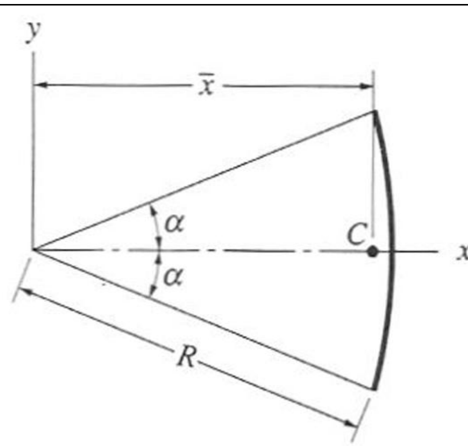
TIP: Pick the xy-axis origin to be at the corner of a right angle (e.g., red mass m_1).

Center of Mass: Continuous

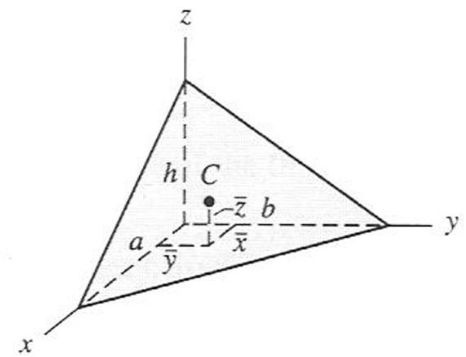
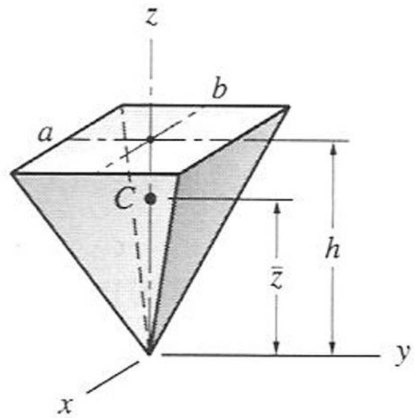
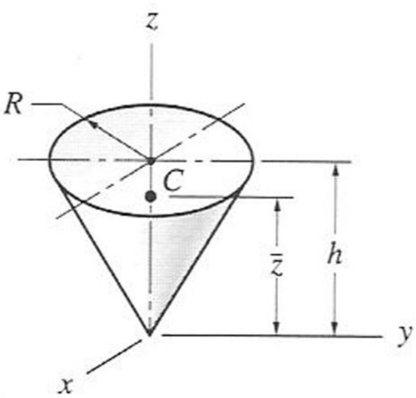
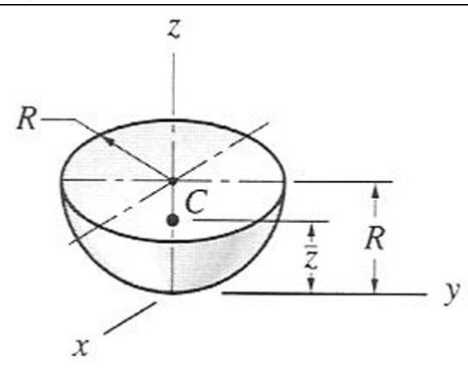
Term	2D	3D
$\bar{x}$	$\bar{x} = \frac{M_y}{M}$	$\bar{x} = \frac{1}{M}\int_0^M x\,dm = \frac{1}{M}\int_0^M x\,\frac{M}{L}dx = \frac{M_{yz}}{M}$
$\bar{y}$	$\bar{y} = \frac{M_x}{M}$	$\bar{y} = \frac{1}{M}\int_0^M y\,dm = \frac{M_{xz}}{M}$
$\bar{z}$	NA	$\bar{z} = \frac{1}{M}\int_0^M z\,dm = \frac{M_{xy}}{M}$ $\bar{z} = \frac{1}{V}\int_{z_{min}}^{z_{max}} z\,dV$
M	$M = \rho\,(Area)$ $M = \rho\int_a^b f(x)\,dx$ $M = \iint_R dm = \iint_R \rho(x,y)\,dA$ $dm = \rho(x,y)\,dy\,dx$	$M = \rho\,(Volume)$ $M = \iiint_Q dm = \iiint_Q \rho(x,y,z)\,dV$ $dm = \rho(x,y,z)\,dz\,dy\,dx$
M_x	$M_x = \rho\iint_R y\,dA = \rho\int_a^b \frac{1}{2}([f(x)]^2)\,dx$	$M_{xy} = \iiint_Q z\,\rho(x,y,z)\,dV$
M_y	$M_y = \rho\iint_R x\,dA = \rho\int_a^b x\,f(x)\,dx$	$M_{yz} = \iiint_Q x\,\rho(x,y,z)\,dV$
M_z	NA	$M_{xz} = \iiint_Q y\,\rho(x,y,z)\,dV$
	$x_{cm} = \frac{\int_0^L x\frac{M}{L}dx}{M} = \frac{1}{L}\frac{x^1}{2}\Big\|_{x=0}^{x=L} = \frac{L}{2}$	

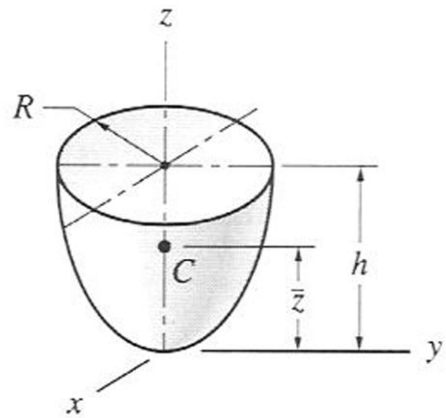
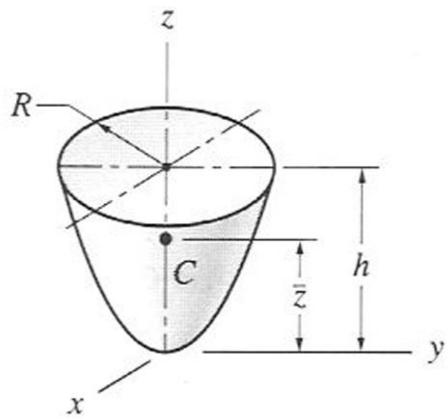
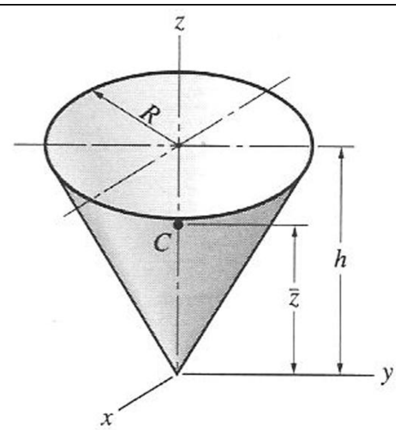
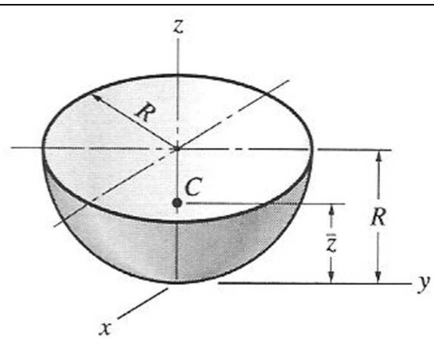
Center of Mass of Common 2D Plane Areas and Curves

Shape	Diagram	Formulas
Triangle		$\bar{x} = \frac{1}{3}(a+b)$ $\bar{y} = \frac{1}{3}h$ $A = \frac{1}{2}bh$
Quarter Circle		$\bar{x} = \frac{4}{3\pi}R$ $\bar{y} = \frac{4}{3\pi}R$ $A = \frac{1}{4}\pi R^2$
Quarter Ellipse		$\bar{x} = \frac{4}{3\pi}a$ $\bar{y} = \frac{4}{3\pi}b$ $A = \frac{1}{4}\pi ab$
Half Parabolic Complement		$\bar{x} = \frac{3}{4}b$ $\bar{y} = \frac{3}{10}h$ $A = \frac{1}{3}bh$

Half Parabola		$\bar{x} = \frac{3}{8}b$ $\bar{y} = \frac{3}{5}h$ $A = \frac{2}{3}bh$
Circular Sector		$\bar{x} = \frac{2R\, sin\, \alpha}{3\alpha}$ $A = \alpha R^2$
Quarter Circle Arc		$\bar{x} = \frac{2}{\pi}R$ $\bar{y} = \frac{2}{\pi}R$ $L = \frac{1}{2}\pi R$
Circular Arc		$\bar{x} = \frac{R\, sin\, \alpha}{\alpha}$ $L = \alpha R$
Note: All formulas shown assume objects of uniform mass density.		

Center of Mass of Common 3D Volumes and Surfaces

Shape	Diagram	Formulas
Right Tetrahedron		$\bar{x} = \frac{1}{4}a$ $\bar{y} = \frac{1}{4}b$ $\bar{z} = \frac{1}{4}h$ $V = \frac{1}{6}abh$
Pyramid		$\bar{z} = \frac{3}{4}h$ $V = \frac{1}{3}abh$
Cone		$\bar{z} = \frac{3}{4}h$ $V = \frac{1}{3}\pi R^2 h$
Hemisphere		$\bar{z} = \frac{5}{8}R$ $V = \frac{2}{3}\pi R^3$

Semi-Ellipsoid of Revolution		$\bar{z} = \frac{5}{8}R$ $V = \frac{2}{3}\pi R^2 h$
Paraboloid of Revolution		$\bar{z} = \frac{2}{3}h$ $V = \frac{1}{2}\pi R^2 h$
Conical Surface		$\bar{z} = \frac{2}{3}h$ $A = \pi R\sqrt{R^2 + h^2}$
Hemispherical Surface		$\bar{z} = \frac{1}{2}R$ $A = 2\pi R^2$
Note: All formulas shown assume objects of uniform mass density.		

Sources

- Mechanics Map – Open Text Project (2026). Center of Mass and Mass Moments of Inertia for Homogeneous Bodies. https://mechanicsmap.psu.edu/websites/centroidtables/centroids3D/centroids3D.html
- University of Utah, Dept. of Mechanical Engineering (2026). ME 1300 - Statics & Strength of Materials, Selected Centroid and Moment of Inertia Shapes. https://my.mech.utah.edu/~me1300/Selected.pdf

See Also

- Harold's Physics Center of Mass Cheat Sheet
- Harold's Physics Moment of Inertia Cheat Sheet

