**Harold’s Calculus 3**

**Multi-Coordinate System**

**Cheat Sheet**

29 November 2022

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Rectangular** | **Polar/Cylindrical** | | **Spherical** | **Parametric** | **Vector** | **Matrix** |
| **Point** | 2D  *3D*  *4D*    • | *or* | |  | *Point (a,b) in Rectangular :*  *,*  *with 1 degree of freedom (df)* |  |  |
| *Polar 🡪 Rect.* | *Rect. 🡪 Polar* |
| **Line** | *Slope-Intercept Form:*  *Point-Slope Form:*  *General Form:*  *Calculus Form:*  *where*  *3-D:* | [http://upload.wikimedia.org/wikipedia/commons/thumb/7/78/Polar_to_cartesian.svg/250px-Polar_to_cartesian.svg.png](http://en.wikipedia.org/wiki/File:Polar_to_cartesian.svg)  http://upload.wikimedia.org/wikipedia/commons/thumb/b/b7/Cylindrical_Coordinates.svg/190px-Cylindrical_Coordinates.svg.png | | http://upload.wikimedia.org/wikipedia/commons/thumb/5/51/Spherical_Coordinates_(Colatitude,_Longitude).svg/360px-Spherical_Coordinates_(Colatitude,_Longitude).svg.png | *where* |  |  |
| **Plane** |  | *s* | |  | *where:*   * *s and t range over all real numbers* * ***v*** *and* ***w*** *are given vectors defining the plane* * *is the vector representing the position of an arbitrary (but fixed) point on the plane* |  | [http://upload.wikimedia.org/wikipedia/commons/thumb/7/7e/Intersecting_planes.svg/220px-Intersecting_planes.svg.png](http://en.wikipedia.org/wiki/File:Intersecting_planes.svg) |
| **Conics** | *General Equation for All Conics:*  *where*  *or*  *Note: If , square hyperbola*  *Rotation:*  *If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *coordinate system:*  *New = (x’, y’), Old = (x, y)*  *rotates through angle from x-axis*  http://www.sensorsmag.com/files/sensor/nodes/2009/6475/Figure9.gif | *General Equation for All Conics:*  *Vertical Axis of Symmetry:*  *Horizontal Axis of Symmetry:*  *p = semi-latus rectum*  *or the line segment running from the focus to the curve in a direction parallel to the directrix*  *Eccentricity:*  Image result for conics | | Image result for conics  557px Conic Sections.svg | | | NA |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Circle** | *General Form:*  *Center:*  *Vertices: NA*  *Focus:*  Equation of a Circle | *Centered at Origin:*  *r = a (constant)*  *Centered at :*  *Hint: Law of Cosines*  *or*  Image result for off center circle in polar coordinates |  | *Center:*  *Focus:* | NA | NA |
| **Sphere** | *General Form:*  > 0  *Cylindrical to Rectangular:*  Spherical to Rectangular: | *Rectangular to Cylindrical:*  *Spherical to Cylindrical:* | *Rectangular to Spherical:*  *Cylindrical to Spherical:* | [https://encrypted-tbn1.gstatic.com/images?q=tbn:ANd9GcQRFtXKyazGXHqZX_E5FPf22QXFOrEkEa0c2XZ6MTKC3ATaXwJW3w](http://www.google.com/imgres?safe=off&sa=N&hl=en&biw=1287&bih=815&tbm=isch&tbnid=D5odqmhVPnxFGM:&imgrefurl=http://maverick.inria.fr/~Xavier.Decoret/resources/xdkwrl/html/a00281.html&docid=9f5rjsFcwRafOM&imgurl=http://maverick.inria.fr/~Xavier.Decoret/resources/xdkwrl/html/images/sphere.gif&w=342&h=334&ei=W692UdyrBsr32wWR3IDQBg&zoom=1&ved=1t:3588,r:38,s:0,i:276&iact=rc&dur=946&page=2&tbnh=177&tbnw=181&start=25&ndsp=30&tx=94&ty=84) | Rectangular:  Cylindrical:  Spherical: | [https://encrypted-tbn3.gstatic.com/images?q=tbn:ANd9GcTcVzEz8siGxLf9_AvM0HFSxumP4P2eM0hQ0AIpN120-0q5Y7TC-Q](http://www.google.com/imgres?safe=off&sa=N&hl=en&biw=1287&bih=815&tbm=isch&tbnid=X5STfIPBciE8CM:&imgrefurl=http://library.thinkquest.org/20991/geo/solids.html&docid=eb6vwUkIGv-EiM&imgurl=http://library.thinkquest.org/20991/media/geo_sphere.gif&w=300&h=300&ei=W692UdyrBsr32wWR3IDQBg&zoom=1&ved=1t:3588,r:13,s:0,i:191&iact=rc&dur=773&page=1&tbnh=177&tbnw=189&start=0&ndsp=25&tx=135&ty=41) |
| **Ellipse** | *General Form:*  *where*  *Center:*  *Vertices:*  *Co-Vertices:*  *Foci:*  *Focus length, c, from center:*  *Eccentricity:*  *If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *coordinate system:*  *New = (x’, y’), Old = (x, y)*  *rotates through angle from x-axis* | *Vertical Axis of Symmetry:*  *Horizontal Axis of Symmetry:*  *relative to center*  http://newportaoit.org/tfuentes/ellipse2.gif | Image result for conics  See the source image  ***Interesting Note:***  *The sum of the distances from each focus to a point on the curve is constant.* | *Center:*  *Rotated Ellipse:*  *= the angle between the x-axis and the major axis of the ellipse*  http://www.sensorsmag.com/files/sensor/nodes/2009/6475/Figure9.gif |  |  |
| **Ellipsoid** |  |  |  |  | http://upload.wikimedia.org/wikipedia/commons/thumb/5/50/Ellipsoid_tri-axial_abc.svg/200px-Ellipsoid_tri-axial_abc.svg.png | *Centered at vector* |
| **Hyperbola** | *General Form:*  *where*  *If , square hyperbola*  *Center:*  *Vertices:*  *Foci:*  *Focus length, c, from center:*  *Eccentricity:*  *If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *coordinate system:*  *New = (x’, y’), Old = (x, y)*  *rotates through angle from x-axis* | Hyperbola  ***Interesting Note:***  *The difference between the distances from each focus to a point on the curve is constant.* | *Vertical Axis of Symmetry:*  *Horizontal Axis of Symmetry:*  *relative to center (h, k)*  Image result for "latus rectum" of a hyperbola  *p = semi-latus rectum*  *or the line segment running from the focus to the curve in the directions* | *Left-Right Opening Hyperbola:*  *Vertex: (h, k)*  *Alternate Form:*  *Up-Down Opening Hyperbola:*  *Vertex: (h, k)*  *Alternate Form:*  *General Form:*  *where A and D have different signs* |  |  |
| **Hyperboloid** |  |  | calculus - Finding the vertex of a two-sheet-hyperboloid - Mathematics  Stack Exchange | |  |  |
| **Parabola** | *Vertical Axis of Symmetry:*  *Vertex:*  *Focus:*  *Directrix:*  *Horizontal Axis of Symmetry:*  *Vertex:*  *Focus:*  *Directrix:*  *General Form:*  *where*  *or*  *If B ≠ 0, then* [*rotate*](http://faculty.eicc.edu/bwood/ma155supplemental/supplemental31.htm) *coordinate system:*  *New = (x’, y’), Old = (x, y)*  *rotates through angle from x-axis* | *Vertical Axis of Symmetry:*  *Horizontal Axis of Symmetry:*  *and*  Parabola | Image result for conics parabola rectum  ***Interesting Note:***  *The distances from a point on the curve to the focus is the same as to the directrix.* | *Vertical Axis of Symmetry:*  *(opens upwards)*  *(opens downwards)*  *Vertex:*  *Horizontal Axis of Symmetry:*  *(opens to the right)*  *(opens to the left)*  *Vertex:*  *Projectile Motion:*  *feet*  *meters*  *General Form:*  *where A and L have the same sign* |  |  |
| **Paraboloid** |  | Paraboloid – Wikipedia |  |  |  |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Limit** |  | http://upload.wikimedia.org/wikipedia/commons/thumb/d/d1/L%C3%ADmite_01.svg/200px-L%C3%ADmite_01.svg.png | | http://upload.wikimedia.org/wikipedia/commons/thumb/6/66/Limit-at-infinity-graph.png/306px-Limit-at-infinity-graph.png |  |  |  |
| **1st Derivative** |  | *Hint: Use Product Rule for* | |  |  | *Unit tangent vector* |  |
| **2nd Derivative** |  |  | |  |  | *Unit normal vector* |  |
| **Integral** | *Fundamental Theorem of Calculus:* | [http://upload.wikimedia.org/wikipedia/commons/thumb/2/2a/Riemann_sum_convergence.png/250px-Riemann_sum_convergence.png](http://en.wikipedia.org/wiki/File:Riemann_sum_convergence.png) | |  | *Riemann Sum:*  *Left Sum:*  *Middle Sum:*  *Right Sum:* |  | |
| **Double Integral** |  |  | |  |  |  |  |
| **Triple Integral** |  |  | |  | *NA* | *NA* | *NA* |
| **Inverse Functions** | *Inverse Function Theorem:* | *if*  *if*  *if*  *if*  *if*  *if* |  | *or*  *or*  *or*  *or*  *or*  *or* | *NA* | *NA* | *NA* |
| **Arc Length** | *Proof:* | *Polar:*  *Where*  *Circle:*  *Proof:* | | *C = πd = 2πr*  http://www.mathwarehouse.com/trigonometry/radians/images/picture-s=r-theta-circle.gif | *Rectangular 2D:*  *Rectangular 3D:*  *Cylindrical:*  *Spherical:* | *s(t)* | *NA* |
| **Curvature** |  | *for r()* | | *NA* | *where f(t) = (x(t), y(t), z(t))* |  | *(See Wikipedia : Curvature)* |
| **Perimeter** | *Square: P = 4s*  *Rectangle: P = 2l + 2w*  *Triangle: P = a + b + c*  *Circle: C = πd = 2πr*  *Ellipse:* | *Ellipse:* | | *Ellipse:* | *NA* | *NA* | *NA* |
| **Area** | *Square: A = s²*  *Rectangle: A = lw*  *Rhombus: A = ½ ab*  *Parallelogram: A = Bh*  *Trapezoid:*  *Kite:*  *Triangle: A = ½ Bh*  *Triangle: A = ½ ab sin(C)*  *Triangle using Heron’s Formula:*  *Equilateral Triangle:*  *Frustum:*  *Circle: A = πr²*  *Circular Sector: A = ½ r²*  *Ellipse: A = πab* | *where*  *Proof:*  *Area of a sector:*  *where arc length*  [http://upload.wikimedia.org/wikipedia/commons/thumb/4/4c/Polar_coordinates_integration_Riemann_sum.svg/220px-Polar_coordinates_integration_Riemann_sum.svg.png](http://en.wikipedia.org/wiki/File:Polar_coordinates_integration_Riemann_sum.svg) | | *NA* | *where and*  *or*  *x(t) = f(t) and y(t) = g(t)*  *Simplified:*  *Proof:*  *y = f(x) = g(t)* |  | *NA* |
| **Lateral Surface Area** | *Cylinder: SA = 2πrh*  *Cone: SA = πrl* | *For rotation about the x-axis:*  *For rotation about the y-axis:* | | *Sphere: SA = 4πr²* | *For rotation about the x-axis:*  *For rotation about the y-axis:* | *NA* | *NA* |
| **Total Surface Area** | *Cube: SA = 6s²*  *Rectangular Box: SA = 2lw + 2wh + 2hl*  *Regular Tetrahedron: SA = 2bh*  *Cylinder: SA = 2πr (r + h)*  *Cone: SA = πr² + πrl = πr (r + l)*  *Sphere: SA = 4πr²* | *Ellipsoid: SA*  *Where p*  *(Knud Thomsen’s Formula)* | | *Ellipsoid: S =* | http://www.numericana.com/answer/G.1.7.xml_gr_1.gif  *where* http://www.numericana.com/answer/G.1.7.xml_gr_2.gif | | |
| **Surface of Revolution** | *For revolution about the x-axis:*  *For revolution about the y-axis:* | *For revolution about the x-axis:*  *For revolution about the y-axis:* | | *Sphere: S = 4πr²* | *For revolution about the x-axis:*  *For revolution about the y-axis:* | *NA* | *NA* |
| **Volume** | *Cube: V = s³*  *Rectangular Prism: V = lwh*  *Cylinder: V = πr²h*  *Triangular Prism: V= Bh*  *Tetrahedron: V= ⅓ Bh*  *Pyramid: V = ⅓ Bh*  *Cone: V = ⅓ Bh = ⅓ πr²h*  *Sphere:*  *Ellipsoid: V = πabc* |  | |  |  |  | *Ellipsoid:* |
| **Volume of Revolution** | **Disk Method**  *Rotation about the x-axis:*  *Rotation about the y-axis:* | cochranmath / Volume of a solid of revolution by plane slicing | | |  |  |  |
| **Washer Method**  *Rotation about the x-axis:* |  | | |  |  |  |
| **Shell Method**  *Rotation about the y-axis:*  *Rotation about the x-axis:* | 2.3 Volumes of Revolution: Cylindrical Shells - Calculus Volume 2 ... | | | This figure has two images. The first is labeled “a” and is of a hollow cylinder around the y-axis. On the front of this cylinder is a vertical line labeled “cut line”. The height of the cylinder is “y=f(x)”. The second figure is labeled “b” and is a shaded rectangular block. The height of the rectangle is “f(x*), the width of the rectangle is “2pix*”, and the thickness of the rectangle is “delta x”. | |  |
| **Moments of Inertia** |  | *NA* | | *NA* |  |  | *(see Wikipedia)* |
| **Center of Mass** | *where*  *1D for Discrete:* | *2D for Discrete:* | | *3D for Discrete:* | *3D for Continuous:*  *where*  *and* | *Where is distance from the axis of rotation, not origin.* |  |
| **Gradient** |  |  | |  | http://upload.wikimedia.org/wikipedia/commons/thumb/3/31/Gradient99.png/350px-Gradient99.png | *where* |  |
| **Line Integral** |  | *NA* | | *NA* | http://upload.wikimedia.org/wikipedia/commons/d/d8/Line-Integral.gif |  |  |
| **Surface Integral** | *Where*  *and* | *NA* | | *NA* | http://upload.wikimedia.org/wikipedia/commons/thumb/8/87/Surface_integral1.svg/220px-Surface_integral1.svg.png |  |  |