Harold's Logic & Sets Cheat Sheet

4 November 2021

The 7 Basic Logical Symbols

Operator	Symbol	Example	English
Intersection	∧, **∧**, ∧, ⋀, ⋀	P ∧ Q	and, overlap
Union	∨, **∨**, ∨, ⋁, ⋁	P ∨ Q	or, both combined
Negation	¬, ⌐	¬P	not
Conditional	→, →, **→**, ⟶	P → Q	If … then …
Biconditional	↔, ↔, **↔**, ⇔, ⟺	P ↔ Q	if and only if (iff)
Universal Quantifier	∀*x*	∀*x P(x)*	for all
Existential Quantifier	∃*x*	∃*x P(x)*	there exists

- The structure of all mathematical statements can be understood using these symbols.
- All mathematical reasoning can be analyzed in terms of the proper use of these symbols.

Logical Connective Laws

Law	Union Example	Intersection Example
Identity Laws	P ∨ F = P	P ∧ T = P
Null Laws	P ∨ T = T	P ∧ F = F
Idempotent Laws	P ∨ P = P	P ∧ P = P
Double Negations or Involution Law	¬ ¬P = P	
Complementary Laws	P ∨ ¬P = T	P ∧ ¬P = F
Commutative Laws	P ∨ Q = Q ∨ P	P ∧ Q = Q ∧ P
Associative Laws	(P ∨ Q) ∨ R = P ∨ (Q ∨ R)	(P ∧ Q) ∧ R = P ∧ (Q ∧ R)
Distributive Laws	P ∧ (Q ∨ R) = (P ∧ Q) ∨ (P ∧ R)	P ∨ (Q ∧ R) = (P ∨ Q) ∧ (P ∨ R)
Uniting Laws	(P ∧ Q) ∨ (P ∧ ¬Q) = P	(P ∨ Q) ∧ (P ∨ ¬Q) = P
Absorption Laws	P ∨ (P ∧ Q) = P	P ∧ (P ∨ Q) = P
De Morgan's Laws	P ∨ Q = ¬(¬P ∧ ¬Q) ¬(P ∨ Q) = ¬P ∧ ¬Q	P ∧ Q = ¬(¬P ∨ ¬Q) ¬(P ∧ Q) = ¬P ∨ ¬Q
Multiplying and Factoring Laws	(P ∨ Q) ∧ (¬P ∨ R) = (P ∧ R) ∨ (¬P ∧ Q)	(P ∧ Q) ∨ (¬P ∧ R) = (P ∨ R) ∧ (¬P ∨ Q)
Consensus Laws	(P ∧ Q) ∨ (Q ∧ R) ∨ (¬P ∧ R) = (P ∧ Q) ∨ (¬P ∧ R)	(P ∨ Q) ∧ (Q ∨ R) ∧ (¬P ∨ R) = (P ∨ Q) ∧ (¬P ∨ R)
Tautology Laws (⊤)	P ∨ (⊤) = ⊤	P ∧ (⊤) = P
	¬(⊤) = ⊥	
Contradiction Laws (⊥)	P ∨ (⊥) = P	P ∧ (⊥) = ⊥
	¬(⊥) = ⊤	

Logical Conditional Connective Laws

Law or Statement	Logical Expression	Is Equivalent To (≡)	Description
Conditional Laws	$P \longrightarrow Q$	$\neg P \lor Q$ $\neg(P \land \neg Q)$	Conditional, If ... Then, Implication • If P, then Q. • P implies Q. • Q, if P. • P only if Q. • Q in case that P. • P is a sufficient condition for Q. • Q is a necessary condition for P.
Biconditional Laws	$P \longleftrightarrow Q$	$(P \longrightarrow Q) \land (Q \longrightarrow P)$ $(P \longrightarrow Q) \land (\neg P \longrightarrow \neg Q)$ $(P \land Q) \lor (\neg P \land \neg Q)$	Bi-conditional, If and only If, iff, XNOR • P iff Q • P if and only if Q • P is a necessary and sufficient condition for Q.
Sufficient Condition	P is a sufficient condition for Q	The truth of P suffices to guarantee the truth of Q.	
Necessary Condition	Q is a necessary condition for P	In order for P to be true, it is necessary for Q to be true also. $\neg Q \longrightarrow \neg P$	
Equivalence	$P \longleftrightarrow Q$	$P \Longrightarrow Q$ $P \equiv Q$	Is equivalent to
Contrapositive	$P \longrightarrow Q$	$\equiv \neg Q \longrightarrow \neg P$	True
Converse*	$P \longrightarrow Q$	$\not\equiv Q \longrightarrow P$	False
Inverse*	$P \longrightarrow Q$	$\not\equiv \neg P \longrightarrow \neg Q$	False

Logical Predicates and Quantifiers

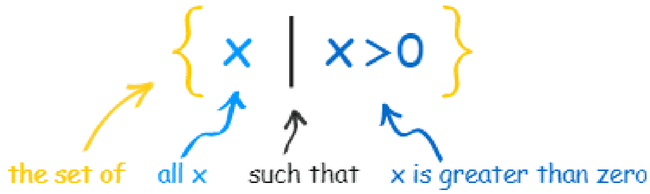
Definition	Logical Expression	Is Equivalent To (≡)	Description
Universe of Discourse	***U***	All possible inputs in a given range	• Universe of Discourse • Universal Set • Universe
Domain of Discourse	$\mathbb{D}$	All possible inputs in a given range	• Domain of Discourse • Universe of Discourse
Logical Statement	*P: "Roxy is a mammal"*	*P*	Must be True or False
Predicate	*P(x): "x is a mammal"*	*P(x)*	Truth depends upon the input variable *x*. $P(x) \neq$ a number.
Example Statements	*Q:* $\forall x \in \mathbb{D}, P(x)$: "x is a mammal"	*"For all x in the domain of discourse, P(x) is a mammal."*	Is either True or False. A quantified predicate turns it into a logical statement.
	T(x, y)	"*x* is a twin of *y*."	Predicate with two input variables
Truth Set (Single Free Variable)	$T = P(x)$	$T = \{a \mid P(a)\}$ $T = \{a \in A \mid P(a)\}$ $a \in T$	The set of all values of x that make the statement *P(x)* true
	Example:	$P(x_1)$, $P(x_2)$, and $P(x_3)$ are True	
Truth Set (Ordered Pair)	$T = P(x, y)$	$\{(a, b) \in A \times B \mid P(a, b)\}$ $(a, b) \in T$	Cross product truth set
	Examples:	$\{(p, n) \in P \times \mathbb{N}$ | the person p has n children$\} = \{$(John, 2), …$\}$ $\{(p, c, n) \in P \times C \times N$ | the person p has lived in the city c for n years$\}$	
Universal Quantifier	$\forall x\ P(x)$ $\forall x \in P(x)$ $\forall x \in \mathbb{D}, P(x)$ $\forall x$, *if x is in* $\mathbb{D}$ *then P(x)*	"For all *x* in the domain, *P(x)* is true" $\forall x \in A\ P(x) \equiv$ $\forall x\ (x \in A \longrightarrow P(x))$	Quantifying predicates • for all • for each member • every • everyone • everybody • everything • x could be anything at all
Existential Quantifier	$\exists x\ P(x)$ $\exists x \in P(x)$ $\exists x \in \mathbb{D}, P(x)$	"There exists x in the domain, such that *P(x)* is true"	Quantifying predicates • there exists an x • some • someone • somebody • at least one value of x • there is at least one x • the truth set is not equal to ∅

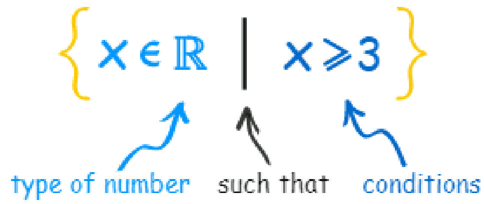
Uniqueness Quantifier	$\exists ! x\ P(x)$	there is a unique x in *P(x)* such that … $\exists x\ (P(x) \land \neg\ y(P(y) \land y \neq x))$ $\exists x\ (P(x) \land \forall y(P(y) \longrightarrow y = x))$ $\exists x\ \forall y(P(y) \leftrightarrow y = x)$ $\exists x\ P(x) \land \forall y\ \forall z((P(y) \land P(z)) \longrightarrow y = z)$	• unique • there is a unique x • there exists exactly one • there is exactly one x such that *P(x)*
Negated Existential Quantifier	$\neg\exists x\ P(x)$ $\nexists x\ P(x)$	$\forall x\ \neg P(x)$	• nobody • no one • not one • there does not exist

Quantifier Laws

Definition	Logical Expression	Is Equivalent To (≡)	Description
Abbreviation	$\exists x\ (x \in A \land \neg P(x))$	$\exists x \in A\ \neg P(x)$	• Simplification
Expanding Abbreviation	$\forall x \in A\ P(x))$	$\forall x\ (x \in A \rightarrow P(x))$	• Complication
Quantifier Negation Laws	$\forall x\ \neg P(x)$	$\neg\exists x\ P(x)$	• Nobody's perfect
	$\neg\forall x\ P(x)$	$\exists x\ \neg P(x)$	• Not everyone is perfect • Someone is imperfect
Conditional Law	$x \in A \rightarrow P(x)$	$x \notin A \lor P(x)$	• $P \longrightarrow Q \equiv \neg P \lor Q$
Subset Negation Law	$x \in A$	$\neg(x \notin A)$	• Swap ∈ with ∉, or vice versa
De Morgan's Law	$\neg(x \notin A \lor P(x))$	$x \in A \land \neg P(x)$	• $\neg\ (P \lor Q) = \neg P \land \neg Q$
Multiple-Quantified Statements	$\forall x\ \forall y$	$\forall y\ \forall x$	• For all objects x and y, …
	$\exists x\ \exists y$	$\exists y\ \exists x$	• There are objects x and y such that, …
	$\neg(\forall x\ \exists y\ P(x, y))$	$\exists x\ \forall y\ \neg P(x, y)$	• Negation of multiply-quantified statements
	$\neg(\exists x\ \forall y\ P(x, y))$	$\forall x\ \exists y\ \neg P(x, y)$	

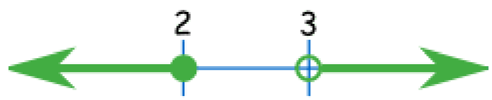
Set-Builder Notation





Set-Builder Notation: $\{ x \in \mathbb{R} \mid x \leq 2 \text{ or } x > 3 \}$

Number Line:



Interval Notation: $(-\infty, 2] \cup (3, +\infty)$

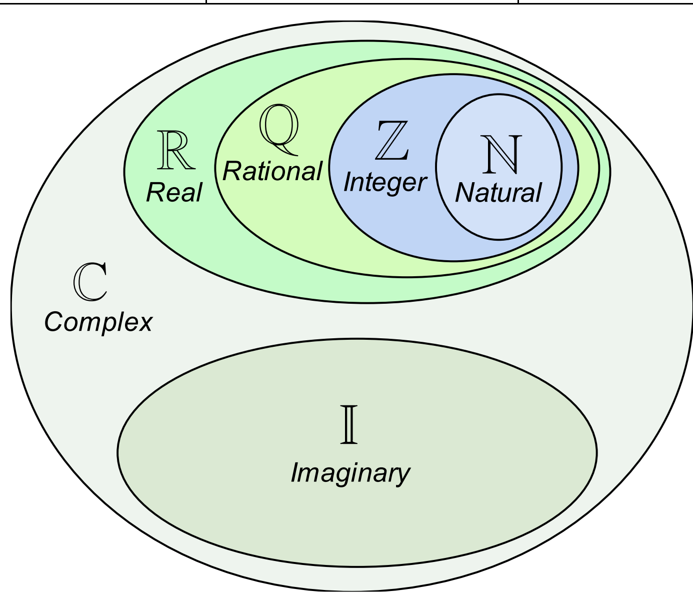
Set Operations

Law	Union Example	Intersection Example
Identity Laws	$A \cup \emptyset = A$	$A \cap \mathbb{U} = A$
Domination Laws	$A \cup \mathbb{U} = \mathbb{U}$	$A \cap \emptyset = \emptyset$
Idempotent Laws	$A \cup A = A$	$A \cap A = A$
Complement Laws	$A \cup A^c = \mathbb{U}$	$A \cap A^c = \emptyset$
Double Complement Law	$(A^c)^c = A$	
Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative Laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
De Morgan's Laws	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$
Absorption Laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Complements of $\mathbb{U}$ and $\emptyset$	$\mathbb{U}^c = \emptyset$	$\emptyset^c = \mathbb{U}$
Set Difference Law	$A \setminus B = A \cap B^c$	

Source

Number Sets

Symbol	Definition	Examples	Equations	Solution
$\emptyset$	**empty** set, set with no members	{ }	$1 = 2$	null
$\mathbb{N}$	**natural** numbers	{0, 1, 2, 3, …} (per ISO 80000-2 2-6.1)	$x - 3 = 0$	$x = 3$
$\mathbb{P}$	**prime** numbers	{2, 3, 5, 7, 11, 13, …}	unofficial	NA
$\mathbb{Z}$	**integers**	{…, −2, −1, 0, 1, 2, …}	$x + 7 = 0$	$x = -7$
$\mathbb{Q}$	**rational** numbers	{0, ¼, ½, ¾, 1}	$4x - 1 = 0$	$x =$ ¼
$\mathbb{A}$	**algebraic** numbers	{5, -7, ½, $\sqrt{2}$}	$2x^2 + 4x - 7 = 0$	x is algebraic
$\mathbb{T}$	**transcendental** numbers	{π, e, e^{π}, sin(x), $\log_b$ a}	$\mathbb{T} = \mathbb{U} - \mathbb{A}$	NA
$\mathbb{R}$	**real** numbers	{3.1415, -1, ⅞, $\sqrt{2}$}	$x^2 - 2 = 0$	$x = \pm\sqrt{2}$
$\mathbb{I}$	**imaginary** numbers	{2i, $\sqrt{-1}$}	$x^2 + 1 = 0$	$x = \pm\sqrt{-1}$ $x = \pm i$
$\mathbb{C}$	**complex** numbers	{1 + 2i, -3.4i, ⅝}	$x^2 - 4x + 5 = 0$	$x = 2 \pm i$
$\mathbb{U}$	**universal** set	{all possible values}	∞	NA
{0}	**zero**	{0}	n = 0	0
$\mathbb{Z}$ - {0}	**non-zero integers**	{-3, -2, -1, 1, 2, 3, …}	n ≠ 0	NA
$\mathbb{Z}^+$	**positive integers**	{1, 2, 3, …}	n > 0	NA
$\mathbb{N} \cup \{0\}$	**non-negative integers**	{0, 1, 2, 3, …}	n ≥ 0	NA
$\mathbb{R}$ - {0} $\mathbb{R} \setminus \{0\}$	**non-zero real** numbers	{-0.001, 0.002}	x ≠ 0	NA
$\mathbb{R}^+$ (0, ∞)	**positive real** numbers	{0.0001, 0.0002, …}	x > 0	NA
[0, ∞)	**non-negative real** numbers	{0, 0.0001, 0.0002, …}	x ≥ 0	NA



Set Notation

Term	Definition	Examples
set	a well-defined collection of distinct mathematical objects	C = {2, 4, 5} denotes a set of three numbers: 2, 4, and 5 D = {(2, 4), (−1, 5)} denotes a set of two ordered pairs of numbers
objects	members, elements	a, 3, (x, y)
tuple	a column of mathematical objects	$\begin{bmatrix} a \\ b \\ c \end{bmatrix}$
n-tuples	$\mathbb{Z}^3$ is the set of all **3-tuples** whose entries are integers	$\mathbb{Z}^3 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a, b, c \in \mathbb{Z} \right\}$
Set-Builder Notation	$Set\ Uppercase\_Letter = \{formula : restrictions\}$	$F = \{n^3 : n \text{ is an integer with } 1 \leq n \leq 100\}$ is the set of cubes of the first 100 positive integers
{ } { }	Denotes a **set**	A = {a, e, i, o, u}
$\vert$:	'**such that**' or 'for which'	B = {x$\mid$ x ∈ ℕ and x ≤ 5 } B = {x: x ∈ ℕ and x ≤ 5 }
⇒ ≡	is **equivalent** to	(C ∩ E) ⇒ (x ∈ C ∧ x ∈ E)
$\vert A \vert$ $n(A)$	**cardinality** of A, the number of elements in set A	if A = {(1,2), (3,4)}, then $\vert A \vert$ = 2
A = B	if and only if they have precisely the **same** elements. A is **equal** to B.	if A = {4, 9} and B = {n^2 : n=2 or n=3}, then A = B
A ⊆ B	if and only if every element of A is also an element of B. A is a **subset** of B.	{1, 8, 1107} ⊆ ℕ
A ⊈ B	A is not a **subset** of B. A is **not contained** in B.	{-1, -8, -1107} ⊈ ℕ
A ⊂ B	A is a **proper subset** of B. B is a subset of A that is not equal to A.	{1, 8, 1107} ⊂ ℕ
A ⊄ B	A is not a **proper subset** of B. A is **not contained** in B.	{-1, -8, -1107} ⊄ ℕ
a ∈ A A ∈ B a ∈ A	a is a member of A, is an **element** of A	¾ ∈ ℚ
a ∉ A	a is not a member of A, is **not** an **element** of A	3.14 ∉ ℤ
A ∩ B A ⋂ B A ∩ B	the set containing elements that are in both A and B. A∩B is the **intersection** of A and B.	if A = {1, 2} and B = {2 ,3}, then A ⋂ B = {2}
A ∪ B A ⋃ B A ∪ B	the set containing elements that are in either A or B or both. A∪B is the **union** of A and B.	if A = {1, 2} and B = {2, 3}, then A ⋃ B = {1, 2, 3}

$A \setminus B$ $A - B$	**Set difference**. The set containing elements that are in A **but not** in B. A\B is "A drop B". A–B is "A difference B".	if $A = \{1, 2\}$ and $B = \{2, 3\}$, then $A \setminus B = \{1\}$
$A \cap B = \emptyset$	A and B are **disjoint** sets. No elements in common.	$A \cap B = \emptyset$

Logical Form of Set Notation

Set Notation	Logical Statement	Description
A	$x \in A$	• Is an element of
$\neg A$	$x \notin A$	• Is not an element of
$A = B$ $A = B$	$A \leftrightarrow B$ $\forall x\, [(x \in A \longrightarrow x \in B) \wedge (x \in B \longrightarrow x \in A)]$ $A \subseteq B \wedge B \subseteq A$	• Equal • Equivalence • Iff • $\stackrel{\text{def}}{=}$
$A \neq B$ $A \neq B$	$\forall x\, (x \in A \wedge x \notin B)$	• Not equal
$A \subseteq B$	$\forall x\, (x \in A \longrightarrow x \in B)$ $\forall x \in A\, (x \in B)$ $x \notin A \setminus B$	• Subset of • $A \cap B = A \longrightarrow A \subseteq B$
$A \nsubseteq B$	$\exists x\, (x \in A \wedge x \notin B)$	• Not a subset of
$A \cap B$	$\forall x\, (x \in A \wedge x \in B)$	• Intersection
$A \cup B$	$\forall x\, (x \in A \vee x \in B)$	• Union
$A \setminus B$	$\forall x\, (x \in A \wedge x \notin B)$	• Difference • But Not
$A \longrightarrow B$	$\forall x\, (x \notin A \vee x \in B)$	• If – Then
$A \cap B = \emptyset$	$\neg \exists x\, (x \in A \wedge x \in B)$ $\forall x \neg (x \in A \wedge x \in B)$ $\forall x\, (x \notin A \vee x \notin B)$ $\forall x\, (x \in A \longrightarrow x \notin B)$	• A nd B are disjoint, having no elements in common
$\mathcal{F}$	$\{A_i \mid i \in I\}$	• Family of sets
$x \in \bigcap \mathcal{F}$	$\{x \mid \forall A \in \mathcal{F}\, (x \in A)\}$ $\{x \mid \forall A\, (A \in \mathcal{F} \longrightarrow x \in A)\}$	• Intersection of family of sets
$x \in \bigcup \mathcal{F}$	$\{x \mid \exists A \in \mathcal{F}\, (x \in A)\}$ $\{x \mid \exists A\, (A \in \mathcal{F} \wedge x \in A)\}$	• Union of family of sets
$\bigcap \mathcal{F}$	$\bigcap_{i \in I} A_i = \{x \mid \forall i \in I\, (x \in A_i)\}$ $\bigcap_{i \in I} A_i = A_1 \cap A_2 \cap A_3 \cap A_4 \cap \ldots$	• Intersection of an indexed family of sets
$\bigcup \mathcal{F}$	$\bigcup_{i \in I} A_i = \{x \mid \exists i \in I\, (x \in A_i)\}$ $\bigcup_{i \in I} A_i = \{x \in I \mid \exists i \in I\, A\, (i, x)\}$ $\bigcap_{i \in I} A_i = A_1 \cup A_2 \cup A_3 \cup A_4 \cup \ldots$	• Union of an indexed family of sets
$x \in \wp(B)$	$x \subseteq B$ $\forall y\, (y \in x \longrightarrow y \in B)$	• Power Set

Logical Form of Numbers

Definition	Logical Statement	Description
Even	$\exists k \in \mathbb{Z}\ (x = 2k)$	• Definition of Even
Odd	$\exists k \in \mathbb{Z}\ (x= 2k + 1)$	• Definition of Odd
Prime	$\forall a, b \in \mathbb{Z}^+ \mid (p \backslash ab \longrightarrow p \backslash a \lor p \backslash b)$	• A positive integer $p > 1$ that has no positive integer divisors other than 1 and p itself is prime. • Here \ means "is a divisor of"
Not Prime	$\exists a \in \mathbb{Z}^+\ \exists b \in \mathbb{Z}^+\ (ab = n \land a < n \land b < n)$	• a and b are factors of n, so not prime
x | y	$\exists k \in \mathbb{Z}\ (kx = y)$ $x \mid y \leftrightarrow \exists k \in \mathbb{Z}\ (y = kx))$	• Divisability • Divides • Divides into • x divides y evenly • x | y to mean "x divides y," • $x \nmid y$ means "x does not divide y"

Logical Form of Geometry

Definition	Logical Statement	Description
Line	$\{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = mx + b\}$ $= \{(0, b), (1, m + b), (2, 2m + b), \ldots\}$	• You can think of the graph of the equation as a picture of its truth set!
Plane	$\mathbb{R} \times \mathbb{R} = \{(x, y) \mid$ x and y are real numbers$\}$	• These are the coordinates of all the points in the plane • $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$
3D Space	$\mathbb{R}^3 = \{(x, y, z) \mid$ x, y and z are real numbers$\}$	• These are the coordinates of all the points in 3D space • $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$
Spacetime	$\mathbb{R}^4 = \{(x, y, z, t) \mid$ x, y, z and t are real numbers$\}$	• These are the coordinates of all the points in 3D space and 1D time • $\mathbb{R}^4 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$

Logical Form of Functions

Definiti on	Logical Statement	Description
Function	$f: x \longrightarrow y$ $\forall x \in X\ \exists! y \in Y\ ((x, y) \in f)$ $f = \{(a, b) \in A \times B \mid b = f(a)\}$	• Function • f is a relation frrom A to B • Example : $f = \{(x, y) \in R \times R \mid y = x^2\}$
Domain	$\mathrm{Dom}(f)$ X	• Domain of f

Range	$\mathrm{Ran}(f)$ $\{f(a) \mid a \in A\}$ Y	• Range ⊆ co-domain • Co-domain • Image of f (linear algebra term)
Surjection	$f = \forall y \in Y \{\exists$ at **least** one $x \in X$ such that $f(x) = y\}$ $\forall y \in Y, \exists x \in X \mid (f(x) = y)$ $\mathrm{Ran}(f) = Y$	• **Onto** • Surjective f • Every y is mapped to by at least one x • No orphan y's • *e.g, y is dating at least one x*
Injection	$f = \forall y \in Y \{\exists$ at **most** one $x \in X$ such that $f(x) = y\}$ $\neg\exists a_1 \in A\ \exists a_2 \in A\ (f(a_1) = f(a_2) \land a_1 \neq a_2)$ $\boldsymbol{\forall a_1, a_2 \in A \mid (f(a_1) = f(a_2) \longrightarrow a_1 = a_2)}$ $f(x) = f(y) \longleftrightarrow x = y$ $f(x) \neq f(y) \longleftrightarrow x \neq y$	• **One-to-one** • Injective f • For any y there is at most one x • Can have orphan y's • *e.g., y is either married or single*
Bijection	$f =$ iff $\forall y \in Y \{\exists$ a **unique** $x \in X$ such that $f(x) = y\}$	• One-to-one correspondence • Bijective f • Invertible f • Iff both surjective and injective • one-to-one and onto • *e.g., Everyone is married to a spouse*
Inverse	$f^{-1}: B \longrightarrow A$ $\forall b \in B\ \exists! a \in A\ ((b, a) \in f^{-1})$ $f(g(x)) = x$ $f^{-1} \circ f = i_A$ and $f \circ f^{-1} = i_B$	• Inverse f • Bijective = surjective and injective

Cartesian Product

Set Notation	Logical Statement	Description
A × B A **×** B A × B A ⨯ B	{(a, b) | a ∈ A ∧ b ∈ B} {(a, b) | a ∈ A, b ∈ B}	• Cartesian product • Cross product

Properties of Cartesian Products

Law	Logical Statement	Description
Distributive	A × (B ∩ C) = (A × B) ∩ (A × C)	• ×∩
Distributive	A × (B ∪ C) = (A × B) ∪ (A × C)	• ×∪
Commutative	(A × B) ∩ (C × D) = (A ∩ C) × (B ∩ D)	• ×∩×
Commutative	(A × B) ∪ (C × D) ⊆ (A ∪ C) × (B ∪ D)	• ×∪×
Domination	A × ∅ = ∅ ∅ × A = ∅	• ×∅ • A, B, C, and D are sets

Set Relations

Set Notation	Logical Statement	Description
$R \subseteq A \times B$	$\forall x\ (x \in R \longrightarrow x \in A \times B)$ $R = \{(a, b) \in A \times B \mid \text{conditions}\}$ $xRy = (x, y) \in R$ Example: $D_r = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid$ x and y differ by less than r$\} \Rightarrow \lvert x - y \rvert < r\}$	• Relation from A to B • R is a subset of the cross product
Dom(R)	$\{a \in A \mid \exists b \in B((a, b) \in R)\}$ $Dom(A) \subseteq A$	• The domain of R is the set containing all the first coordinates of its ordered pairs
Ran(R)	$\{b \in B \mid \exists a \in A\ ((a, b) \in R)\}$ $Ran(B) \subseteq B$	• The range of R is the set containing all the second coordinates of its ordered pairs
R^{-1}	$\{(y, x) \in Y \times X \mid (x, y) \in R\}$ $(y, x) \in R^{-1} \longleftrightarrow (x, y) \in R$ $(x, y) \in R^{-1} \longrightarrow (x, y) \in R$	• The inverse of R is the relation R^{-1} from B to A with the order of the coordinates of each pair reversed
$S \circ R$ $S \circ R$	$\{(a, c) \in A \times C \mid \exists b \in B\ ((a, b) \in R \text{ and } (b, c) \in S)\}$ $(x, z) \in S \circ R \longleftrightarrow \exists y \mid (x, y) \in R \text{ and } (y, z) \in S$ $S \circ R = \{(a, c) \in A \times C \mid \exists b \in B\ (aRb \text{ and } bSc)\}$ xRy and ySc $\{(a, c) \in A \times C \mid \exists b \in B\ (aRb \wedge bSc)\}$	• The composition of S and R is the relation $S \circ R$ from A to C • xRy and ySc, meaning $R{:}x \longrightarrow R{:}y \longrightarrow S{:}x \longrightarrow S{:}y$, so $(R{:}x, S{:}y)$ • Ring operator
i_A	$\{(x, y) \in A \times A \mid x = y\}$ $\{(x, x) \mid x \in A\}$	• Identity relation

Order Properties of Set Relation

Property	Logical Statement	Description
Reflexive	xRx $(x, x) \in R$ $\forall x \in A\ (xRx)$ $\forall x \in A\ ((x, x) \in R)$	• $i_A \subseteq R$ where i_A is the identity relation of set A or $i_A = \{(x, x) \mid x \in A\}$ • Directed graph: Loop
Symmetric	$xRy \longrightarrow yRx$ $\forall x \in A\ \forall y \in A\ (xRy \longrightarrow yRx)$	• $R = R^{-1}$ • Directed graph: 2-way arrow (edges come in pairs)
Antisymmetric	$(xRy \wedge yRx) \longrightarrow (x = y)$ $\forall x \in A\ \forall y \in A\ ((xRy \wedge yRx) \longrightarrow (x = y))$	• Equivalence
Asymmetric	$xRy \longrightarrow \neg\,(yRx)$ $\forall x \in A\ \forall y \in A\ \forall z \in A\ (xRy \longrightarrow \neg\,(yRx))$	• Fails the vertical line test, so not a proper function, f(x) • Directed graph: 1-way arrow
Transitive	$(xRy \wedge yRz) \longrightarrow xRz$ $\forall x \in A\ \forall y \in A\ \forall z \in A\ ((xRy \wedge yRz) \longrightarrow xRz)$	• $R \circ R \subseteq R$ • Similar to $S \circ R$ • Directed graph: Two routes from vertex A to vertex B, 1-hop and 2-hops
Total	$xRy \vee yRx$ $\forall x \in A\ \forall y \in A\ (xRy \vee yRx)$	• Either-or
Density	$xRy \longrightarrow \exists z \mid xRz \wedge zRy$ $\forall x \in A\ \forall y\ (xRy) \longrightarrow \exists z \mid xRz \wedge zRy$	• A middle-man exists
Binary	$R^{-1} \circ R$ = Relation on set A $R \circ R^{-1}$ = Relation on set C	• Relation on set <set> • Binary relation on set <set>
Identity	$i_A = \{(x, y) \in A \times A \mid x = y\}$ $i_A = \{(x, x) \mid x \in A\}$	• Similar to a diagonal matrix

Relations

Property	i_A	Equivalence (=)	Partial Order (Poset)	Total Order (Linear)
Reflexive	✓	✓	✓	
Symmetric	✓	✓		
Antisymmetric			✓	✓
Asymmetric				
Transitive	✓	✓	✓	✓
Total				✓
Density				
Binary Relation		✓		

Logical Truth Tables

P	Q	Conjuntion (and) ∧	NAND ⊼	Disjuntion (or) ∨	NOR ⊽	XOR ⊻	XNOR ⊙	Negation (not) ¬P
F	F	F	T	F	T	F	T	
F	T	F	T	T	F	T	F	T
T	F	F	T	T	F	T	F	F
T	T	T	F	T	F	F	T	

P	Q	Material Implication (If … Then) →	Biconditional (Iff) ↔	Tautology (True) ⊤	Contradiction (False) ⊥
F	F	T	T	T	F
F	T	T	F	T	F
T	F	F	F	T	F
T	T	T	T	T	F

Blank Truth Tables

Inputs				Output		
P	Q	R	S	X	Y	Z
F	F	F	F			
F	F	F	T			
F	F	T	F			
F	F	T	T			
F	T	F	F			
F	T	F	T			
F	T	T	F			
F	T	T	T			
T	F	F	F			
T	F	F	T			
T	F	T	F			
T	F	T	T			
T	T	F	F			
T	T	F	T			
T	T	T	F			
T	T	T	T			

Inputs			Output	
P	Q	R	X	Y
F	F	F		
F	F	T		
F	T	F		
F	T	T		
T	F	F		
T	F	T		
T	T	F		
T	T	T		

Inputs		Output
P	Q	X
F	F	
F	T	
T	F	
T	T	

Mathematical Number Sets → Computer Science Data Types

Symbol	Definition	C Data Type	C++ Data Type
$\emptyset$	**empty** set, set with no members	void	void
$\mathbb{N}$	**natural** numbers	enum unsigned unsigned char unsigned short unsigned int unsigned long unsigned long long	enum unsigned unsigned char unsigned short unsigned int unsigned long unsigned long long
$\mathbb{Z}$	**integers**	char short int long long long	char short int long long long
$\mathbb{Q}$	**rational** numbers	NA	std::ratio<1, 10>
$\mathbb{R}$	**real** numbers	float double long double	float double long double
$\mathbb{I}$	**imaginary** numbers	(see complex below) double complex z1; im = cimag(z1);	(see complex below) std::complex <double> z1; im = std::imag(z1);
$\mathbb{C}$	**complex** numbers	#include <complex.h> float complex double complex long double complex	#include <complex> std::complex<float> std::complex <double> std::complex <long double>

Links

- https://www.storyofmathematics.com/set-notation
- https://byjus.com/maths/set-theory-symbols/