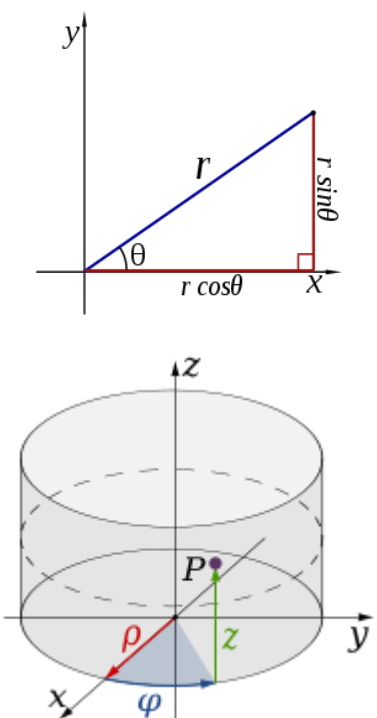
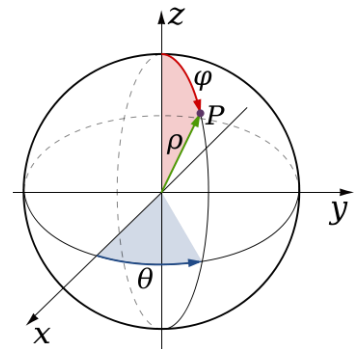


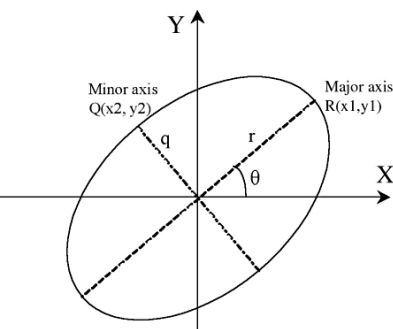
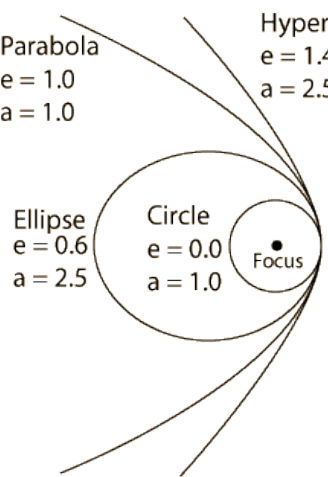
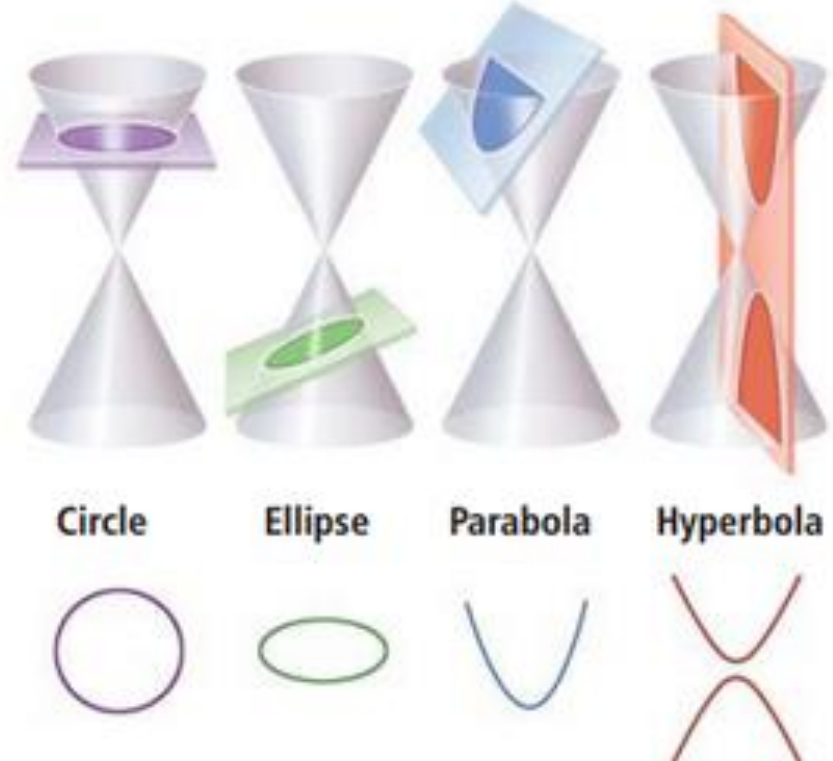
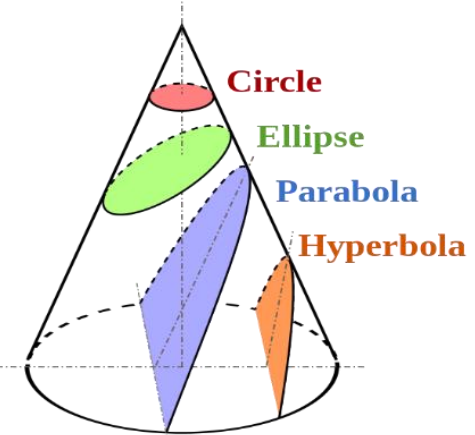
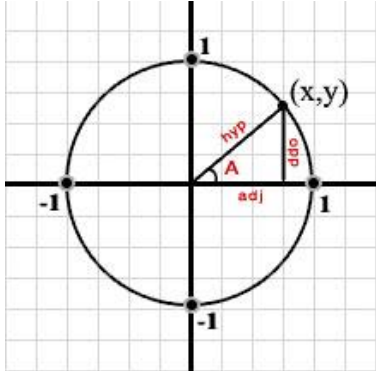
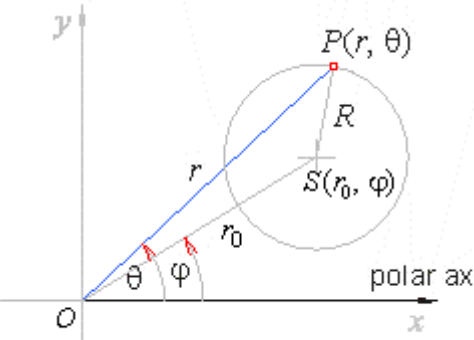
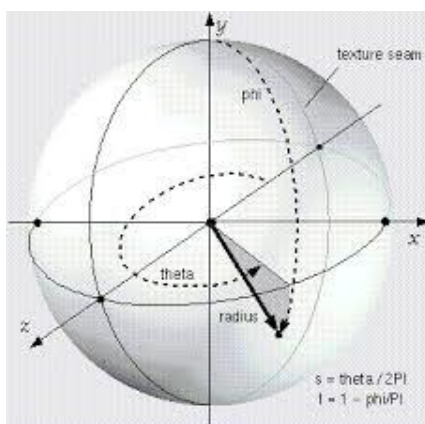
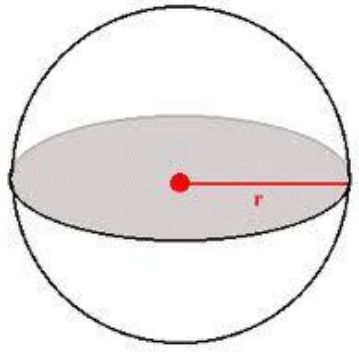
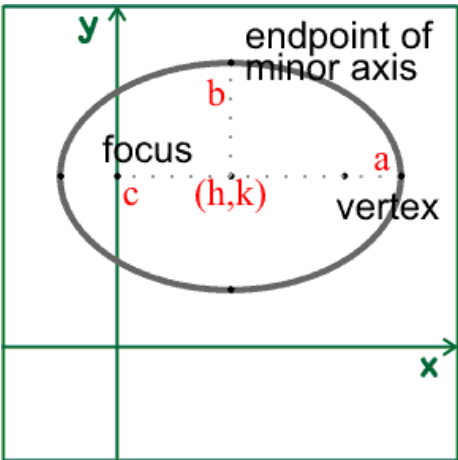
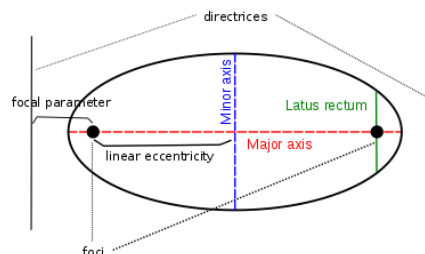
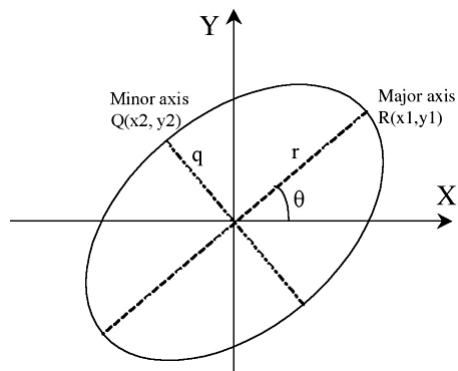
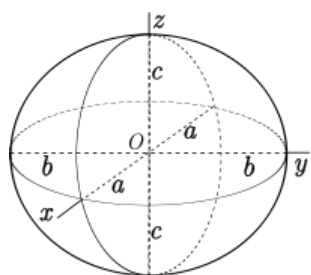


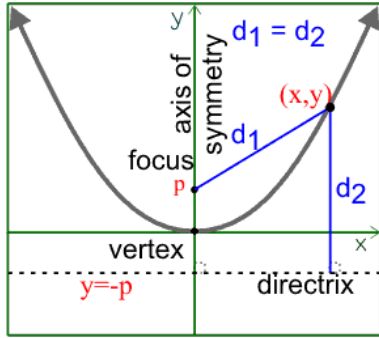
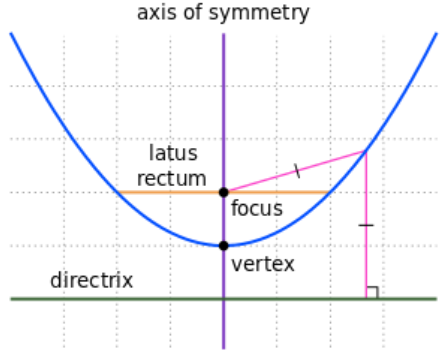
Harold's Calculus 3
Multi-Coordinate System
"Cheat Sheet"
 28 August 2020

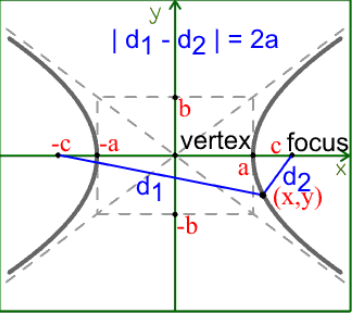
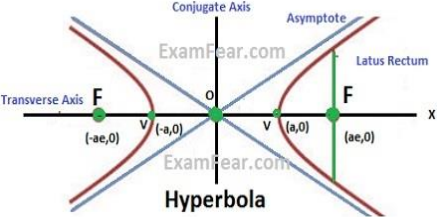
	Rectangular	Polar/Cylindrical	Spherical	Parametric	Vector	Matrix
Point	2D $f(x) = y$ (x, y) (a, b) 3D $f(x, y) = z$ (x, y, z) 4D $f(x, y, z) = w$ (x, y, z, w) •	(r, θ) or $r \angle \theta$ <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <i>Polar</i> → <i>Rect.</i> $x = r \cos \theta$ $y = r \sin \theta$ $z = z$ $\tan \theta = \frac{y}{x}$ </div> <div style="width: 45%;"> <i>Rect.</i> → <i>Polar</i> $r^2 = x^2 + y^2$ $r = \pm \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ </div> </div>	(ρ, θ, ϕ) $x = \rho \sin \phi \cos \theta$ $y = \rho \sin \phi \sin \theta$ $z = \rho \cos \phi$ $\rho^2 = r^2 + z^2$ $\rho^2 = x^2 + y^2 + z^2$ $\tan \theta = \left(\frac{y}{x}\right)$ $\phi = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$ $\phi = \cos^{-1}\left(\frac{z}{\rho}\right)$	<i>Point (a,b) in Rectangular :</i> $x(t) = a$ $y(t) = b$ $\langle a, b \rangle$ <i>t = 3rd variable, usually time,</i> <i>with 1 degree of freedom (df)</i>	$\vec{r} = \langle x_0, y_0, z_0 \rangle$ $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$	$[a] [x] = [b]$
Line	<i>Slope-Intercept Form:</i> $y = mx + b$ <i>Point-Slope Form:</i> $y - y_0 = m(x - x_0)$ <i>General Form:</i> $Ax + By + C = 0$ where A and $B \neq 0$ <i>Calculus Form:</i> $f(x) = f'(a)x + f(0)$ where $m = f'(a)$ 3-D: $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$ <hr style="border: 2px solid blue; width: 100px; margin-left: 0;"/>			$\langle x, y \rangle = \langle x_0, y_0 \rangle + t \langle a, b \rangle$ $\langle x, y \rangle = \langle x_0 + at, y_0 + bt \rangle$ where $\langle a, b \rangle = \langle x_2 - x_1, y_2 - y_1 \rangle$ $x(t) = x_0 + ta$ $y(t) = y_0 + tb$ $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{a}$	$\vec{r} = \vec{r}_0 + t \vec{v}$ $= \langle x_0, y_0, z_0 \rangle$ $+ t \langle a, b, c \rangle$ 	$[a \ b] \begin{bmatrix} x \\ y \end{bmatrix} = [c]$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$

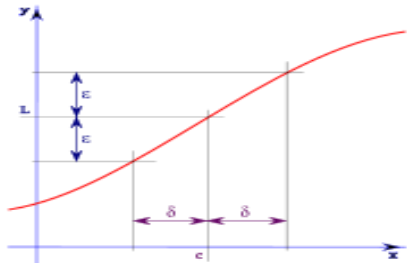
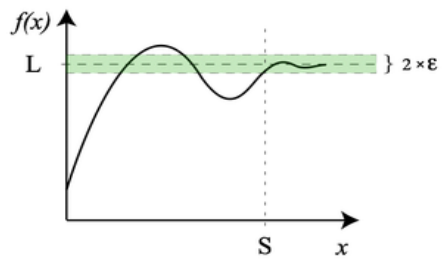
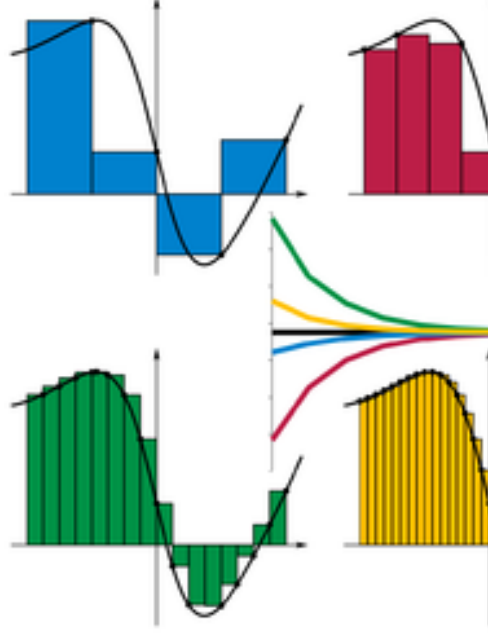
	Rectangular	Polar/Cylindrical	Spherical	Parametric	Vector	Matrix
Plane	$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ $ax + by + cz = d$ where $d = ax_0 + by_0 + cz_0$ $f(x, y) = Ax + By + C$	$(r, \theta, \text{constant})$ $(0 \leq r < \infty)$ $(0 \leq \theta < 2\pi)$ where r and θ take on all values in their domains	$(\rho, \theta, \text{constant})$ $(0 \leq \rho < \infty)$ $(0 \leq \theta < 2\pi)$ where ρ and θ take on all values in their domains	$\mathbf{r} = \mathbf{r}_0 + s\mathbf{v} + t\mathbf{w}$ where: <ul style="list-style-type: none"> s and t range over all real numbers \mathbf{v} and \mathbf{w} are given vectors defining the plane \mathbf{r}_0 is the vector representing the position of an arbitrary (but fixed) point on the plane 	$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$	
Conics	<p><i>General Equation for All Conics:</i></p> $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ <p>where</p> <p>Line: $A = B = C = 0$</p> <p>Circle: $A = C$ and $B = 0$</p> <p>Ellipse: $AC > 0$ or $B^2 - 4AC < 0$</p> <p>Parabola: $AC = 0$ or $B^2 - 4AC = 0$</p> <p>Hyperbola: $AC < 0$ or $B^2 - 4AC > 0$</p> <p>Note: If $A + C = 0$, square hyperbola</p> <p><i>Rotation:</i></p> <p>If $B \neq 0$, then <u>rotate</u> coordinate system:</p> $\cot 2\theta = \frac{A - C}{B}$ $x = x' \cos \theta - y' \sin \theta$ $y = y' \cos \theta + x' \sin \theta$ <p>New = (x', y'), Old = (x, y) rotates through angle θ from x-axis</p> 	<p><i>General Equation for All Conics:</i></p> <p><i>Vertical Axis of Symmetry:</i></p> $r = \frac{p}{1 - e \cos \theta}$ <p><i>Horizontal Axis of Symmetry:</i></p> $r = \frac{p}{1 - e \sin \theta}$ <p>where $p = \begin{cases} a(1 - e^2) \\ 2d \\ a(e^2 - 1) \end{cases}$ for $\begin{cases} 0 \leq e < 1 \\ e = 1 \\ e > 1 \end{cases}$</p> <p>$p$ = semi-latus rectum or the line segment running from the focus to the curve in a direction parallel to the directrix</p> <p><i>Eccentricity:</i></p> <p>Circle $e = 0$</p> <p>Ellipse $0 < e < 1$</p> <p>Parabola $e = 1$</p> <p>Hyperbola $e > 1$</p> <p>Parabola $e = 1.0$ $a = 1.0$</p> <p>Hyperbola $e = 1.4$ $a = 2.5$</p> <p>Ellipse $e = 0.6$ $a = 2.5$</p> <p>Circle $e = 0.0$ $a = 1.0$</p> <p>Focus</p> 	 <p>Circle</p> <p>Ellipse</p> <p>Parabola</p> <p>Hyperbola</p>  <p>Circle</p> <p>Ellipse</p> <p>Parabola</p> <p>Hyperbola</p>	NA		

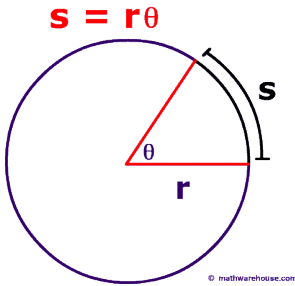
	Rectangular	Polar/Cylindrical	Spherical	Parametric	Vector	Matrix
Circle	$x^2 + y^2 = r^2$ $(x - h)^2 + (y - k)^2 = r^2$ <p>General Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where $A = C$ and $B = 0$</p> <p>Center: (h, k) Focus: (h, k)</p> 	<p>Centered at Origin: $r = a$ (constant) $\theta = \theta$ $[0, 2\pi]$ or $[0, 360^\circ]$</p> <p>Centered at (r_0, ϕ): $r^2 + r_0^2 - 2rr_0 \cos(\theta - \phi) = R^2$</p> <p>Hint: Law of Cosines or</p> $r = r_0 \cos(\theta - \phi) + \sqrt{a^2 - r_0^2 \sin^2(\theta - \phi)}$ 	$\rho = \text{constant}$ $\theta = \theta$ $[0, 2\pi]$ $\phi = \text{constant} = 0$	$x(t) = r \cos(t) + h$ $y(t) = r \sin(t) + k$ $[t_{min}, t_{max}] = [0, 2\pi]$ <p>Center: (h, k) Focus: (h, k)</p>	NA	NA
Sphere	$x^2 + y^2 + z^2 = r^2$ $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$ <p>Focus and center: (h, k, l)</p> <p>General Form: $Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$ where $A = B = C > 0$</p> <p>Cylindrical to Rectangular: $x = r \cos(\theta)$ $y = r \sin(\theta)$ $z = z$</p> <p>Spherical to Rectangular: $x = r \sin \theta \cos \phi$ $y = r \sin \theta \sin \phi$ $z = r \cos \theta$</p>	<p>Rectangular to Cylindrical: $r = \sqrt{x^2 + y^2}$</p> <p>Spherical to Cylindrical: $\rho = r \sin(\theta)$ $\phi = \phi$ $z = r \cos(\theta)$</p>	$\rho = \text{constant}$ $\theta = \theta$ $[0, 2\pi]$ $\phi = \phi$ $[0, 2\pi]$ <p>Rectangular to Spherical: $r = \sqrt{x^2 + y^2 + z^2}$ $\theta = \arccos\left(\frac{z}{r}\right)$ $\phi = \arctan\left(\frac{y}{x}\right)$</p> <p>Cylindrical to Spherical: $r = \sqrt{\rho^2 + z^2}$ $\theta = \arctan\left(\frac{\rho}{z}\right) = \arccos\left(\frac{z}{r}\right)$ $\phi = \phi$</p>		<p>Rectangular: $\mathbf{r} \equiv \begin{bmatrix} x \\ y \\ z \end{bmatrix}$</p> <p>Cylindrical: $\mathbf{r} \equiv \begin{bmatrix} r \cos(\theta) \\ r \sin(\theta) \\ z \end{bmatrix}$</p> <p>Spherical: $\mathbf{r} \equiv \begin{bmatrix} r \sin \theta \cos \phi \\ r \sin \theta \sin \phi \\ r \cos \theta \end{bmatrix}$</p>	

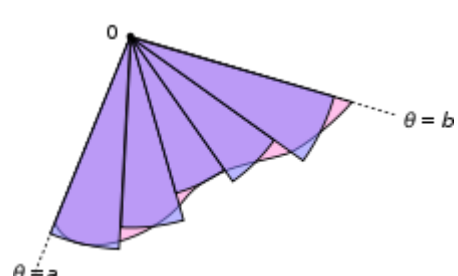
	Rectangular	Polar/Cylindrical	Spherical	Parametric	Vector	Matrix
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ <p>General Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where $B^2 - 4AC < 0$ or $AC > 0$</p> <p>Center: (h, k) Vertices: $(h \pm a, k)$ and $(h, k \pm b)$ Foci: $(h \pm c, k)$</p> <p>Focus length, c, from center: $c = \sqrt{a^2 - b^2}$</p> <p>Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a}$</p> <p>If $B \neq 0$, then <u>rotate</u> coordinate system: $\cot 2\theta = \frac{A-C}{B}$ $x = x' \cos \theta - y' \sin \theta$ $y = y' \cos \theta + x' \sin \theta$</p> <p>New = (x', y'), Old = (x, y) rotates through angle θ from x-axis</p>	<p>Vertical Axis of Symmetry: $r = \frac{a(1-e^2)}{1 \pm e \cos \theta}$</p> <p>Horizontal Axis of Symmetry: $r = \frac{a(1-e^2)}{1 \pm e \sin \theta}$</p> <p>Eccentricity: $0 < e < 1$</p> $r(\theta) = \frac{ab}{\sqrt{(b \cos \theta)^2 + (a \sin \theta)^2}}$ <p>relative to center (h, k)</p> 	 <p>Interesting Note: The <u>sum</u> of the distances from each focus to a point on the curve is constant. $d_1 + d_2 = k$</p>	$x(t) = a \cos(t) + h$ $y(t) = b \sin(t) + k$ $[t_{min}, t_{max}] = [0, 2\pi]$ <p>Center: (h, k)</p> <p>Rotated Ellipse: $x(t) = a \cos t \cos \theta - b \sin t \sin \theta + h$ $y(t) = a \cos t \sin \theta + b \sin t \cos \theta + k$</p> <p>$\theta$ = the angle between the x-axis and the major axis of the ellipse</p> 		
Ellipsoid	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} + \frac{(z-l)^2}{c^2} = 1$	$\frac{r^2 \cos^2 \theta}{a^2} + \frac{r^2 \sin^2 \theta}{b^2} + \frac{z^2}{c^2} = 1$	$\frac{r^2 \cos^2 \theta \sin^2 \phi}{a^2} + \frac{r^2 \sin^2 \theta \sin^2 \phi}{b^2} + \frac{r^2 \cos^2 \phi}{c^2} = 1$	$x(t, u) = a \cos(t) \cos(u) + h$ $y(t, u) = b \cos(t) \sin(u) + k$ $z(t, u) = c \sin(t) + l$ $[t_{min}, t_{max}] = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $[u_{min}, u_{max}] = [-\pi, \pi]$ <p>Center: (h, k, l)</p>		$(\mathbf{x} - \mathbf{v})^T A^{-1} (\mathbf{x} - \mathbf{v}) = 1$ <p>Centered at vector \mathbf{v}</p>

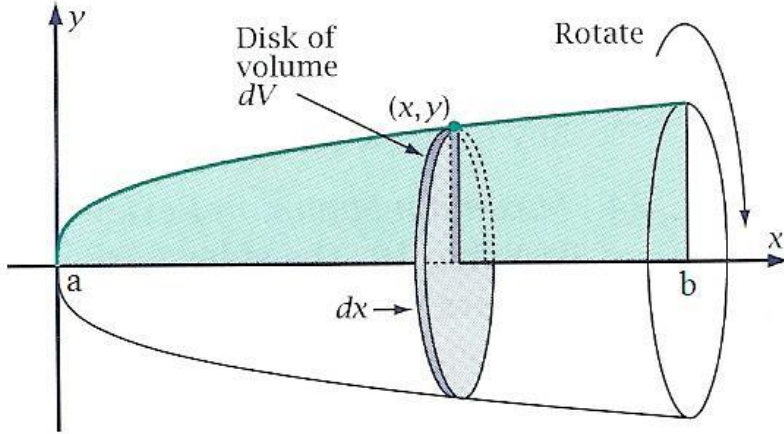
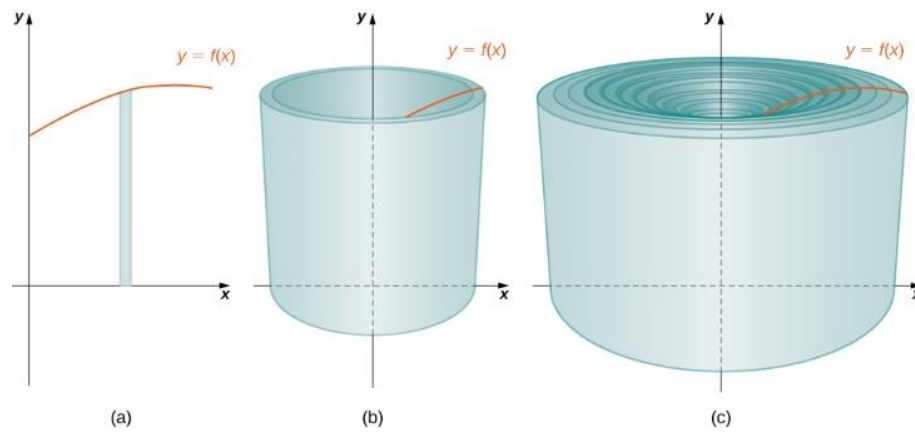
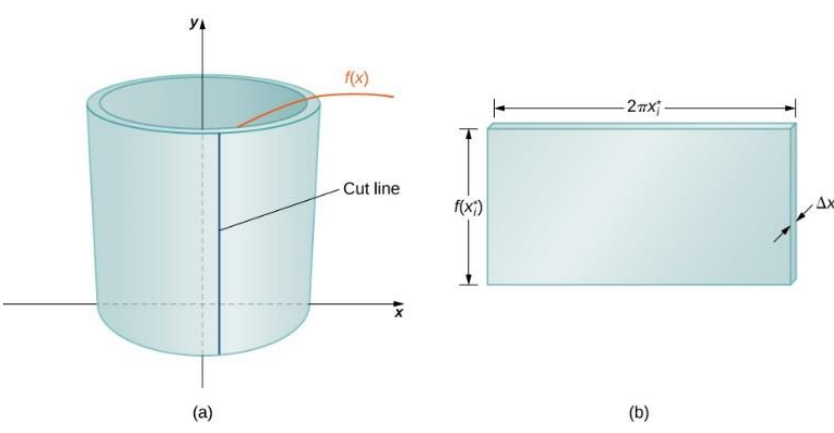
	Rectangular	Polar/Cylindrical	Spherical	Parametric	Vector	Matrix
Parabola	<p><i>Vertical Axis of Symmetry:</i> $x^2 = 4py$ $(x - h)^2 = 4p(y - k)$ Vertex: (h, k) Focus: $(h, k + p)$ Directrix: $y = k - p$</p> <p><i>Horizontal Axis of Symmetry:</i> $y^2 = 4px$ $(y - k)^2 = 4p(x - h)$ Vertex: (h, k) Focus: $(h + p, k)$ Directrix: $x = h - p$</p> <p><i>General Form:</i> $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where $B^2 - 4AC = 0$ or $AC = 0$</p> <p>If $B \neq 0$, then <i>rotate</i> coordinate system: $\cot 2\theta = \frac{A - C}{B}$ $x = x' \cos \theta - y' \sin \theta$ $y = y' \cos \theta + x' \sin \theta$</p> <p><i>New = (x', y'), Old = (x, y)</i> <i>rotates through angle θ from x-axis</i></p>	<p><i>Vertical Axis of Symmetry:</i> $r = \frac{ed}{1 \pm e \cos \theta}$</p> <p><i>Horizontal Axis of Symmetry:</i> $r = \frac{ed}{1 \pm e \sin \theta}$</p> <p><i>Eccentricity: $e = 1$</i> <i>and $d = 2p$</i></p> 		<p><i>Vertical Axis of Symmetry:</i> $x(t) = 2pt + h$ $y(t) = pt^2 + k$ (<i>opens upwards</i>) $y(t) = -pt^2 - k$ (<i>opens downwards</i>) $[t_{min}, t_{max}] = [-c, c]$ Vertex: (h, k)</p> <p><i>Horizontal Axis of Symmetry:</i> $y(t) = 2pt + k$ $x(t) = pt^2 + h$ (<i>opens to the right</i>) $x(t) = -pt^2 - h$ (<i>opens to the left</i>) $[t_{min}, t_{max}] = [-c, c]$ Vertex: (h, k)</p> <p><i>Projectile Motion:</i> $x(t) = x_0 + v_x t + \left(\frac{1}{2}\right) a_x t^2$ $y(t) = y_0 + v_y t - 16t^2$ <i>feet</i> $y(t) = y_0 + v_y t - 4.9t^2$ <i>meters</i> $v_x = v \cos \theta$ $v_y = v \sin \theta$</p> <p><i>General Form:</i> $x = At^2 + Bt + C$ $y = Lt^2 + Mt + N$ <i>where A and L have the same sign</i></p>		
Nose Cone	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = \frac{(z - l)^2}{c^2}$					

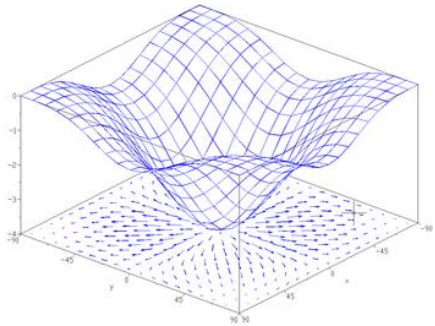
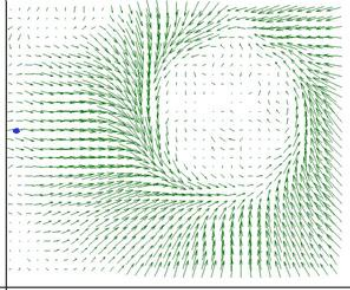
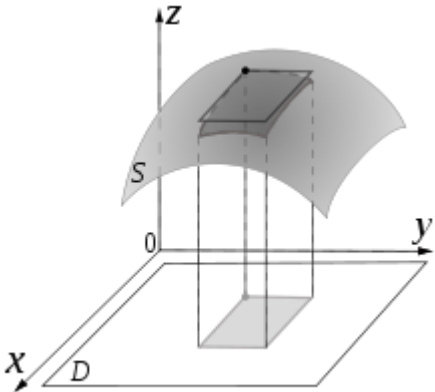
	Rectangular	Polar/Cylindrical	Spherical	Parametric	Vector	Matrix
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ <p>General Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where $B^2 - 4AC > 0$ or $AC < 0$</p> <p>If $A + C = 0$, square hyperbola</p> <p>Center: (h, k) Vertices: $(h \pm a, k)$ Foci: $(h \pm c, k)$</p> <p>Focus length, c, from center: $c = \sqrt{a^2 + b^2}$</p> <p>Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a} = \sec \theta$</p> $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} - \frac{(z-l)^2}{c^2} = 1$ $-\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} + \frac{(z-l)^2}{c^2} = 1$ <p>If $B \neq 0$, then <u>rotate</u> coordinate system: $\cot 2\theta = \frac{A - C}{B}$ $x = x' \cos \theta - y' \sin \theta$ $y = y' \cos \theta + x' \sin \theta$</p> <p>New = (x', y'), Old = (x, y) rotates through angle θ from x-axis</p>	<p>Vertical Axis of Symmetry: $r = \frac{a(e^2 - 1)}{1 \pm e \cos \theta}$</p> <p>Horizontal Axis of Symmetry: $r = \frac{a(e^2 - 1)}{1 \pm e \sin \theta}$</p> <p>Eccentricity: $e > 1$ where $e = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a} = \sec \theta > 1$ relative to center (h, k)</p> $-\cos^{-1}\left(-\frac{1}{e}\right) < \theta < \cos^{-1}\left(-\frac{1}{e}\right)$  <p>$p =$ semi-latus rectum or the line segment running from the focus to the curve in the directions $\theta = \pm \frac{\pi}{2}$</p> <p>Interesting Note: The <u>difference</u> between the distances from each focus to a point on the curve is constant. $d_1 - d_2 = k$</p>		<p>Left-Right Opening Hyperbola: $x(t) = a \sec(t) + h$ $y(t) = b \tan(t) + k$ $[t_{min}, t_{max}] = [-c, c]$ Vertex: (h, k)</p> <p>Alternate Form: $x(t) = \pm a \cosh(t) + h$ $y(t) = b \sinh(t) + k$</p> <p>Up-Down Opening Hyperbola: $x(t) = a \tan(t) + h$ $y(t) = b \sec(t) + k$ $[t_{min}, t_{max}] = [-c, c]$ Vertex: (h, k)</p> <p>Alternate Form: $x(t) = a \sinh(t) + h$ $y(t) = \pm b \cosh(t) + k$</p> <p>General Form: $x(t) = At^2 + Bt + C$ $y(t) = Dt^2 + Et + F$ where A and D have different signs</p>		

	Rectangular	Polar/Cylindrical	Spherical	Parametric	Vector	Matrix
Limit	$\lim_{x \rightarrow c} f(x) = L$					
1st Derivative	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$ $f'(x) = \frac{dy}{dx} = y' = D_x$	$\frac{dy}{dx} = \frac{dy}{d\theta} = \frac{dr}{d\theta} \sin \theta + r \cos \theta$ $\frac{dy}{dx} = \frac{dr}{d\theta} \cos \theta - r \sin \theta$ <p>Hint: Use Product Rule for $y = r \sin \theta$ $x = r \cos \theta$</p>		$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}, \quad \text{provided } \frac{dx}{dt} \neq 0$	$\frac{d}{dt}(\vec{r}) = \vec{r}'$ <p>Unit tangent vector $\vec{T}(t) = \frac{\vec{r}'(t)}{\ \vec{r}'(t)\ }$ where $\vec{r}'(t) \neq \vec{0}$</p>	
2nd Derivative	$f''(x) = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d^2y}{dx^2} = y'' = D_{xx}$	$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{d\theta} \left(\frac{dy}{dx} \right) \frac{d\theta}{dx}$		$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dt} \left(\frac{dy}{dx} \right) \frac{dt}{dx} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \frac{1}{\frac{dx}{dt}}$	<p>Unit normal vector $\vec{N}(t) = \frac{\vec{T}'(t)}{\ \vec{T}'(t)\ }$ where $\vec{T}'(t) \neq \vec{0}$</p>	
Integral	<p>Fundamental Theorem of Calculus:</p> $F(x) = \int_a^b f(x) dx = F(b) - F(a)$			<p>Riemann Sum:</p> $S = \sum_{i=1}^n f(y_i)(x_i - x_{i-1})$ <p>Left Sum:</p> $S = \left(\frac{1}{n}\right) \left[f(a) + f\left(a + \frac{1}{n}\right) + f\left(a + \frac{2}{n}\right) + \dots + f\left(b - \frac{1}{n}\right) \right]$ <p>Middle Sum:</p> $S = \left(\frac{1}{n}\right) \left[f\left(a + \frac{1}{2n}\right) + f\left(a + \frac{3}{2n}\right) + \dots + f\left(b - \frac{1}{2n}\right) \right]$ <p>Right Sum:</p> $S = \left(\frac{1}{n}\right) \left[f\left(a + \frac{1}{n}\right) + f\left(a + \frac{2}{n}\right) + \dots + f(b) \right]$	$\int_a^b \vec{r}(t) dt = \left\langle \int_a^b f(t) dt, \int_a^b g(t) dt, \int_a^b h(t) dt \right\rangle$	
Double Integral	$\int_a^b \int_{c(y)}^{d(y)} f(x, y) dx dy$	<p>Same as rectangular, but $f(x, y) \rightarrow f(\rho \cos \phi, \rho \sin \phi)$</p>				

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Triple Integral	$\int_a^b \int_{c(z)}^{d(z)} \int_{e(y,z)}^{g(y,z)} f(x,y,z) dx dy dz$	Same as rectangular, but $f(x,y,z) \rightarrow f(\rho \cos \phi, \rho \sin \phi, z)$	Same as rectangular, but $f(x,y,z) \rightarrow f(\rho \cos \theta \sin \phi, \rho \sin \theta \sin \phi, \rho \cos \phi)$	NA	NA	NA
Inverse Functions	If $f(x) = y$, then $f^{-1}(y) = x$ Inverse Function Theorem: $f^{-1}(f'(a)) = \frac{1}{f'(a)}$	$y = \sin \theta$ $\theta = \sin^{-1} y$ $y = \cos \theta$ $\theta = \cos^{-1} y$ $y = \tan \theta$ $\theta = \tan^{-1} y$ $y = \csc \theta$ $\theta = \csc^{-1} y$ $y = \sec \theta$ $\theta = \sec^{-1} y$ $y = \cot \theta$ $\theta = \cot^{-1} y$	$\theta = \arcsin y$ $\theta = \arccos y$ $\theta = \arctan y$ $\theta = \operatorname{arccsc} y$ $\theta = \operatorname{arcsec} y$ $\theta = \operatorname{arccot} y$	NA	NA	NA
Arc Length	$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$ Proof: $\Delta s = \sqrt{(x-x_0)^2 + (y-y_0)^2}$ $\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ $ds = \sqrt{dx^2 + dy^2}$ $ds = \sqrt{dx^2 + dy^2} \left(\frac{dx^2}{dx^2}\right)$ $ds = \sqrt{dx^2 + \left(\frac{dy}{dx}\right)^2 dx^2}$ $ds = \sqrt{dx^2 \left(1 + \left(\frac{dy}{dx}\right)^2\right)}$ $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ $L = \int ds$	Polar: $L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$ Where $r = f(\theta)$ Circle: $L = s = r\theta$ Proof: $L = (\text{fraction of circumference}) \cdot \pi \cdot (\text{diameter})$ $L = \left(\frac{\theta}{2\pi}\right) \pi (2r) = r\theta$	$C = \pi d = 2\pi r$ 	Rectangular 2D: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ Rectangular 3D: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$ Cylindrical: $L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\theta}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$ Spherical: $L = \int_{t_1}^{t_2} \sqrt{\left(\frac{d\rho}{dt}\right)^2 + \rho^2 \sin^2 \phi \left(\frac{d\theta}{dt}\right)^2 + \rho^2 \left(\frac{d\phi}{dt}\right)^2} dt$	$L = \int_a^b \ \vec{r}'(t)\ dt$ $s(t) = \int_0^t \ \vec{r}'(u)\ du$	NA
Curvature	$\kappa = \frac{ y'' }{(1+y'^2)^{3/2}}$	$\kappa(\theta) = \frac{ r^2 + 2r'^2 - rr'' }{(r^2 + r'^2)^{3/2}}$ for $r(\theta)$	NA	$\kappa = \frac{\sqrt{(z''y' - y''z')^2 + (x''z' - z''x')^2 + (y''x - x''y')^2}}{(x'^2 + y'^2 + z'^2)^{3/2}}$ where $f(t) = (x(t), y(t), z(t))$	$\kappa = \left \frac{d\vec{T}}{ds} \right $ $\kappa = \frac{\ \vec{T}'(t)\ }{\ \vec{r}'(t)\ }$ $\kappa = \frac{\ \vec{r}'(t) \times \vec{r}''(t)\ }{\ \vec{r}'(t)\ ^3}$	(See Wikipedia : Curvature)
Perimeter	Square: $P = 4s$ Rectangle: $P = 2l + 2w$ Triangle: $P = a + b + c$ Circle: $C = \pi d = 2\pi r$ Ellipse: $C \approx \pi(a + b)$	Ellipse: $C \approx 2\pi \sqrt{\frac{a^2+b^2}{2}}$ $C \approx \pi [3(a+b) - \sqrt{(3a+b)(a+3b)}]$ $C \approx \pi(a+b) \left(1 + \frac{3h}{10 + \sqrt{4-3h}}\right)$	Ellipse: $C = 4a \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} d\theta$ $h = \frac{(a-b)^2}{(a+b)^2}$ & $k^2 = \left(1 - \frac{b^2}{a^2}\right)$	NA	NA	NA

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Area	<p>Square: $A = s^2$ Rectangle: $A = lw$ Rhombus: $A = \frac{1}{2} ab$ Parallelogram: $A = Bh$ Trapezoid: $A = \frac{(B_1 + B_2)}{2} h$ Kite: $A = \frac{d_1 d_2}{2}$ Triangle: $A = \frac{1}{2} Bh = \frac{1}{2} ab \sin(C)$ Heron's Formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ Equilateral Triangle: $A = \frac{1}{4}\sqrt{3}s^2$ Frustum: $A = \frac{1}{3}\left(\frac{B_1+B_2}{2}\right)h$ Circle: $A = \pi r^2$ Circular Sector: $A = \frac{1}{2} r^2 \theta$ Ellipse: $A = \pi ab$</p>	$A = \int_{\alpha}^{\beta} \frac{1}{2} [f(\theta)]^2 d\theta$ <p>where $r = f(\theta)$</p> <p>Proof: Area of a sector: $A = \int s dr = \int r \Delta\theta dr = \frac{1}{2} r^2 \Delta\theta$ where arc length $s = r \Delta\theta$</p> 	NA	$A = \int_{\alpha}^{\beta} g(t) f'(t) dt$ <p>where $f(t) = x$ and $g(t) = y$ or $x(t) = f(t)$ and $y(t) = g(t)$</p> <p>Simplified: $A = \int_{\alpha}^{\beta} y(t) \frac{dx(t)}{dt} dt$</p> <p>Proof: $\int_a^b f(x) dx$ $y = f(x) = g(t)$ $dx = \frac{df(t)}{dt} dt = f'(t) dt$</p>	$A = \iint_D \left \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right du dv$	NA
Lateral Surface Area	<p>Cylinder: $SA = 2\pi rh$ Cone: $SA = \pi rl$ $SA = 2\pi \int_a^b f(x) \sqrt{1 + [f'(x)]^2} dx$</p>	<p>For rotation about the x-axis: $S = \int 2\pi y ds$</p> <p>For rotation about the y-axis: $S = \int 2\pi x ds$</p> $ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$ $r = f(\theta), \quad \alpha \leq \theta \leq \beta$	Sphere: $SA = 4\pi r^2$	<p>For rotation about the x-axis: $S = \int 2\pi y ds$</p> <p>For rotation about the y-axis: $S = \int 2\pi x ds$</p> $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ <p>if $x = f(t), y = g(t), \alpha \leq t \leq \beta$</p>	NA	NA
Total Surface Area	<p>Cube: $SA = 6s^2$ Rectangular Box: $SA = 2lw + 2wh + 2hl$ Regular Tetrahedron: $SA = 2bh$ Cylinder: $SA = 2\pi r(r+h)$ Cone: $SA = \pi r^2 + \pi rl = \pi r(r+l)$ Sphere: $SA = 4\pi r^2$</p>	<p>Ellipsoid: $SA \approx$ $4\pi \left(\frac{a^p b^p + a^p c^p + b^p c^p}{3} \right)^{1/p}$ <p>Where $p \approx 1.6075$, Relative Error $\leq 1.061\%$ (Knud Thomsen's Formula)</p> </p>	Ellipsoid: $S =$	$\int_0^{2\pi} \int_0^{\pi} \sin[\theta] \sqrt{b^2 c^2 \sin^2[\theta] \cos^2[\phi] + a^2 c^2 \sin^2[\theta] \sin^2[\phi] + a^2 b^2 \cos^2[\theta]} d\theta d\phi =$ $2\pi \left(c^2 + \frac{b c^2}{\sqrt{a^2 - c^2}} \text{EllipticF}[\theta, m] + b \sqrt{a^2 - c^2} \text{EllipticE}[\theta, m] \right)$ <p>where $m = \frac{a^2 (b^2 - c^2)}{b^2 (a^2 - c^2)}; \theta = \text{ArcSin}\left[\sqrt{1 - \frac{c^2}{a^2}}\right]; a \geq b \geq c$</p>		
Surface of Revolution	<p>For revolution about the x-axis: $A = 2\pi \int_a^b f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$</p> <p>For revolution about the y-axis: $A = 2\pi \int_a^b x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$</p>	<p>For revolution about the x-axis: $A = 2\pi \int_{\alpha}^{\beta} r \cos \theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$</p> <p>For revolution about the y-axis: $A = 2\pi \int_{\alpha}^{\beta} r \sin \theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$</p>	Sphere: $S = 4\pi r^2$	<p>For revolution about the x-axis: $A_x = 2\pi \int_a^b y(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$</p> <p>For revolution about the y-axis: $A_y = 2\pi \int_a^b x(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$</p>	NA	NA

	Rectangular	Polar/Cylindrical	Spherical	Parametric	Vector	Matrix
Volume	Cube: $V = s^3$ Rectangular Prism: $V = lwh$ Cylinder: $V = \pi r^2 h$ Triangular Prism: $V = Bh$ Tetrahedron: $V = \frac{1}{3} Bh$ Pyramid: $V = \frac{1}{3} Bh$ Cone: $V = \frac{1}{3} Bh = \frac{1}{3} \pi r^2 h$ Sphere: $V = \frac{4}{3} \pi r^3$ Ellipsoid: $V = \frac{4}{3} \pi abc$ $\iiint f(x, y, z) dx dy dz$	$\iiint f(r \cos \theta, r \sin \theta, z) r dz dr d\theta$	$\iiint f \left(\begin{matrix} \rho \sin \varphi \cos \theta \\ \rho \sin \varphi \sin \theta \\ \rho \cos \varphi \end{matrix} \right) \dots \rho^2 \sin \varphi d\rho d\varphi d\theta$			Ellipsoid: $V = \frac{4}{3} \pi \sqrt{\det(A^{-1})}$
Volume of Revolution	Disk Method $V = \int_a^b (\text{area of circle}) d(\text{thickness})$ Rotation about the x-axis: $V = \int_a^b \pi [f(x)]^2 dx$ Rotation about the y-axis: $V = \int_c^d \pi x^2 dy$					
	Washer Method Rotation about the x-axis: $V = \int_a^b \pi \{ [f(x)]^2 - [g(x)]^2 \} dx$	$V = V_{\text{Outer Disk}} - V_{\text{Inner Disk}}$				
	Shell Method $V = \int_a^b (\text{circumference}) (\text{height}) dx$ Rotation about the y-axis: $V = \int_a^b 2\pi x f(x) dx$ Rotation about the x-axis: $V = \int_c^d 2\pi y g(y) dy$					

	Rectangular	Polar/Cylindrical	Spherical	Parametric	Vector	Matrix
Moments of Inertia	$I = \sum_{i=1}^N m_i r_i^2 = \int_0^a m r^2 dr$	NA	NA		$I = \iiint_V \rho(\mathbf{r}) d(\mathbf{r})^2 dV(\mathbf{r})$	(see Wikipedia)
Center of Mass	$\mathbf{R} = \frac{1}{M} \sum_{i=1}^N m_i \mathbf{r}_i$ where $M = \sum_{i=1}^N m_i$ 1D for Discrete: $x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$	2D for Discrete: $M_y = \sum_{i=1}^N m_i x_i$ $M_x = \sum_{i=1}^N m_i y_i$ $\bar{x} = \frac{M_y}{M}, \bar{y} = \frac{M_x}{M}$	3D for Discrete: $x_{cm} = \bar{x} = \frac{1}{M} \sum_{i=1}^N m_i x_i$ $y_{cm} = \bar{y} = \frac{1}{M} \sum_{i=1}^N m_i y_i$ $z_{cm} = \bar{z} = \frac{1}{M} \sum_{i=1}^N m_i z_i$	3D for Continuous: $\bar{x} = \frac{1}{M} \int_0^M x dm$ $\bar{y} = \frac{1}{M} \int_0^M y dm$ $\bar{z} = \frac{1}{M} \int_0^M z dm$ where $M = \int_0^M dm$ and $dm = \rho dz dy dx$	$\mathbf{R} = \frac{1}{M} \int \mathbf{r} dm$ $\mathbf{R} = \frac{1}{M} \iiint_V \rho(\mathbf{r}) \mathbf{r} dV$ Where \mathbf{r} is distance from the axis of rotation, not origin.	
Gradient	$\nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$	$\nabla f(\rho, \phi, z) = \frac{\partial f}{\partial \rho} \mathbf{e}_\rho + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \mathbf{e}_\phi + \frac{\partial f}{\partial z} \mathbf{e}_z$	$\nabla f(r, \theta, \phi) = \frac{\partial f}{\partial r} \mathbf{e}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \mathbf{e}_\theta + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \mathbf{e}_\phi$		$(\nabla f(\mathbf{x})) \cdot \mathbf{v} = D_{\mathbf{v}} f(\mathbf{x})$ $\nabla f = \frac{\partial f_i}{\partial x_j} \mathbf{e}_i \mathbf{e}_j$ where $f = (f_1, f_2, f_3)$	
Line Integral	$\int_C f ds = \int_a^b f(\mathbf{r}(t)) \mathbf{r}'(t) dt$	NA	NA		$\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ $= \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt$	
Surface Integral	$\int_S f dS = \iint_T f(\mathbf{x}(s, t)) \left \frac{\partial \mathbf{x}}{\partial s} \times \frac{\partial \mathbf{x}}{\partial t} \right ds dt$ Where $\mathbf{x}(s, t) = (x(s, t), y(s, t), z(s, t))$ and $\left(\frac{\partial \mathbf{x}}{\partial s} \times \frac{\partial \mathbf{x}}{\partial t} \right) = \left(\frac{\partial(y, z)}{\partial(s, t)}, \frac{\partial(z, x)}{\partial(s, t)}, \frac{\partial(x, y)}{\partial(s, t)} \right)$	NA	NA		$\int_S \mathbf{v} \cdot d\mathbf{S} = \int_S (\mathbf{v} \cdot \mathbf{n}) dS = \iint_T \mathbf{v}(\mathbf{x}(s, t)) \cdot \left(\frac{\partial \mathbf{x}}{\partial s} \times \frac{\partial \mathbf{x}}{\partial t} \right) ds dt$	