Harold's Exponential Growth and Decay Cheat Sheet

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Discrete	Continuous
$A = P\left(1+\frac{r}{n}\right)^{nt}$	$A = Pe^{rt}$
Simple Interest: $A = P + I = P + Prt = P(1+rt)$ A = Amount after time t P = Original amount, such as principle e = The natural number (~2.718) r = Rate of growth/loss, e.g. interest rate (15% = 0.15) t = Elapsed time n = Divides time into periods per time unit	$e \approx 2.71828\ 18284\ 59045\ 23536 \ldots$ $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}$ $A = \lim_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt} = Pe^{rt}$ e = $\sum_{i=0}^{\infty}\frac{x^n}{n\,!} = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\cdots$
Savings Account Example: P = \$100 r = 8% = 0.08 t = 1 year n = 4 (quarterly) $A = \$100\left(1+\frac{0.08}{4}\right)^{4(1)}$ = \$108.24	**Savings Account Example:** $A = \$100\ e^{0.08(1)}$ = \$108.33 If n = 1, A = \$108.00 (+0¢) Annually If n = 4, A = \$108.24 (+24¢) Quarterly If n = 12, A = \$108.29 (+5¢) Monthly If n = 365, A = \$108.33 (+4¢) Daily If n = ∞, A = \$108.33 (+0¢) Continuously
Compounded interest after 3 years: $A(3) = P\ (1+8\%)\ (1+8\%)\ (1+8\%)$ $A(3) = P(1+0.08)^3$ = 1.26 * P	(See calculus derivation on page 2)
$A = P\left(1+\frac{r}{n}\right)^{nt}$ $P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$ $r = n\left[\left(\frac{A}{P}\right)^{1/nt} - 1\right]$ $t = \frac{1}{n}\frac{\ln(\frac{A}{P})}{\ln\left(1+\frac{r}{n}\right)}$ n = ?	$A = Pe^{rt}$ $P = \frac{A}{e^{rt}}$ $r = \frac{1}{t}\ln\left(\frac{A}{P}\right)$ $t = \frac{1}{r}\ln\left(\frac{A}{P}\right)$ n = ∞

Calculus Derivation

Assume the rate of growth or decay is proportional to the amount of substance (P).

$$\frac{dP}{dt} \propto P$$

$$\frac{dP}{dt} = k\,P$$

Separate variables and integrate:

$$\frac{dP}{P} = k\,dt$$

$$\int \frac{dP}{P} = \int k\,dt$$

$$\ln|P| = kt + c$$

Solve for *P(t)*:

$$e^{\ln|P|} = e^{kt+c}$$

$$|P| = e^{c}\,e^{kt} = Ce^{kt}$$

At t=0 (initial condition):

$$P(0) = Ce^{k*0} = C*1 = P_0$$

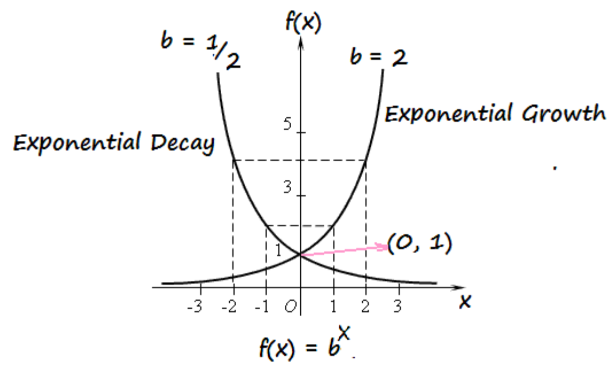
Therefore,

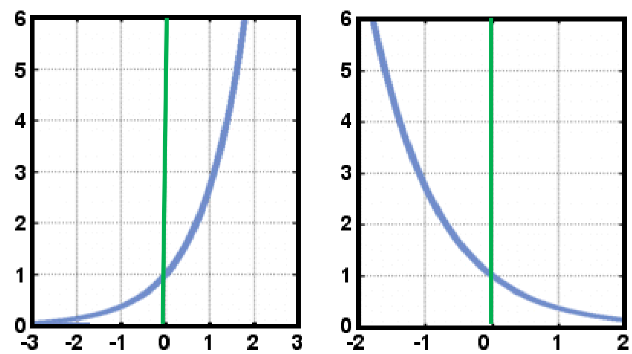
$$P(t) = P_0\,e^{kt}$$

or

$$A = Pe^{rt}$$

Graphs





Left: Exponential Growth (k or r positive)
Right: Exponential Decay (k or r negative)

Chemistry

A = $Amount\ remaining\ after\ time\ t$
$A_0 = Amount\ starting\ with\ at\ time\ t = 0$
T = $Half\ Life\ (time\ units)$
t = $time\ (time\ units)$

Half-Life

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{T}}$$