**Harold’s Calculus 3**

**Multi-Coordinate System**

**Cheat Sheet**

29 November 2022

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|  | **Rectangular** | **Polar/Cylindrical** | **Spherical** | **Parametric** | **Vector** | **Matrix** |
| **Point** | 2D*3D**4D*• |  *or*  |  | *Point (a,b) in Rectangular :**,**with 1 degree of freedom (df)* |  |  |
| *Polar 🡪 Rect.* | *Rect. 🡪 Polar* |
| **Line** | *Slope-Intercept Form:**Point-Slope Form:**General Form:**Calculus Form:**where* *3-D:* | http://upload.wikimedia.org/wikipedia/commons/thumb/7/78/Polar_to_cartesian.svg/250px-Polar_to_cartesian.svg.pnghttp://upload.wikimedia.org/wikipedia/commons/thumb/b/b7/Cylindrical_Coordinates.svg/190px-Cylindrical_Coordinates.svg.png | http://upload.wikimedia.org/wikipedia/commons/thumb/5/51/Spherical_Coordinates_(Colatitude,_Longitude).svg/360px-Spherical_Coordinates_(Colatitude,_Longitude).svg.png | *where* |  |  |
| **Plane** |  | *s* |  | *where:** *s and t range over all real numbers*
* ***v*** *and* ***w*** *are given vectors defining the plane*
* *is the vector representing the position of an arbitrary (but fixed) point on the plane*
 |  | http://upload.wikimedia.org/wikipedia/commons/thumb/7/7e/Intersecting_planes.svg/220px-Intersecting_planes.svg.png |
| **Conics** | *General Equation for All Conics:**where* *or* *Note: If , square hyperbola**Rotation:**If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *coordinate system:**New = (x’, y’), Old = (x, y)**rotates through angle from x-axis*http://www.sensorsmag.com/files/sensor/nodes/2009/6475/Figure9.gif | *General Equation for All Conics:**Vertical Axis of Symmetry:**Horizontal Axis of Symmetry:**p = semi-latus rectum**or the line segment running from the focus to the curve in a direction parallel to the directrix**Eccentricity:*Image result for conics | Image result for conics557px Conic Sections.svg | NA |

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| **Circle** | *General Form:**Center:* *Vertices: NA**Focus:* Equation of a Circle | *Centered at Origin:**r = a (constant)**Centered at :**Hint: Law of Cosines**or*Image result for off center circle in polar coordinates |  | *Center:* *Focus:*  | NA | NA |
| **Sphere** | *General Form:* > 0*Cylindrical to Rectangular:*Spherical to Rectangular: | *Rectangular to Cylindrical:**Spherical to Cylindrical:* | *Rectangular to Spherical:**Cylindrical to Spherical:* | https://encrypted-tbn1.gstatic.com/images?q=tbn:ANd9GcQRFtXKyazGXHqZX_E5FPf22QXFOrEkEa0c2XZ6MTKC3ATaXwJW3w | Rectangular:Cylindrical:Spherical: | https://encrypted-tbn3.gstatic.com/images?q=tbn:ANd9GcTcVzEz8siGxLf9_AvM0HFSxumP4P2eM0hQ0AIpN120-0q5Y7TC-Q |
| **Ellipse** | *General Form:**where* *Center:* *Vertices:* *Co-Vertices:* *Foci:* *Focus length, c, from center:**Eccentricity:**If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *coordinate system:**New = (x’, y’), Old = (x, y)**rotates through angle from x-axis* | *Vertical Axis of Symmetry:**Horizontal Axis of Symmetry:**relative to center* http://newportaoit.org/tfuentes/ellipse2.gif | Image result for conicsSee the source image***Interesting Note:****The sum of the distances from each focus to a point on the curve is constant.* | *Center:* *Rotated Ellipse:* *= the angle between the x-axis and the major axis of the ellipse*http://www.sensorsmag.com/files/sensor/nodes/2009/6475/Figure9.gif |  |  |
| **Ellipsoid** |  |  |  |  | http://upload.wikimedia.org/wikipedia/commons/thumb/5/50/Ellipsoid_tri-axial_abc.svg/200px-Ellipsoid_tri-axial_abc.svg.png | *Centered at vector*  |
| **Hyperbola** | *General Form:**where* *If , square hyperbola**Center:* *Vertices:* *Foci:* *Focus length, c, from center:**Eccentricity:**If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *coordinate system:**New = (x’, y’), Old = (x, y)**rotates through angle from x-axis* | Hyperbola***Interesting Note:****The difference between the distances from each focus to a point on the curve is constant.* | *Vertical Axis of Symmetry:**Horizontal Axis of Symmetry:**relative to center (h, k)*Image result for "latus rectum" of a hyperbola*p = semi-latus rectum* *or the line segment running from the focus to the curve in the directions*  | *Left-Right Opening Hyperbola:**Vertex: (h, k)**Alternate Form:**Up-Down Opening Hyperbola:**Vertex: (h, k)**Alternate Form:**General Form:**where A and D have different signs* |  |  |
| **Hyperboloid** |  |  | calculus - Finding the vertex of a two-sheet-hyperboloid - Mathematics  Stack Exchange |  |  |
| **Parabola** | *Vertical Axis of Symmetry:**Vertex:* *Focus:* *Directrix:* *Horizontal Axis of Symmetry:**Vertex:* *Focus:* *Directrix:* *General Form:**where* *or* *If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *coordinate system:**New = (x’, y’), Old = (x, y)**rotates through angle from x-axis* | *Vertical Axis of Symmetry:**Horizontal Axis of Symmetry:**and* Parabola | Image result for conics parabola rectum***Interesting Note:****The distances from a point on the curve to the focus is the same as to the directrix.* | *Vertical Axis of Symmetry:* *(opens upwards)* *(opens downwards)**Vertex:* *Horizontal Axis of Symmetry:* *(opens to the right)* *(opens to the left)**Vertex:* *Projectile Motion:* *feet* *meters**General Form:**where A and L have the same sign* |  |  |
| **Paraboloid** |  | Paraboloid – Wikipedia |  |  |  |  |

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| **Limit** |  | http://upload.wikimedia.org/wikipedia/commons/thumb/d/d1/L%C3%ADmite_01.svg/200px-L%C3%ADmite_01.svg.png | http://upload.wikimedia.org/wikipedia/commons/thumb/6/66/Limit-at-infinity-graph.png/306px-Limit-at-infinity-graph.png |  |  |  |
| **1st Derivative** |  | *Hint: Use Product Rule for* |  |  | *Unit tangent vector* |  |
| **2nd Derivative** |  |  |  |  | *Unit normal vector* |  |
| **Integral** | *Fundamental Theorem of Calculus:* | http://upload.wikimedia.org/wikipedia/commons/thumb/2/2a/Riemann_sum_convergence.png/250px-Riemann_sum_convergence.png |  | *Riemann Sum:**Left Sum:**Middle Sum:**Right Sum:* |  |
| **Double Integral** |  |  |  |  |  |  |
| **Triple Integral** |  |  |  | *NA* | *NA* | *NA* |
| **Inverse Functions** | *Inverse Function Theorem:* | *if* *if* *if* *if* *if* *if*  |  | *or* *or* *or* *or* *or* *or*  | *NA* | *NA* | *NA* |
| **Arc Length** | *Proof:* | *Polar:**Where* *Circle:**Proof:* | *C = πd = 2πr*http://www.mathwarehouse.com/trigonometry/radians/images/picture-s=r-theta-circle.gif | *Rectangular 2D:**Rectangular 3D:**Cylindrical:**Spherical:* | *s(t)* | *NA* |
| **Curvature** |  | *for r()* | *NA* | *where f(t) = (x(t), y(t), z(t))* |  | *(See Wikipedia : Curvature)* |
| **Perimeter** | *Square: P = 4s**Rectangle: P = 2l + 2w**Triangle: P = a + b + c**Circle: C = πd = 2πr**Ellipse:*  | *Ellipse:*  | *Ellipse:*  | *NA* | *NA* | *NA* |
| **Area** | *Square: A = s²**Rectangle: A = lw**Rhombus: A = ½ ab**Parallelogram: A = Bh**Trapezoid:* *Kite:* *Triangle: A = ½ Bh**Triangle: A = ½ ab sin(C)**Triangle using Heron’s Formula:**Equilateral Triangle:* *Frustum:* *Circle: A = πr²**Circular Sector: A = ½ r²**Ellipse: A = πab* | *where* *Proof:**Area of a sector:**where arc length* http://upload.wikimedia.org/wikipedia/commons/thumb/4/4c/Polar_coordinates_integration_Riemann_sum.svg/220px-Polar_coordinates_integration_Riemann_sum.svg.png | *NA* | *where and* *or**x(t) = f(t) and y(t) = g(t)**Simplified:**Proof:**y = f(x) = g(t)* |  | *NA* |
| **Lateral Surface Area** | *Cylinder: SA = 2πrh**Cone: SA = πrl* | *For rotation about the x-axis:**For rotation about the y-axis:* | *Sphere: SA = 4πr²* | *For rotation about the x-axis:**For rotation about the y-axis:* | *NA* | *NA* |
| **Total Surface Area** | *Cube: SA = 6s²**Rectangular Box: SA = 2lw + 2wh + 2hl**Regular Tetrahedron: SA = 2bh**Cylinder: SA = 2πr (r + h)**Cone: SA = πr² + πrl = πr (r + l)**Sphere: SA = 4πr²* | *Ellipsoid: SA* *Where p* *(Knud Thomsen’s Formula)* | *Ellipsoid: S =* | http://www.numericana.com/answer/G.1.7.xml_gr_1.gif*where* http://www.numericana.com/answer/G.1.7.xml_gr_2.gif |
| **Surface of Revolution** | *For revolution about the x-axis:**For revolution about the y-axis:* | *For revolution about the x-axis:**For revolution about the y-axis:* | *Sphere: S = 4πr²* | *For revolution about the x-axis:**For revolution about the y-axis:* | *NA* | *NA* |
| **Volume** | *Cube: V = s³**Rectangular Prism: V = lwh**Cylinder: V = πr²h**Triangular Prism: V= Bh**Tetrahedron: V= ⅓ Bh**Pyramid: V = ⅓ Bh**Cone: V = ⅓ Bh = ⅓ πr²h**Sphere:* *Ellipsoid: V = πabc* |  |  |  |  | *Ellipsoid:* |
| **Volume of Revolution** | **Disk Method***Rotation about the x-axis:**Rotation about the y-axis:* | cochranmath / Volume of a solid of revolution by plane slicing |  |  |  |
| **Washer Method***Rotation about the x-axis:* |  |  |  |  |
| **Shell Method***Rotation about the y-axis:**Rotation about the x-axis:* | 2.3 Volumes of Revolution: Cylindrical Shells - Calculus Volume 2 ... | This figure has two images. The first is labeled “a” and is of a hollow cylinder around the y-axis. On the front of this cylinder is a vertical line labeled “cut line”. The height of the cylinder is “y=f(x)”. The second figure is labeled “b” and is a shaded rectangular block. The height of the rectangle is “f(x*), the width of the rectangle is “2pix*”, and the thickness of the rectangle is “delta x”. |  |
| **Moments of Inertia** |  | *NA* | *NA* |  |  | *(see Wikipedia)* |
| **Center of Mass** | *where* *1D for Discrete:* | *2D for Discrete:* | *3D for Discrete:* | *3D for Continuous:**where* *and*  | *Where is distance from the axis of rotation, not origin.* |  |
| **Gradient** |  |  |  | http://upload.wikimedia.org/wikipedia/commons/thumb/3/31/Gradient99.png/350px-Gradient99.png | *where*  |  |
| **Line Integral** |  | *NA* | *NA* | http://upload.wikimedia.org/wikipedia/commons/d/d8/Line-Integral.gif |  |  |
| **Surface Integral** | *Where* *and* | *NA* | *NA* | http://upload.wikimedia.org/wikipedia/commons/thumb/8/87/Surface_integral1.svg/220px-Surface_integral1.svg.png |  |  |