**Harold’s AP Calculus Notes**

**Cheat Sheet**

23 April 2024

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| **Limits** |  |
| **Definition of Limit**  Let *f* be a function defined on an open interval containing and let be a real number. The statement:  means that for each there exists a such that  if  **Tip:**  Direct substitution: Plugin and see if it provides a legal answer. If so then *.* |  |
| **The Existence of a Limit**  The limit of as approaches *c* is *L* if and only if: | and |
| **Definition of Continuity**  A function  **is continuous** at if for every there exists a such that and.  **Tip:**  Rearrange to have as a factor. Since we can find an equation that relates both and together. | **Prove that is a continuous function.**      Since  So, given , we can **choose**  in the Definition of Continuity. So, substituting the chosen for we get:  Since both conditions are met, is continuous. |
| **Two Special Trig Limits** |  |

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| **Derivatives** | (See Larson’s 1-pager of common derivatives) |
| **Definitions of a Derivative of a Function**  (Slope Function) |  |
| **Derivatives Notation** |  |
| **1. Chain Rule** |  |
| **2. Constant Rule** |  |
| **3. Constant Multiple Rule** |  |
| **4. Sum and Difference Rule** |  |
| **5. Product Rule** |  |
| **6. Quotient Rule** |  |
| **7. Power Rule** |  |
| **8. General Power Rule** |  |
| **9. Power Rule for x** |  |
| **10. Absolute Value** |  |
| **11. Natural Exponential Rule** |  |
| **12. General Natural Exponential Rule** |  |
| **13. Exponential Rule** |  |
| **14. General Exponential Rule** |  |
| **15. Natural Logorithm Rule** |  |
| **16. General Natural Logorithm Rule** |  |
| **17. Logorithm Rule** |  |
| **18. General Logorithm Rule** |  |
| **19. Sine** |  |
| **20. Cosine** |  |
| **21. Tangent** |  |
| **22. Cotangent** |  |
| **23. Secant** |  |
| **24. Cosecant** |  |
| **25. Arcsine** |  |
| **26. Arccosine** |  |
| **27. Arctangent** |  |
| **28. Arccotangent** |  |
| **29. Arcsecant** |  |
| **30. Arccosecant** |  |
| **31. Hyperbolic Sine** |  |
| **32. Hyperbolic Cosine** |  |
| **33. Hyperbolic Tangent** |  |
| **34. Hyperbolic Cotangent** |  |
| **35. Hyperbolic Secant** |  |
| **36. Hyperbolic Cosecant** |  |
| **37. Hyperbolic Arcsine** |  |
| **38. Hyperbolic Arccosine** |  |
| **39. Hyperbolic Arctangent** |  |
| **40. Hyperbolic Arccotangent** |  |
| **41. Hyperbolic Arcsecant** |  |
| **42. Hyperbolic Arccosecant** |  |

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| **Physics** | **Translational Motion** |
| **Position Function** |  |
| **Velocity Function** |  |
| **Acceleration Function** |  |
| **Jerk Function** |  |
| **Gravitational Constant (g)** |  |

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| **Analyzing the Graph of a Function** | (See Harold’s Illegals and Graphing Rationals Cheat Sheet) |
| **x-Intercepts (Zeros or Roots)** |  |
| **y-Intercept** |  |
| **Domain** | Valid values |
| **Range** | Valid values |
| **Continuity** | No division by 0, no negative square roots or logs |
| **Vertical Asymptotes (VA)** | = division by 0 or undefined |
| **Horizontal Asymptotes (HA)** | and |
| **Infinite Limits at Infinity** | and |
| **Differentiability** | Limit from both directions arrives at the same slope |
| **Relative Extrema** | Create a table with domains: |
| **Concavity** | If , then **cup up**  If, then **cup down** |
| **Points of Inflection** | , then concavity changes |

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| **Graphing with Derivatives** |  |
| **Test for Increasing and Decreasing Functions** | 1. If , then *f* is **increasing** (slope up) ↗  2. If , then *f* is **decreasing** (slope down) ↘  3. If , then *f* is constant (zero slope) → |
| **First Derivative Test** | 1. If changes from – to + at , then has a *relative* **minimum** at  2. If changes from + to - at , then has a *relative* **maximum** at  3. If , is + + or -, then is neither |
| **Second Deriviative Test**  Let , and exists, then | 1. If , then *f* has a relative **minimum** at  2. If , then *f* has a relative **maximum** at  3. If , then the test fails |
| **Test for Concavity** | 1. If for all , then the graph is concave **up** ⋃  2. If for all , then the graph is concave **down** ⋂ |
| **Inflection Points**  (Change in concavity) | If is a point of inflection of , then either  1. or  2. does not exist at |

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| **Tangent Lines** |  |
| **General Form** |  |
| **Slope-Intercept Form** |  |
| **Point-Slope Form** |  |
| **Calculus Form** |  |
| **Slope** |  |

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| **Differentiation & Differentials** | | |  |
| **Rolle’s Theorem** | | Assume is continuous on the closed interval , and is differentiable on the open interval .  If , then there exists at least one number in such that . |  |
| **Mean Value Theorem** | | If meets the conditions of Rolle’s Theorem, then you can find ‘’. |  |
| **Intermediate Value Theorem** | | Assume is a continuous function with the interval as its domain.  If takes **values** and at each end of the interval, then it also takes any **value** between and at some point within the interval. |  |
| **Calculating Differentials**  (Tanget line approximation) | | so  Example: |  |
| **Newton’s Method** | | Finds zeros of , or finds if .  Example: | Newton's Method in Calculus | Formula, Equation & Examples - Lesson |  Study.com |
| **Related Rates** |  | Steps to solve:   1. Identify the known variables and rates of change. 2. Construct an equation relating these quantities.   (Often uses the Pythagorean Theorem.)   1. Differentiate both sides of the equation. 2. Solve for the desired rate of change. 3. Substitute the known rates of change and quantities into the equation. | http://i1104.photobucket.com/albums/h330/mathclassroom/Calculus/ladder.png |
| **L’Hôpital’s Rule** | |  | Guillaume de l'Hôpital (1661 - 1704) - Biography - MacTutor History of  Mathematics |

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| **Numerical Methods** | | | | | |  | |
| **Riemann Sum** | http://tutorial.math.lamar.edu/Classes/CalcI/AreaProblem_files/image010.gif | | | | | where  and  and  Types:   * Left Sum (LHS) * Middle Sum (MHS) * Right Sum (RHS) | |
| **Midpoint Rule**  (Middle Sum) | ApproxDef_G1 | | | | | where  and  Error Bounds: | |
| **Trapezoidal Rule** | ApproxDef_G2 | | | | | where  and  Error Bounds: | |
| **Simpson’s Rule** | ApproxDef_G3 | | | | | Where is even  and  and  Error Bounds: | |
| **TI-84 Plus** | | http://www.gosale.com/product_images/4948000/texas-instruments-ti-84-plus-4948122.jpg | | | | [MATH] fnInt(f(x),x,a,b), [MATH] [1] [ENTER]  Example: [MATH] fnInt(x^2,x,0,1) | |
| **TI-Nspire CAS** | | http://i5.walmartimages.com/dfw/dce07b8c-6b5e/k2-_10014b04-7e9e-4157-a2ec-ab5b03fa1234.v1.jpg | | | | [MENU] [4] Calculus [3] Integral  [TAB] [TAB]  [X] [^] [2] [TAB]  [TAB] [X] [ENTER]  Shortcut: [ALPHA] [WINDOWS] [4] | |
| **Integration** | | | | (See Harold’s Fundamental Theorem of Calculus Cheat Sheet) | | |
| **Basic Integration Rules**  (Integration is the “inverse” of differentiation, and vice versa.) | | | |  | | |
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| **1. The Constant Multiple Rule** | | | |  | | |
| **2. The Sum and Difference Rule** | | | |  | | |
| **The Power Rule** | | | |  | | |
| **The General Power Rule** | | | | If then | | |
| **Reimann Sum** | | | |  | | |
| **Definition of a Definite Integral**  (Area under curve) | | | |  | | |
| **Swap Bounds** | | | |  | | |
| **Additive Interval Property** | | | |  | | |
| **First Fundamental Theorem of Calculus** | | | |  | | |
| **Second Fundamental Theorem of Calculus** | | | |  | | |
| **Mean Value Theorem for Integrals** | | | |  | | |
| **Average Value of a Function** | | | |  | | |
| **Integration Methods** | | | | |  | |
| **1. Memorized** | | | | | See Ron Larson’s 1-pager of common integrals | |
| **2. U-Substitution** | | | | | Set  \_\_\_\_\_ \_\_\_\_\_ | |
| **3. Integration by Parts** | | | | | \_\_\_\_\_ \_\_\_\_\_  \_\_\_\_\_ \_\_\_\_\_  Pick ‘’ using the **LIATE** Rule:  **L –**[**Logarithmic**](http://en.wikipedia.org/wiki/Logarithmic_function) :  **I –**[**Inverse Trig.**](http://en.wikipedia.org/wiki/Inverse_trigonometric_function):  **A –**[**Algebraic**](http://en.wikipedia.org/wiki/Polynomial):  **T –**[**Trigonometric**](http://en.wikipedia.org/wiki/Trigonometric_functions):  **E –**[**Exponential**](http://en.wikipedia.org/wiki/Exponential_function): | |
| **4. Partial Fractions** | | | | | where are polynomials.  **Case 1:** If degree of  then do long division first.  **Case 2:** If degree of  then do partial fraction expansion. | |
| **5a. Trig Substitution for** | | | | | Substutution:  Identity: | |
| **5b. Trig Substitution for** | | | | | Substutution:  Identity: | |
| **5c. Trig Substitution for** | | | | | Substutution:  Identity: | |
| **6. Computer Algebra Systems (CAS)** | | | | | [TI-Nspire CX CAS Graphing Calculator](http://education.ti.com/en/us/products/calculators/graphing-calculators/ti-nspire-cx-cas-handheld/tabs/overview)  [TI –Nspire CAS](https://itunes.apple.com/us/app/ti-nspire-cas/id545351700?mt=8) iPad app | |
| **7. Numerical Methods** | | | | | Riemann Sum, Midpoint Rule, Trapezoidal Rule, Simpson’s Rule, various quadrature rules, TI-84 Calculator, etc. | |
| **8. WolframAlpha** | | | | | [WolframAlpha](http://www.wolframalpha.com) is the Google of mathematics. Shows steps. Free. | |
| **9. AI Chatbot** | | | | | OpenAI [ChatGPT](https://openai.com/chatgpt), Microsoft [Copilot](https://copilot.microsoft.com/), Google [Gemini](https://gemini.google.com), etc. | |
| **Partial Fractions** | | | (See Harold’s Partial Fraction Decomposition Cheat Sheets) | | | |
| **Condition** | | | where are polynomials  and degree of  If degree of then do long division first | | | |
| **Example Expansion** | | |  | | | |
| **Typical Solution** | | |  | | | |

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| **Sequences & Series** | (See Harold’s Series Cheat Sheet) |
| **Sequence** | (Limit)  Example: () |
| **Geometric Series** | only if  where is the **radius** of convergence  and is the **interval** of convergence |

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| **Convergence Tests** | (See Harold’s Series Convergence Tests Cheat Sheet) | |
| **Series Convergence Tests** | 1. Divergence or Term 2. Geometric Series 3. p-Series 4. Alternating Series 5. Integral | 1. Ratio 2. Root 3. Direct Comparison 4. Limit Comparison 5. Telescoping Series |

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| **Taylor Series** | (See Harold’s Taylor Series Cheat Sheet) |
| **Taylor Series** | where  ( is the worst case scenario or max. value of in the range.)  and |