Harold's Cryptology Cheat Sheet

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Definitions

Term	Definition
Cryptology	The study of cryptography and cryptoanalysis
Cryptography	Methods of encipherment (secret techniques)
Cryptoanalysis	Methods of decipherment (code breaking)
Plain	Plain text message to be encrypted
Cipher	Encrypted text message to be decrypted
Key	Secret string or set of numbers used to encrypt plain text
Stegenography	Information hiding in files, like JPG images

Text to Numbers Encoding

Letter	Number	Letter	Number
A	00	**N**	13
B	01	**O**	14
C	02	**P**	15
D	03	**Q**	16
E	04	**R**	17
F	05	**S**	18
G	06	**T**	19
H	07	**U**	20
I	08	**V**	21
J	09	**W**	22
K	10	**X**	23
L	11	**Y**	24
M	12	**Z**	25
		<space>	26

Cipher Methods

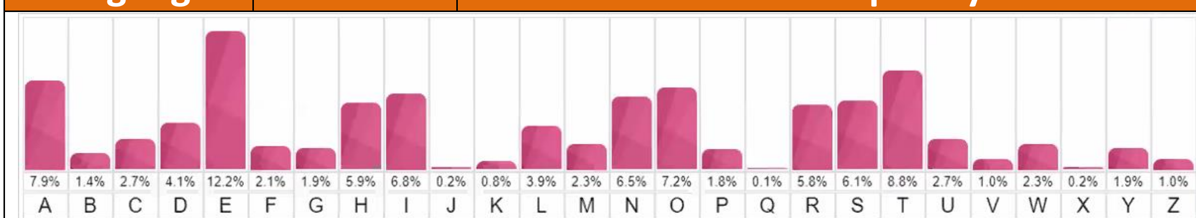
Method	Concept	Example	How
Shift	m = plain text message c = cipher text e.g., (“A” = 0, “B” = 1, ...) $0 \le a, b < n$ $\gcd(a, n) = 1$ (coprime)		Modular Arithmetic for all three (see Harold’s Modular Arithmetic Cheat Sheet)
	Multiply and shift, then wrap	**Affine Ciphers**	$\boldsymbol{c = (am + b)\ MOD\ n}$ $m = (\boldsymbol{a^{-1}}(\boldsymbol{c - b}))\ \boldsymbol{MOD\ n}$ To find a^{-1}, solve for *r*: $(c - b) \cdot r \equiv 1\ (mod\ n)$ Since $r = a^{-1}$, then $a^{-1}am = m\ (mod\ n)$
	Shift and wrap	**Caesar Cipher**	$\boldsymbol{c = (m + b)\ MOD\ n}$ $m = (\boldsymbol{c - b})\ \boldsymbol{MOD\ n}$ Same as Affine with *a* = 1.
	Multiply and wrap	**Decimation Cipher**	$\boldsymbol{c = (am)\ MOD\ n}$ Same as Affine with *b* = 0 and “A” = “A”.
Substitution	Replacement (simple)	Mixed Alphabet with Key Words	**Key**: Unique letters of the key word in order, without repetitions **Plain**: A B C … X Y Z **Cipher**: <Key> followed by remaining letters of the alphabet, without repetitions
		Keyword Columnar Transposition Substitution	1. Row 1: Key word unique chars (# cols) 2. Rows 2-n: Remaining unique chars in rows of a fixed column table 3. Add a padding character as needed 4. Cipher text is simply reading columns top down in alphabetical order
Transposition	Rearranged	Columnar Transposition	1. Agree upon number of columns 2. Rows 1-n: Use clear text to write out rows of a fixed column table 3. Add a padding character as needed 4. Cipher text is simply reading columns top down in order left to right

Spreadsheet Example – Caesar Cipher

Operation		A	B	C	D	E	F	G	H	I	J
Plain Text	1	S	K	Y	I	S	C	L	E	A	R
Plain Text as #	2	18	10	24	8	18	2	11	4	0	17
Cipher Text as #	3	25	27	5	15	25	9	18	11	7	24
Cipher Text	4	Z	R	F	P	Z	J	S	L	H	Y

Function	Description	Excel Formula
CODE(“A”)	Converts an ASCII character into a number	`A2=CODE(A1) - CODE(“A”)`
MOD(n, m)	Adds a fixed offset to each number (n) then mods it by m	`A3=MOD(A2 + 7, 26)`
CHAR(65)	Converts a number into an ASCII character	`A4=CHAR(A3 + CODE(“A”))`
Combined	All three functions combined into one	`A4=CHAR(MOD(CODE(A1) - CODE(“A”) + 7, 26) + CODE(“A”))`

Frequency Analysis



Language	Combos	Letter Frequency
English	Letters	ETOANIS RHCUL ETAOI NSHRD LCUMW FGYPB VKXJQZ (Texts) ESIAR NTOLC DUGPM HBYFV KWZXJQ (Dictionaries) ETAON RISHD LFCMU GYPWB VKJXZQ (40K sample) ETAOI NSRHD LUCMF YWGPB VKXQJZ ETAOI NSRHL DCUMF PGWYB VKXJQZ
	Diagrams	TH HE AN RE ER IN ON AT ND ST ES EN OF TE ED OR TI HI AS TO TH HE IN EN NT RE ER AN TI ES ON AT SE ND OR AR AL TE CO DE TO RA ET ED IT SA EM RO
	Double Letters	LL EE SS OO TT FF RR NN PP CC
	Trigrams	THE AND THA ENT ING ION TIO FOR NDE HAS NCE EDT TIS OFT STH MEN
French	Letters	ESAIT NRUOL DCMPV ÉQFBG HJÀXZ ÈÊYÇK ÛÙÂW
Italian	Letters	EAION LRTSC DPUMV GZFBH ÀQÈÚW ÍYJKX ÒÉÇÆ
German	Letters	ENSRI ATDHU LGCOM WBFKZ ÜÖßJY XQÀÈÚ ÍÒÉ
Spanish	Letters	EAOSR NIDLC TUMPB GYÍVQ ÓHFZJ ÉÁÑXÚ ÜWK

RSA Algorithm

Term	Definition
RSA	Public key cryptosystem developed by Rivest, Adelman, and Shamir in 1978.
Key Prep	1. Bob selects two large prime numbers, p and q. 2. Bob computes N = pq and ϕ = (p-1) (q-1) 3. Bob finds an integer e such that gcd (e, ϕ) = 1. 4. Bob computes the multiplicative inverse of e mod ϕ: an integer d such that (ed mod ϕ) = 1. 5. Public (encryption) key: N and e. 6. Private (decryption) key: d.
Example	1. Bob selects two primes: p = 31 q = 59 2. Compute: N = p · q = 31 · 59 = 1829 ϕ = (p - 1) · (q - 1) = 30 · 58 = 1740 3. Find integer e such that gcd (e, ϕ) = 1 Guess e = 859 and check: gcd (859, 1740) = 1 If the first guess is not relatively prime to ϕ, try another. 4. Using Euclid's algorithm, find A and B such that A · 859 + B · 1740 = 1 79 · 859 + (-39) · 1740 = 1 79 · 859 = 1 mod 1740 d = 79 is the inverse of 859 mod 1740 5. Public key: (e, N) e = 859 N = 1829 6. Private key: (d, N) d = 79 N = 1829
Encryption	$c = m^e \ mod\ N$ Public key:
Decryption	$m = c^d \ mod\ N$ Private key:
Number Theory Fact	Let p and q be prime numbers and pq = N. Suppose that m ∈ $\mathbf{Z}_N$ and gcd (m, N) = 1. Then $m^{(p-1)(q-1)}$ mod N = 1.
Theorem: Validity of the RSA Cryptosystem	If m ∈ $\mathbf{Z}_N$ and gcd (m, N) = 1, then RSA encryption and decryption applied to m always yield m as the unique result.

Sources:

- SNHU MAT 230 - Discrete Mathematics, zyBooks.
- SNHU MAT 260 - Cryptology, Invitation to Cryptology, 1st Edition, Thomas Barr, 2001.

