Harold's Finances
"Cheat Sheet"
16 February 2015

One-Time Investments	Formulas	
Variables	$A = FV = Amount\ after\ time\ t$ $P = PV = Original\ amount\ or\ principle$ $e = Euler's\ number\ (\sim 2.71828\ 18284\ 59045)$ $r = Annual\ interest\ rate.\ Rate\ of\ growth/loss, e.g. 15\% = 0.15$ $t = Time\ in\ years$ $k = Number\ of\ times\ per\ year$ $n = kt = Number\ of\ payments, compoundings$ $i = \left(\frac{r}{k}\right) = Effective\ interest\ rate$	
Simple interest	$A = P + I$ $A = P + Prt$ $A = P(1 + rt)$	
Interest compounded annually	$A = P(1 + r)^t$ $A = P(1 + i)^n$	
Interest compounded *k* times per year	$A = P\left(1 + \frac{r}{k}\right)^{kt}$	
Interest compounded continuously	$A = Pe^{rt}$	
Savings Account Example: $P = \$100$ $r = 8\% = 0.08$ $t = 1\ year$ $k = 4\ (quarterly)$	$If\ k = 1,\ A = \$108.00\ (+0¢)$ *Annually* $If\ k = 4,\ A = \$108.24\ (+24¢)$ *Quarterly* $If\ k = 12,\ A = \$108.29\ (+5¢)$ *Monthly* $If\ k = 365, A = \$108.33\ (+4¢)$ *Daily* $If\ k = \infty,\ A = \$108.33\ (+0¢)$ *Continuously*	
Solving for …	**Discrete**	**Continuous**
Solving for future value or amount	$A = P(1+i)^n = P\left(1 + \frac{r}{k}\right)^{kt}$	$A = Pe^{rt}$
Solving for present value or principle	$P = \frac{A}{(1+i)^n} = \frac{A}{\left(1 + \frac{r}{k}\right)^{kt}}$	$P = \frac{A}{e^{rt}}$
Solving for annual interest rate	$r = ki = k\left[\sqrt[n]{\frac{A}{P}} - 1\right] = k\left[\left(\frac{A}{P}\right)^{1/kt} - 1\right]$	$r = \frac{1}{t} ln\left(\frac{A}{P}\right)$
Solving for effective interest rate	$i = \frac{r}{k} = \sqrt[n]{\frac{A}{P}} - 1 = \left(\frac{A}{P}\right)^{1/kt} - 1$	$i = rt = ln\left(\frac{A}{P}\right)$
Solving for time	$t = \frac{1}{k}\frac{ln\left(\frac{A}{P}\right)}{ln(1+i)} = \frac{1}{k}\frac{ln\left(\frac{A}{P}\right)}{ln\left(1 + \frac{r}{k}\right)}$	$t = \frac{1}{r} ln\left(\frac{A}{P}\right)$
Solving for number of times per year	$k = Unknown$	$k = \infty$
Solving for number of payments	$n = kt = \frac{ln\left(\frac{A}{P}\right)}{ln(1+i)} = \frac{ln\left(\frac{A}{P}\right)}{ln\left(1 + \frac{r}{k}\right)}$	$n = \infty$

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Regular Payments	Formulas
Variables	$FV = Future\ Value$ $PV = Present\ Value$ $R = Regular\ payment\ amount$ $r = Annual\ interest\ rate$ $k = Number\ of\ times\ per\ year$ $i = \left(\frac{r}{k}\right) = Effective\ interest\ rate$ $n = kt = Number\ of\ payments, compoundings$
Future value of an annuity	$FV = R\left[\frac{(1+i)^n - 1}{i}\right] = R\left[\frac{\left(1+\frac{r}{k}\right)^n - 1}{\left(\frac{r}{k}\right)}\right]$
Current value of a loan, Remaining balance	$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right] = \left[\frac{1-\left(1+\frac{r}{k}\right)^{-n}}{\left(\frac{r}{k}\right)}\right]$
Annual percentage yield, Effective rate	$APY = (1+i)^k - 1$ $APY = \left(1+\frac{r}{k}\right)^k - 1$
House Payment Example: $PV = \$300{,}000\ (home\ loan)$ $r = 3.5\% = 0.035$ $t = 30\ years$ $k = 12\ (monthly)$ $n = kt = 360\ payments$	$R = PV\left[\frac{i}{1-(1+i)^{-n}}\right]$ $i = \frac{r}{k} = \frac{0.035}{12} = 0.00291\overline{6}$ $R = \$300{,}000\left[\frac{0.00291\overline{6}}{1-(1.00291\overline{6})^{-360}}\right]$ $\boldsymbol{R = \$1,347.13/month}$
Example cost analysis	$\underline{t = 30\ years}$: $Cost\ of\ loan = nR = (360)(\$1{,}347.13) = \$484{,}966.80$ $Interest\ paid = nR - PV = \$484{,}966.80 - \$300{,}000 = \$184{,}966.80$ $\underline{t = 15\ years}$: $Cost\ of\ loan = nR = (180)(\$2{,}144.65) = \$386{,}037.00$ $Interest\ paid = nR - PV = \$386{,}037.00 - \$300{,}000 = \$86{,}037.00$

Solving for …	Future Value	Present Value
Solving for value	$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$	$PV = R\left[\frac{1-(1+i)^{-n}}{i}\right]$
Solving for payment amount	$R = FV\left[\frac{i}{(1+i)^n - 1}\right]$	$R = PV\left[\frac{i}{1-(1+i)^{-n}}\right]$
Solving for number of payments	$n = kt = \frac{ln\left(\frac{FV}{R}i + 1\right)}{ln(1+i)}$	$n = kt = -\frac{ln\left(1 - \frac{PV}{R}i\right)}{ln(1+i)}$
Solving for annual interest rate	$r = ki = \left(\frac{i}{t}\right)\frac{ln\left(\frac{FV}{R}i + 1\right)}{ln(1+i)}$	$r = ki = -\left(\frac{i}{t}\right)\frac{ln\left(1 - \frac{PV}{R}i\right)}{ln(1+i)}$
Solving for number of times per year	$k = \frac{r}{i} = \frac{n}{t} = \left(\frac{1}{t}\right)\frac{ln\left(\frac{FV}{R}i + 1\right)}{ln(1+i)}$	$k = \frac{r}{i} = \frac{n}{t} = -\left(\frac{1}{t}\right)\frac{ln\left(1 - \frac{PV}{R}i\right)}{ln(1+i)}$
Solving for effective interest rate	$i = \frac{r}{k} = \frac{rt}{n} = Unknown$	

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