

Harold's
Illegals and Graphing Rationals
"Cheat Sheet"
 24 October 2019

Function	When is it Undefined?	When is it Defined?	Domain
$\frac{1}{x}$	Division by zero (0)	$x \neq 0$	$(-\infty, 0) \cup (0, \infty)$
$f(x) \frac{(x-c)}{(x-c)}$	Hole at $x = c$ (artificial)	$x \neq c$	$(-\infty, c) \cup (c, \infty)$
\sqrt{x}	Negative square roots	$x \geq 0$	$[0, \infty)$
$\sqrt[2n]{x}$ $x^{1/2n}$	Negative even roots	$x \geq 0$	$[0, \infty)$
$\ln x$ $\log x$	Negative logarithms	$x > 0$	$(0, \infty)$
$\tan x$ $\sec x$	Trig functions with division by zero when $\cos x = 0$	$x \neq \frac{\pi}{2} \pm n\pi$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\cot x$ $\csc x$	Trig functions with division by zero when $\sin x = 0$	$x \neq n\pi$	$(0, \pi) \cup (\pi, 2\pi)$
$\sin^{-1} x$ $\cos^{-1} x$	Inverse trig functions where $ x > 1$	$ x \leq 1$	$[-1, 1]$
$\sec^{-1} x$ $\csc^{-1} x$	Inverse trig functions where $ x < 1$	$ x \geq 1$	$(-\infty, -a] \cup [a, \infty)$
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	Conic Section: Ellipse boundaries $ x > a$ $ y > b$	$ x \leq a$ $ y \leq b$	$[-a, a]$ $(-b \leq y \leq b)$
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	Conic Section: Hyperbola boundary $ x < a$	$ x \geq a$	$(-\infty, -a] \cup [a, \infty)$

Graphing Rational Functions	How to Obtain
$f(x) = \frac{P(x)}{Q(x)} = \frac{ax^n + \dots + b}{cx^m + \dots + d}$	Reorder from highest to lowest power
Horizontal Asymptote (HA)	<p style="text-align: center;">Left:</p> Case 1: $n > m$, $\lim_{x \rightarrow \infty} \frac{ax^n}{cx^m} \rightarrow \infty$ Case 2: $n = m$, $\lim_{x \rightarrow \infty} \frac{ax^n}{cx^m} = \frac{a}{c}$ (line) Case 3: $n < m$, $\lim_{x \rightarrow \infty} \frac{ax^n}{cx^m} = 0$
Slant Asymptote (SA)	Case 4: $n = m+1$, $\frac{ax^n}{cx^m} \rightarrow \frac{a}{c}x + k$ Do synthetic or long division to determine k
y-intercept	Right: Plug in $x = 0$ to get b/d
x-intercepts (roots)	Top: Find roots of $P(x)$, check for undefined
Vertical Asymptote (VA)	Bottom: Find roots of $Q(x)$, check for holes
Domain	All x except for VAs
Range	Depends upon Domain