

**Harold's**  
**Graphing Rationals and Illegals**  
**"Cheat Sheet"**  
 23 August 2022

Graphing Rational Functions	How to Obtain	
$f(x) = \frac{P(x)}{Q(x)} = \frac{\boxed{ax^n + \dots + b}}{\boxed{cx^m + \dots + d}}$	Reorder from highest to lowest degree/power	
Horizontal Asymptote (HA)	<b>Left:</b>	
	Case 1: $n > m$	$\lim_{x \rightarrow \infty} \frac{ax^n}{cx^m} \rightarrow \infty$
	Case 2: $n = m$	$\lim_{x \rightarrow \infty} \frac{ax^n}{cx^m} = \frac{a}{c}$ (line)
	Case 3: $n < m$	$\lim_{x \rightarrow \infty} \frac{ax^n}{cx^m} = 0$
Slant Asymptote (SA)	Case 4: $n = m+1$	$\frac{ax^n}{cx^m} \rightarrow \frac{a}{c}x + k$
	Use synthetic or long division to determine $k$	
y-intercept	<b>Right:</b> Plug in $x = 0$ to get $y = \frac{b}{d}$	
x-intercepts (roots)	<b>Top:</b> Factor to find roots of $P(x)$ , check for holes	
Vertical Asymptotes (VA)	<b>Bottom:</b> Factor to find roots of $Q(x)$ , check for holes	
Domain	All $x$ except for VAs and holes	
Range	Depends upon Domain	

Function	When is it Undefined?	When is it Defined?	Domain
$\frac{1}{x}$	Division by zero (0)	$x \neq 0$	$(-\infty, 0) \cup (0, \infty)$
$f(x) \frac{(x-c)}{(x-c)}$	Hole at $x = c$ (uncommon)	$x \neq c$	$(-\infty, c) \cup (c, \infty)$
$\sqrt{x}$	Negative square roots	$x \geq 0$	$[0, \infty)$
$\sqrt[2n]{x}$ $x^{1/2n}$	Negative even roots	$x \geq 0$	$[0, \infty)$
$\ln x$ $\log x$	Negative logarithms	$x > 0$	$(0, \infty)$
$\tan x$ $\sec x$	Trig functions with division by zero when $\cos x = 0$	$x \neq \frac{\pi}{2} \pm n\pi$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\cot x$ $\csc x$	Trig functions with division by zero when $\sin x = 0$	$x \neq n\pi$	$(0, \pi) \cup (\pi, 2\pi)$
$\sin^{-1} x$ $\cos^{-1} x$	Inverse trig functions where $ x  > 1$	$ x  \leq 1$	$[-1, 1]$
$\sec^{-1} x$ $\csc^{-1} x$	Inverse trig functions where $ x  < 1$	$ x  \geq 1$	$(-\infty, -a] \cup [a, \infty)$